10th World Congress on Structural and Multidisciplinary Optimization
May 19 -24, 2013, Orlando, Florida, USA

Topology Optimization of Phononic Crystals for Self-collimation of Elastic waves

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1. Abstract

Phononic crystals (PCs), which can be viewed as an elastic version of an artificial crystalline structure, have recently attracted much attention because the characteristics of wave propagation can be effectively manipulated with proper engineering of PC structures. While there were several earlier works to use the topology optimization method [1] to design phononic or photonic crystals for various purposes [2]-[4], there is no investigation to use the topology optimization for self-collimating PCs in spite of their practical usefulness. While the design problem involves the curvature of the equi-frequency contour (EFC), the curvature-related problems have not been studied so far. In this study, we first investigate several important issues arising in the process of topology optimization formulation. Then, we suggest a curvature-based formulation for designing self-collimating PCs and present several important case studies.

Self-collimating PCs are the structures that exhibit a self-collimation phenomenon. It refers to the non-diffractive propagation of a wave in the form of a narrow beam unlike wave spread taking place in a homogeneous isotropic medium [5], [6]. Due to their zero-diffraction wave propagation characteristic, they can be used as special lenses, diodes, beam bends/splitters and so on [7]-[9]. Previous works concerned with the design of self-collimating PCs have mainly worked with lattice parameters such as period and lattice symmetry [10]-[12]. Obviously, any change in the topological layout of a unit cell will not be allowed with these approaches. In contrast, we directly look for optimal material distributions of a two-phase unit cell by setting up an appropriate topology optimization formulation. The specific design goal is to find the unit cell of a PC that collimates bulk elastic shear-vertical waves along the target direction.

The collimation characteristics in PCs can be analyzed through their EFCs, which plot a set of propagating wave vectors at a specific frequency. Note that the direction of wave propagation is always perpendicular to the EFC in lossless media. This means that the propagating direction and the slope of an EFC are interrelated and that the diffraction properties are closely related to the curvature of an EFC. Based on this observation, we utilize the geometrical properties of the EFCs of PCs in formulating a topology optimization problem. For the topology optimization, dispersion analysis including the determination of EFCs at a specific frequency is performed by the finite element analysis incorporating the Bloch solution form [13]. To facilitate the analysis of the geometrical properties of EFCs, the wave equation is casted into a quadratic eigenvalue problem and the objective function is so defined as to flatten a segment of an EFC corresponding to a certain range of wave vectors. The constraint equation ensures that the normal vectors of the EFC segment are in a specified direction that is identical to the designated propagating direction. The sensitivities of the objective and constraint functions are derived by using the adjoint variable method [14]. Case studies to design various collimating PCs are considered to verify the effectiveness of the developed method.

2. Keywords: Phononic Crystals (PCs), Topology Optimization, Equi-frequency Contour (EFC).

3. Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) grant (No: 20120005693) funded by the Korea Ministry of Education, Science and Technology (MEST) contracted through IAMD at Seoul National University and the WCU program (No. R31-2008-000-10083-0) through NRF funded by MEST.

4. References

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