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The variable thickness sheet problem revisited

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1. Abstract
The optimal structural design problem which we consider is the well-studied variable thickness sheet (VTS) problem in which compliance is minimized with a constraint limiting the volume of the structure [1]. The loads are static and linear elasticity is assumed. The variable thickness sheet problem was first solved numerically by the use of finite elements in the early 1970s. Large-scale variable thickness sheets problem have since been solved on two-dimensional design domains by e.g. sequential convex optimization methods.
The objective of this presentation is to show that by using modern methods from various fields it is possible to numerically solve large-scale variable thickness sheet problems on 3D design domains using only a modest number of function evaluations. Although the variable thickness sheet problem has only marginal industrial interest it shares many of the challenges of other topology optimization problems which are frequently solved. The techniques reported herein may thus prove very useful when developing efficient optimization methods for other classes of structural topology optimization problems.
We first present a brief overview of some of the many various available equivalent formulations of the problem and state the relationships between them. For each problem formulation we list the most important properties. Based on the properties we suggest the two most promising problem formulations and proceed with numerical experiments on these. Our choices are the primal nested and the primal simultaneous analysis and design formulations, respectively.
Our choice of optimization method is a modern primal-dual interior point method recently proposed for general nonlinear optimization problems [2]. The particular method is chosen for its robustness, excellent rate of convergence, and the observed property that the number of iterations is often almost independent of the problem size. Our implementation of the method does not require that the stiffness matrix or the primal-dual saddle-point systems are assembled or factorized. Instead, these matrices are only involved in matrix-vector multiplications as part of Krylov subspace methods for solving the equilibrium equations and computing the search direction in the interior point method. In particular we show that the nested primal problem formulation can be efficiently solved without explicitly computing or storing the Hessian of compliance. The numerical experiments also indicate that for VTS problems the simultaneous analysis and design formulation can be solved in the same number of interior point iterations as the nested formulation. Since the linear elasticity equilibrium equations do not need to be solved, this formulation gives a computational advantage.
The use of Krylov subspace methods for solving the saddle-point system in interior point methods for general problems has become an increasingly popular approach. The main difficulty is the inherent and severe ill-conditioning of the saddle-point matrix as the optimum is approached. The stiffness matrix also experience increasing ill-conditioning as the optimization method modifies the design. We propose special purpose preconditioners for both systems and numerically show that they are powerful enough to deal with the increasing ill-conditioning.
An extensive set of numerical experiments on 3D design domains suggest that the combination of techniques result in a very robust and efficient method capable of solving very large-scale variable thickness sheet problems.
2. Keywords: Structural optimization, interior point methods, Krylov subspace methods, preconditioners

References
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