

A Design Method for Optimal Truss Structures with Certain Redundancy Based on Combinatorial Rigidity Theory

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Abstract

The concept of redundancy in structures is central in many design philosophies, and its importance has long been recognized by structural engineers. Generally speaking, because a non-redundant structure immediately collapses when only one member is damaged or lost, it is obvious that redundancy is essential to the safety of structures. Several definitions of redundancy have been proposed, for example, in terms of the collapse load, the number of plastic hinges, and the probability of system failure, but there is still not the common definition of redundancy. In this paper, we define, as redundancy, the margin of the number of members until the collapse of the entire structure when some components are damaged.

The truss structures designed by Truss Topology Design (TTD) method don't have redundancy. In the TTD problem, we deal with the selection of optimal configuration for pin-jointed trusses, in particular, the optimization of the connectivity of the nodes by the members, in which volume and/or compliance are minimized. In general, it is known that such truss structures are statically determinate and not redundantly rigid, that is, if just one member is damaged or lost, the entire structure cannot support loads. Therefore, it is important to take redundancy of structures into consideration in the TTD.

We present a new practical design method for finding a redundant TTD based on combinatorial rigidity theory. In this study, a truss structure is said to be a "2-edge-rigid truss" if we need to remove at least two members from the truss structure not to be rigid. This means the truss structure is still rigid even if any one of the members is damaged or lost. It is difficult to find a 2-edge-rigid TTD under general conditions; however, we can easily find a 2-edge-rigid TTD under a certain condition by using a method based on combinatorial rigidity theory. Presenting method enables to find an approximately optimal TTD with low computational cost. In the method, (1) as a relaxation problem, we solve the TTD problem without 2-edge-rigid constraint (step1), after that, (2) we find a 2-edge-rigid TTD by adding a minimum number of members to the solution of step1 (step2). Finally, (3) under the truss topology of step2, we determine the optimal section areas of the members of the truss structure (step3). The solution of step1 corresponds to the lower bound of the objective value of the global optimal solution, and that of step3 correspond to the upper bound of the objective value. Hence, we can also estimate the accuracy of the solution from the difference between the two solutions.

In numerical examples, we can find redundantly rigid truss structures which are still rigid even if any one of the members is damaged or lost. The objective value of the solution is less than one percent greater than that of the lower bound. Therefore, we can conclude that the method is effective to design an optimal redundant truss structure.