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Extension of Michell's theory to exact stress-based multi-load truss optimization

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The historic aspects of this presentation can be explained as follows. Single-load truss topology optimization was first considered over a century ago by Michell (1904), but nobody has extended Michell's theory to exact elastic sress-constrained multi-load truss design ever since. The aim of this presentation is to fill this significant gap in our knowledge.

All past attempts in the literature (e.g. by Hemp, Prager and the first author) used 'plastic design', which has a highly limited applicability. Plastic design is based on a constraint on the ultimate load (collapse load) of structures having a rigid-plastic or elastic-plastic material. It requires fulfillment of the equilibrium conditions only.

On the other hand, in 'elastic design' we must also satisfy elastic compatibility. For a single load condition, optimal plastic and optimal elastic truss designs yield the same solution, due to statical determinacy of the optimal topology. This is not so in general for trusses with multiple load conditions.

It is to be noted that 'classical' Michell structures are rather unpractical, because (i) they do not take buckling into consideration, (ii) are based on a single load condition, (iii) have usually an infinite number of members, (iv) are unstable structures (i.e. mechanisms), and (v) assume the same permissible stress in tension and compression.

In spite of these shortcomings, classical Michell trusses have been studied by some of the best brains in exact topology optimization, such as Prager, Hemp, Strang and Kohn, with other important contributions by Cox, A.S.L. Chan, H.S.Y. Chan, McConnel, Lewiński, Sokół, Rozvany, Ming Zhou, Gollub, Pichugin, Tyas, Gilbert, Melchers and Dewhurst. The degree of interest in this classical field is indicated by the fact that the 2011 ISSMO/Springer prize was awarded to Pichugin et al. for an outstanding paper on Michell trusses.

The shortcomings listed above are being gradually overcome. Optimal truss layouts for unequal permissible stresses in tension and compression have been derived by Rozvany (1996), Graczykowski and Lewiński (2006a and b, 2007a and b, 2010), and Pichugin et al. (2012). Such solutions can also be used as approximations for handling point (i) above. As to point (iii), Michell trusses with a finite number of members have been investigated already by Prager and several other authors later. The proper treatment of shortcoming (ii) is first presented in the current paper. This also overcomes objection (iv), because multi-load optimal trusses are usually not mechanisms.

All exact analytical results in this paper, and methods presented for their derivation, are new, they are based on analytically derived optimality criteria and accurately confirmed by a numerical truss topology optimization program of the second author. This program, based on the adaptive ground structure approach, can at present handle up to four billion potential truss elements. Some basic properties of optimal multi-load trusses will also be outlined, and optimal plastic and elastic truss designs compared. The multi-load optimal topologies derived in this paper represent useful benchmarks for checking on numerical methods in topology optimization.

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