On multigrid-CG for efficient topology optimization

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In typical topology optimization procedures, the computational effort involved in repeated solutions of the analysis equations dominates the computational cost of the whole process. This motivates the search for efficient approaches aimed at reducing the computational effort invested in the analysis. We focus on the use of Preconditioned Conjugate Gradients (PCG) with a multigrid (MG) V-cycle as the preconditioning step. The proposed procedure is particularly suitable for performing 3-D topology optimization on a standard PC, within high-level programming environments such as MATLAB. Furthermore, it paves the way for integrating 3-D topology optimization in CAD software (e.g. Rhino) as well as in applications for mobile devices (e.g. the TopOpt app).

Multigrid methods are well-known for their efficiency in solving discretized differential equations, with computation time scaling linearly with the problem size [4]. Multigrid methods can also serve as effective preconditioners for Krylov subspace solvers such as PCG [2]. The resulting procedure, termed MGCG, shows excellent convergence properties and is also very suitable for parallel processing [1, 3]. Numerical experiments conducted in this study show that MGCG is also very effective as the FE linear solver within structural topology optimization procedures. This is despite the non-smooth distribution of density and stiffness in the design domain, which is an inherent property of topology optimization problems that aim to generate distinct void-material layouts.

The main concern of the current study is on investigating the utilization of MGCG to compute *approximations* to the solutions of the analysis equations. Namely, we wish to reduce computational effort by reducing the number of CG iterations. We observe that in many cases, with only a few CG iterations and without satisfying typical convergence criteria, the same optimized topology and structural performance are obtained as with a standard procedure. At the same time computational effort is reduced by roughly one order of magnitude. The approximate strain energy density distribution resulting from the coarse-grid solution is accurate enough for determining the new material distribution if no 'small' topological features are present - meaning structural members that appear on the fine grid but cannot be realized on the coarser levels. Therefore it is suggested that the number of CG iterations should ensure that small features are captured, meaning it should correspond to a) filter size; and b) number of MG levels. The resulting number of CG iterations will be smaller than required for satisfying the standard CG convergence measure, i.e. the solution of the analysis equations will be approximate. Nevertheless, the required accuracy in the analysis can be related to the progress of optimization, represented by the proximity to a KKT point. According to numerous numerical experiments, the result of the optimization process can be the same as with the fully accurate approach, with significant reduction in computational cost.

References

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