4.1 limitation on use of uniaxial stress-strain data

- Rate of loading
  - High rate of loading increases apparent yield stress and apparent Young’s modulus
  - More brittle behavior
- Low temperature – more brittle
- High temperature – sensitive to load duration – creep (Deborah number) \( De = \frac{t_c}{t_p} \)
- Multi-axial state of stress – 3D yield criterion required
4.2 Nonlinear material behavior

- Fracture can occur in the elastic or plastic range.
- We focus on elastic, plastic, and fracture failure
Elastic-perfectly plastic model

• Popular in civil engineering as basis for limit load calculations
3-bar truss example

- Calculate ultimate load with yield stress $Y$ and elastic perfectly plastic behavior

\[ \varepsilon_B = \frac{v}{L} \quad \varepsilon_A = \varepsilon_C = \frac{v}{4L} \]
\[ n_B = \frac{EA}{L} v \quad n_A = n_C = \frac{EA}{4L} v = 0.25n_B \]

- Equilibrium
\[ n_A = n_C \quad p = n_B + 0.5(n_A + n_C) \]
Progression of yielding

• Solution for internal forces
  \[ n_A = n_C = 0.2p \quad n_B = 0.8p \]

• Member B yields first at \( p = 1.25YA \)

• Members A and C continue to absorb load elastically until \( n_A = n_B = n_C = YA \). Then from vertical equilibrium \( p = 2YA \)

• Is this model (elastic-perfectly plastic) conservative?
Limit analysis

- Lower bound theorem: *If any stress distribution throughout the structure can be found which is everywhere in equilibrium internally and balances the existing loads, and at the same time does not violate the yield conditions, these loads will be carried safely by the structure.*
Now with both loads

- Need to formulate optimization problem

\[ A_A = A_B = A_C = A \]

- Maximize \( p \) such that equations of equilibrium are satisfied and no stresses exceed the yield stress

\[ -AY \leq n_A, n_B, n_C \leq AY \]

\[ \frac{\sqrt{3}}{2}(n_A - n_C) - p = 0 \]

\[ n_B + 0.5(n_A + n_C) - p = 0 \]
Non-dimensional form

- Since all variables are loads, need a characteristic load to divide them by
- Why is $AY$ a good choice? $n'_A = n_A / AY$, $p' = p/AY$

\[-AY \leq n_A, n_B, n_C \leq AY \implies -1 \leq n'_A, n'_B, n'_C \leq 1\]

\[\frac{\sqrt{3}}{2}(n'_A - n'_C) - p' = 0\]

\[n'_B + 0.5(n'_A + n'_C) - p' = 0\]
### Solution with EXCEL Solver

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<th>nC</th>
<th>p</th>
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**Worksheet:** [3-bar.xls]Sheet1  
**Report Created:** 1/25/2007 5:31:16 PM

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Elastic-strain hardening model

- Some steel alloys have more gradual softening.
- Reloading from A' produces elastic linear behavior up to C (the hardening phenomenon).
Reading assignment

Sections 4.3-4: Question: What is the main determinant of whether we will use the maximum principal stress criterion or the maximum shear stress criterion?