Chapter 6

Classical Control Design
 WHY DESIGN?

Feedback Control System Has to Satisfy Several Conditions in Order to Perform in Some Prescribed Optimal Sense.

Most Systems Need Some Adjustment in Order to Satisfy all Conditions (or Comprise Among Conflicting and Demanding Specifications).

May Need to Change the Structure of the System and Redesign to Meet Performance Requirements.

Suitable Control System:

Stable

Acceptable response to input commands

Less sensitive to system parameter changes

Minimal steady-state error for input command

Able to eliminate the effect of undesirable disturbances

Design via gain adjustment.

Design via cascaded or feedback filters
a. Sample root locus, showing possible design point via gain adjustment (A) and desired design point that cannot be met via simple gain adjustment (B);

b. responses from poles at A and B
Compensation techniques:

a. cascade

b. feedback

Other Compensation: Output or Load Input

Design via gain adjustment. Design via cascaded or feedback filters.
### Types of cascade compensators (continued on next slide)

<table>
<thead>
<tr>
<th>Function</th>
<th>Compensator</th>
<th>Transfer function</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| Improve steady-state error | PI          | $K\frac{s + z_c}{s}$ | 1. Increases system type.  
2. Error becomes zero.  
3. Zero at $-z_c$ is small and negative.  
4. Active circuits are required to implement. |
| Improve steady-state error | Lag         | $K\frac{s + z_c}{s + p_c}$ | 1. Error is improved but not driven to zero.  
2. Pole at $-p_c$ is small and negative.  
3. Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$.  
4. Active circuits are not required to implement. |
| Improve transient response | PD          | $K(s + z_c)$ | 1. Zero at $-z_c$ is selected to put design point on root locus.  
2. Active circuits are required to implement.  
3. Can cause noise and saturation; implement with rate feedback or with a pole (lead). |
| Improve transient response | Lead       | $K\frac{s + z_c}{s + p_c}$ | 1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus.  
2. Pole at $-p_c$ is more negative than zero at $-z_c$.  
3. Active circuits are not required to implement. |
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</table>
| Improve steady-state error and  | PID         | $K\frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{s}$                           | 1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error.  
2. Lead zero at $-z_{\text{lead}}$ improves transient response.  
3. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin.  
4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus.  
5. Active circuits required to implement.  
6. Can cause noise and saturation; implement with rate feedback or with an additional pole. |
| transient response              | Lag-lead    | $K\frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{(s + p_{\text{lag}})(s + p_{\text{lead}})}$ | 1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error.  
2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response.  
3. Lag pole at $-p_{\text{lag}}$ is small and negative.  
4. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$.  
5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus.  
6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$.  
7. Active circuits are not required to implement. |
Finding open-loop parameter on the root locus to meet design specifications

Design Specifications
\( r = \alpha_n = 2 \text{rad} / \text{sec} \)
\( \zeta = 0.45 \)

\[
1 + K \frac{(s + 3)}{s(s + 1)(s + 2)(s + \alpha)} = 0
\]

Angle Criterion:
\[
\angle(s_o + 3) - \angle(s_o) - \angle(s_o + 1) - \angle(s_o + 2) - \angle(s_o + \alpha) = 180^\circ
\]

\( s_o = -0.90 + j1.79 \implies (s_o + 3) = 2.1 + j1.79 \implies \theta_2 = 40.4^\circ, \text{ etc.} \)

\[40.4^\circ - (116.7^\circ + 86.8^\circ + 58.4^\circ + \theta_\alpha) = 180^\circ\]

\[\theta_\alpha = -401.5^\circ \implies -41.5^\circ \implies \alpha = \text{not achievable due to resulting angle not realistic, i.e., in this case the reduced angle must be between 0 and 180 degrees so that directed line segment is above real axis.}\]

Hence, no (+K) root locus exists for design point selected.
Finding open-loop parameter on the root locus to meet design specifications → Change to zero compensator

Design Specifications
\[ r = \alpha_n = 2 \text{ rad / sec} \]
\[ \zeta = 0.45 \]

\[ 1 + K \frac{(s + 3)(s + \alpha)}{s(s + 1)(s + 2)} = 0 \]

Angle Criterion:
\[ \angle(s_o + 3) + \angle(s_o + \alpha) - \angle(s_o) - \angle(s_o + 1) - \angle(s_o + 2) = 180^\circ \]
\[ s_o = -0.90 + j1.79 \]
\[ (s_o + 3) = 2.1 + j1.79 \Rightarrow \theta_2 = 40.4^\circ, \text{ etc.} \]
\[ 40.4^\circ + \theta_\alpha - (116.7^\circ + 86.8^\circ + 58.4^\circ) = 180^\circ \]
\[ \theta_\alpha = 401.5^\circ \Rightarrow 41.5^\circ \Rightarrow \alpha = [1.79/(\tan(41.5^\circ))] + 0.9 \]
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Finding open-loop parameter on the root locus to meet design specifications

**Change to zero compensator**

\[
1 + \frac{K (s + 3)(s + \alpha)}{s(s + 1)(s + 2)} = 0
\]

**Design Specifications**

\[
r = \omega_n = 2 \text{ rad/sec} \\
\zeta = 0.45
\]
Alternate Approach to Two Parameter Design Using Root Locus:

**Characteristic Equation:** \[ 1 + \frac{\beta s}{s^3 + s^2 + \alpha} = 0 \]

The diagram shows the root locus for different values of \( \alpha \):
- \( \beta \) root locus for \( \alpha = 1.5 \)
- \( \beta \) root locus for \( \alpha = 5.0 \)

The poles for \( \alpha = 1.5 \) and \( \beta = 0 \):
- Pole: \( 0.299 + 0.923i \)

The poles for \( \alpha = 5.0 \) and \( \beta = 0 \):
- Pole: \( 0.553 + 1.44i \)
2nd Order Approximated Performance Design Specifications:
- Overlay Design Regions on Root Locus Plots
- Select Desired Closed-Loop Root Locations
a. System

b. Root Locus

Closed-Loop Root Location that Satisfies the Damping Ratio Specification.
Making second-order approximations

- **Open-loop pole**
- **Closed-loop pole**
- **Closed-loop zero**
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System

\[
R(s) \quad + \quad E(s) \quad \rightarrow \quad \frac{K(s + 1.5)}{s(s + 1)(s + 10)} \rightarrow C(s)
\]

Root Locus

\( \zeta = 0.8 \)

\(-4.6 + j3.45, K = 39.64\)

\(-1.19 + j0.90, K = 12.79\)

\(-0.87 + j0.66, K = 7.36\)

\(X\) = Closed-loop pole

\(X\) = Open-loop pole
Characteristics of the system at different $K$ values:

<table>
<thead>
<tr>
<th>Case</th>
<th>Closed-loop poles</th>
<th>Closed-loop zero</th>
<th>Gain</th>
<th>Third closed-loop pole</th>
<th>Settling time</th>
<th>Peak time</th>
<th>$K_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.87 \pm j0.66$</td>
<td>$-1.5 + j0$</td>
<td>7.36</td>
<td>$-9.25$</td>
<td>4.60</td>
<td>4.76</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>$-1.19 \pm j0.90$</td>
<td>$-1.5 + j0$</td>
<td>12.79</td>
<td>$-8.61$</td>
<td>3.36</td>
<td>3.49</td>
<td>1.9</td>
</tr>
<tr>
<td>3</td>
<td>$-4.60 \pm j3.45$</td>
<td>$-1.5 + j0$</td>
<td>39.64</td>
<td>$-1.80$</td>
<td>0.87</td>
<td>0.91</td>
<td>5.9</td>
</tr>
</tbody>
</table>
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Second- and third-order responses for:

a. Case 2 No Pole-Zero Cancellation

b. Case 3 ~Pole-Zero Cancellation

Case 2 response

Case 3 response
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Classical PID Control