9.8 Mesh revision methods

- Three mesh refinement methods

Figure 9.8-1. (a) Initial mesh for a square plate with in-plane corner load $P$. (b) A possible $h$ refinement. (c) A possible $p$ refinement. (d) A possible $r$ refinement.
Gradient (stress) recovery and smoothing

- Element smoothing

- Elements with largest difference between original and smoothed stresses are candidates for mesh refinement.

Figure 9.9-1. Element stress field $\sigma$ and element-smoothed stress field $\sigma^*$ in a quadrilateral element, depicted normal to the element plane.
Nodal averaging

• First average at nodes (when is it not appropriate?)

\[ (\sigma_e)_{ave} = \frac{1}{n} \sum_{i=1}^{n} (\sigma_e)_i \]

• Then interpolate in element

\[ \sigma^* = \begin{bmatrix} N \end{bmatrix} \{ \sigma_n^* \} \]

• Alternatively, used averaged strains to smooth displacements

• Example: One dimensional bar with two-node elements. Use strains and beam element shape functions

\[ u^* = N_1 u_1 + N_2 \varepsilon_{x1}^* + N_3 u_2 + N_4 \varepsilon_{x2}^* \quad \text{And} \quad \sigma_x^* = E \varepsilon_x^* = E \frac{du^*}{dx} \]
Example of two methods of nodal averaging

Figure 9.9-2. Uniform bar modeled by linear elements, showing a possible axial displacement field, the resulting element-by-element stress field, and smoothed stress field from (a) Eq. 9.9-2, and (b) Eq. 9.9-3.
Patch recovery

- Select patch of elements

Figure 9.9-3. Patch recovery using plane elements. △, □ = locations where stress (or another gradient) is sampled. ● = nodes. ○ = nodes where stresses $\sigma^*$ are most accurately provided by the patch. Element names are explained in Chapter 3.
Stress representation in Patch

- Stress representation
  \[ \sigma^* = \begin{bmatrix} P \end{bmatrix}\{a\} \]

- Where
  
  **Bilinear**: \( [P] = \begin{bmatrix} 1 & x & y & xy \end{bmatrix} \)  
  \( \{a\} \) contains \( a_1 \) through \( a_4 \)

  **Quadratic**: \( [P] = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 \end{bmatrix} \)  
  \( \{a\} \) contains \( a_1 \) through \( a_6 \)

- Minimize difference at sampling points
  \[ F_p = \sum_{i=1}^{nsp} \left( \sigma^* - \sigma \right)_i^2 \]

- By least squares
  \[ [A] = \sum_{i=1}^{nsp} [P]_i^T [P]_i \]
  
  \[ [A]\{a\} = \{b\} \quad \text{where} \]
  \[ \{b\} = \sum_{i=1}^{nsp} [P]_i^T \sigma_i \]

- See text for treatment of stresses on boundary
Loubignac stress recovery