Probabilistic Sensitivity Analysis for Novel Second-Order Reliability Method (SORM) Using Generalized Chi-Squared Distribution

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ABSTRACT

Gradient-based deterministic design optimizations require an evaluation of sensitivity of constraints with respect to design variables. Likewise, gradient-based reliability-based design optimizations (RBDO) require evaluation of sensitivity of probabilistic constraints at the most probable point (MPP) owing to the fact that probabilistic constraints are evaluated at the MPP. While conventional SORM that improves FORM in terms of accuracy still contains three types of error – (1) approximating a general nonlinear limit state function by a quadratic function at the MPP in standard normal U-space, (2) approximating the quadratic function at most probable point in standard normal U-space, and (3) approximating the quadratic function U-space by a parabolic surface, – the recently proposed novel SORM contains only the first type of the errors. To develop RBDO utilizing the novel SORM for reliability analysis, sensitivity of probabilistic constraints at the MPP is required. Thus, this study presents the sensitivity analysis of the novel SORM for more accurate RBDO. The analytic derivation of the sensitivity first involves third-order derivative of a performance function or derivative of the Hessian matrix, which is however not available. Therefore, during the analytic derivation, it is assumed that the Hessian matrix does not change due to the change of the distribution

parameter, thus its eigenvalues and eigenvectors do not change either accordingly. The calculation of sensitivity based on the derivation requires evaluation of probability density function (PDF) of linear combination of non-central chi-square variables, which can be obtained using either noncentral chi-squared or general chi-squared distribution. This study utilizes the general chi-squared distribution, whose calculation is divided into two cases: when eigenvalues of the Hessian matrix are positive and when they are negative. The accuracy of the proposed probabilistic sensitivity derivation is compared with the finite difference method (FDM) using numerical examples. The numerical examples demonstrate that the analytic sensitivity of the novel SORM agrees very well with the sensitivity obtained by FDM when a performance function is quadratic in U-space and input variables are normally distributed. It will be further tested the sensitivity of higher order performance function in terms of how the proposed assumption – the Hessian is constant – affects the accuracy of the sensitivity.

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