WHAT ARE TWIN PRIMES AND HOW ARE THEY FOUND?

If one looks at the first few primes one has the sequence-

P={2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53, 59, 61,67,71,73,79,83,89,97,101,103,107,109,…}

The distance between several of these is just two. When this happens, as it does for-

T={[3,5],[5,7],[11,13],[17,19],29,31],[41,43],[59,61],[71,73,],[101,103],[107,109]}

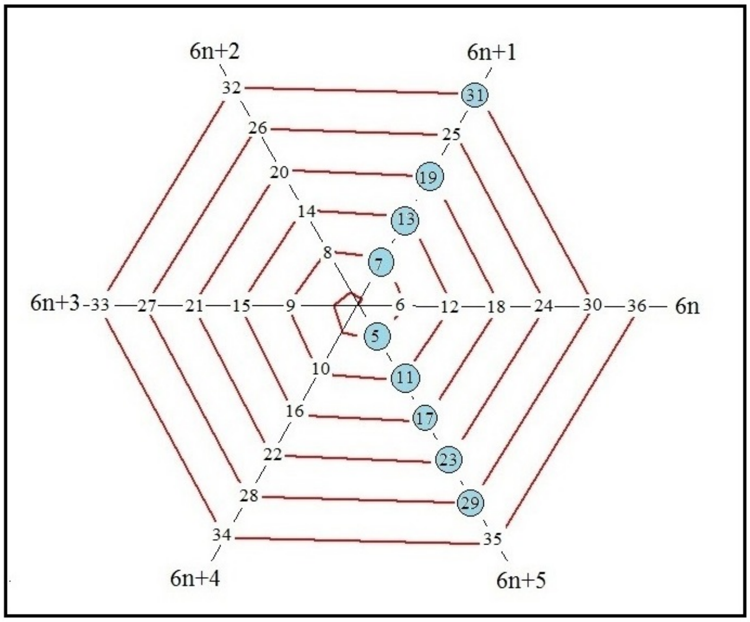
, one has what are called <u>Twin Primes</u>. In this we do not include the even prime of 2. Adding up the two numbers in a twin primes, we get-

A={8,12,24,36,60,80,120,144,204,216}

All of these are even numbers divisible by two.

It is our purpose here to look at all twin primes and discover any heretofore unknown characteristics of these numbers using our earlier discovered fact that all primes greater than 3 must lie along the radial lines 6n+1 or 6n-1 (equivalent 6n+5) at the crossing points with a hexagonal integer spiral as shown-

PRIMES AND TWIN PRIMES ALONG THE HEXAGONAL INTEGER SPIRAL



-PRIMES ALONG DIAGONALS 6n+1, AND 6n-1 TWIN PRIMES-[7,5], [13,11], [19,17], [31,29]

Unlike in an Ulam Spiral where primes are scattered almost randomly throughout the r-θ plane , the present representation clearly puts all primes and twin primes along just two diagonals. The twin primes are essentially the two primes intersected by single straight vertical line. We note that twin primes have the form-

$[p_n, p_{n-1}] = [6m+1, 6m-1]$

So adding and subtracting things yields-

$$p_n + p_{n-1} = 12m \quad and \quad p_n - p_{n-1} = 2$$

The second of these results is nothing new stating simply the result shown by A above. However, the first result is new and is an important find. It says in effect that all twin primes with n=4 or greater satisfy-

$$(p_n + p_{n-1}) \bmod(12) = 0 \quad or \quad the \quad equivalent \quad (p_n - 1) \bmod(6) = 0$$

Let us construct a table verifying these points-

n	p_n	p_{n-1}	$[p_n+p_{n-1}]\bmod(12)$	$[p_n-1] \bmod(6)=0$

4	7	5	0	0
6	13	11	0	0
8	19	17	0	0
11	31	29	0	0
14	43	41	0	0
18	61	59	0	0
21	73	71	0	0
27	103	101	0	0
29	109	107	0	0

Note that not all ns(and hence ms) yield twin-primes but the values of n given do produce them.
With the $[p_n\text{-}1]$ mod(6) operation , it now becomes easy to find twin primes. For example, if we ask what are the twin primes lying between p_{1000}=7919 and p_{1020}=8111, we can get the answer through the simple computer command-

for n from 1000 to 1020 do {n,ithprime(n), ithprime(n-1),ithprime(n)-ithprime(n-1),(ithprime(n)-1)mod(6)}od;

It produces the three twin-prime pairs-

[7951,7949] , [8011,8009] , [8089,8087]

In these calculations we have discarded all terms for which the difference in the neighboring primes is not 2 and for which the mod operation does not yield zero.

In looking further at the values of p_n and p_{n-1} in he above table one notices that all double primes can be expressed in terms of the base pair [7,5]. We have that [13,11]=[7+6,5+6] and also[19,17]=[7+12,5+12] etc. This suggests that all twin primes can be expressed as-

$[p_n , p_{n-1}]=[7+6m , 5+6m]$ for those integers m where both ps are prime

Thus a few twin pairs are [19,17], [61,59], and [349,347]. A more extensive table using the program-

for m from 0 to 50 do {m,7+6*m,5+6*m,isprime(7+6*m),isprime(5+6*m)}od;

follows-

TWIN PRIME DETERMINATION USING $P_n = 7+6m$ AND $P_{n-1} = 5+6m$

{0, 5, 7, *true*}
{1, 11, 13, *true*}
{2, 17, 19, *true*}
{4, 29, 31, *true*}
{6, 41, 43, *true*}
{9, 59, 61, *true*}
{11, 71, 73, *true*}
{16, 101, 103, *true*}
{17, 107, 109, *true*}
{22, 137, 139, *true*}
{24, 149, 151, *true*}
{29, *true*, 179, 181}
{31, *true*, 191, 193}
{32, *true*, 197, 199}

From this list we have removed those rows where either p_n or p_{n-1} are not primes. This is an extremely simple program for finding twin primes in the vicinity of any positive integer. The program works more efficiently than the earlier calculations in that one is no longer dependent on knowing what the nthprime is. Looking in the range $m=10^9\pm30$ we quickly find just two twin-primes-

[5999999851, 5999999849] and [6000000049, 6000000047]

One can go to even higher numbers. Looking in the trillion number range of $10^{12}+50$ to $10^{12}+100$, we find the two twin primes-

[6000000000523, 6000000000521] and [6000000000583, 6000000000581]

The density of twin primes (like that of primes) are observed to decrease rapidly with increasing m but the calculation remains simple since the identification of prime numbers is an easy task for most electronic computers while identifying the ithprime above 10^{15} is not.

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