

# **h-Adaptive Extended Finite Element Method for Structural Optimization**

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## **Abstract**

Structural optimization has become a powerful tool to inspire engineers for optimal design. In recent years, the Level Set Method (LSM) is gaining popularity in structural optimization circle due to its nature of geometric boundary tracking. In the level set framework, the evolution of boundary is driven by boundary velocity derived from Shape Design Sensitivity Analysis (SDSA) with the result of finite element analysis. However, the conventional Finite Element Method (FEM) is troublesome either in handling moving boundaries between solid material and voids or in handling topological changes, as the finite element meshes need to conform to the boundaries of geometry. With the advent of Extended Finite Element Method (XFEM), shape functions of conventional FEM are extended with enrichment functions, which make XFEM suitable for representing aforementioned moving boundaries, and the tedious and time consuming re-meshing process is avoided. In the level set based structural optimization, the boundary velocity is of crucial importance nearby the interface. To shorten the optimization process and to obtain accurate SDSA result, it is important to adjust underlying meshes adequately. Higher resolution of finite element meshes in the vicinity of the boundary interface and relatively lower resolution in the regions away from the interface should be adopted for a significant decrease of computing time cost while ensuring the accuracy. This paper aims to develop an accurate and efficient XFEM scheme for structural optimization in the level set framework.

In this paper, based on previous studies on conventional XFEM with fixed fine meshes, h-Adaptive Extended Finite Element Method is investigated and developed in both two and three-dimensions. The underlying meshes are described depicted by Quadtree (2D) or Octree (3D) data structure which is capable of well managing multilevel adaptive mesh. Another benefit of this data structure is post-processing of XFEM is convenient. Starting from regular fine meshes, the adaptive meshes are generated to fit but not necessary to conform to structural geometry by coarsening and reducing initial fine meshes. During this process, the blending elements are produced with hanging nodes and the adaptive meshes are restrained to the 1-irregular mesh. The hanging node is associated with degree of freedom and treated by modifying corresponding shape functions which should satisfy Partition of Unity (POU) property. The imposition of Neumann boundary conditions is quite straightforward and the same as conventional FEM, while the imposition of Dirichlet boundary conditions is different. This study employs Nitsche's method to enforce Dirichlet boundary conditions. The effectiveness and validation of proposed h-adaptive XFEM are demonstrated through several benchmarks.

By incorporating the h-Adaptive XFEM method into level set based structural optimization, the uniform fine meshes are used to achieve level set evolution and the adaptive meshes are updated accordingly along with the propagation of structural boundary at each optimization step, which is advantageous over both conventional FEM and conventional XFEM. The reliability and effectiveness of the above mentioned framework are verified by several test cases.

**Keywords:** h-Adaptive XFEM, Level set, Nitsche's method, hanging nodes, structural optimization