Reduction of the *Free Material Design* problem to a locking material formulation

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Abstract.
The *Free Material Design* (FMD) problem concerns the optimum distribution of the elastic moduli within a given feasible domain to make the body as stiff as possible under the isoperimetric condition bounding the integral of the trace of the Hooke tensor. In the case of multiple loads the minimization concerns the weighted sum of the compliances corresponding to the subsequent loads, applied non-simultaneously, see [1]. Both the problems lead to saddle-point formulations with the displacement fields assumed as the primal behavioral unknowns, see [2,3].

An essential simplification of the FMD formulation for a single load condition is achieved in [4] by rearranging the problem to the form involving the stress fields along with the design variables, and then eliminating the latter thus reducing the FMD problem to a variational problem with one unknown stress field, viz. to a minimization of an integral of the Euclidean norm of the stress, the minimization being taken over the trial stresses being statically admissible. The stress-based setting thus obtained is equivalent to a locking material problem. The formulation dual to it assumes a form of maximization of the virtual work over the kinematically admissible virtual displacements such that the associated strains belong pointwise to a locking locus set, here a ball with respect to the same norm.

The present paper extends the above approach to the multiple load case. The problem is reduced to the minimization of the integral of the sum of singular values of the matrix composed of trial stress fields corresponding to the subsequent load conditions, the minimization being taken over the trial stresses equilibrating the subsequent load cases. This new result encompasses the single load case [4] as well as the two load case discussed by the present authors at WCSMO 9 and announced in [5]. The numerical results are constructed with using the SVD decomposition technique to interpolate the stress fields satisfying the equilibrium conditions. The main feature of the approach is the integrand in the stress minimization being of linear growth. The solutions, being non-smooth, cut off the domains where the material is unnecessary, thus circumventing a posteriori procedures to detect places where the holes should be designed. The holes are created along with the solution construction.

References

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