How to route a pipe – Discrete approaches for physically correct routing

Sonja Mars[‡], **Jakob Schelbert** [† *1], **Lars Schewe**[†1]

[‡]Technische Universität Darmstadt, Discrete Optimization
Dolivostr. 15, 64293 Darmstadt
smars @opt.tu-darmstadt.de

[†]FAU Erlangen-Nuremberg, Discrete Optimization
Cauerstr. 11, 91058 Erlangen
{jakob.schelbert, lars.schewe} @math.uni-erlangen.de

Keywords: global optimization, mixed-integer second-order-cone programming, timoshenko beam, topology optimization

ABSTRACT

We present a mixed-integer non-linear appoach for routing a pipe through a power plant. Finding a globally optimal solution is a difficult task due to the fact that one has many possibilities for a given discretization of the design space following the ground structure approach. The problem combines discrete aspects and non-linear constraints that model the physics of the pipe. Due to the discrete constraints that force the pipe to form a path or even a Steiner tree, conventional truss topology optimization methods are not directly applicable.

In the past time the wide field of truss design with linear elasticity has been discussed from a mostly continuous optimization point of view. Models for the problem can be written as linear or quadratic problems, or with the use of semidefinite inequalities, depending on additional restrictions (see [BS04] for an overview of different models). On the other hand, some research has been done modeling the problem with discrete variables, as in [AS07] or [SS03].

In our real-world application a rough outline of the admissible region and a start and end point are given. In addition to the self-weight of the pipe we are also asked to place hangers that provide support for the pipe. The task is find a cheap practical solution for an engineer to start with considering several constraints that model technical restrictions.

For our approach we consider a non-linear problem formulation, namely a second-order-cone problem which – as one adds integer variables – is in general a subject of current research. We show that introducing integer variables to the SOCP-model preserves the convexity of the relaxed problem. Furthermore we investigate how adding path constraints changes the properties of the problem. To the best knowledge of the authors this was not done before and yields some difficulties for solving the model. Due to the properties of our solution the physical model of trusses is not enough to express all desired characteristics of the pipe such as moments and shear forces. To address this we consider linear Timoshenko beams for our pipe as well as the self-weight of the beams. Path-constraints, which need additional variables, compel the solution of our model to have the desired structure. Here a flow formulation is used that is very common in combinatorial optimization. An artificial flow is routed from the source to the sink and thus ensures the pipe not to split up as a traditional truss would do to achieve a more stiff structure.

[1]funded by the German federal ministry of education and research with project LeOpIn under grant 03MS637A

For solving big real-world instances we also introduce a decomposition scheme that splits the problem into a master problem consisting of finding the cheapest path/Steiner tree and a sub problem in which a mixed-integer SOCP is solved to place the hangers to fulfill physical constraints.

We give numerical results for some real-world test scenarios.

References

[AS07] Wolfgang Achtziger and Mathias Stolpe, *Truss topology optimization with discrete design variables—guaranteed global optimality and benchmark examples*, Structural and Multidisciplinary Optimization **34** (2007), no. 1, 1–20.

[BS04] M. P. Bendsoe and O. Sigmund, *Topology optimization: Theory, methods and applications*, Springer, 2004.

[SS03] M. Stolpe and K. Svanberg, *Modelling topology optimization problems as linear mixed 0–1 programs*, International Journal for Numerical Methods in Engineering **57** (2003), no. 5, 723–739.