Comparison between reliability based and robust topology optimization considering uncertainties in the geometry

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Abstract

The aim of this work is to identify, to discuss and to compare the applicability limits of different strategies for obtaining topology optimized designs with uncertainties in the geometry. Traditionally, topology optimization is performed under deterministic assumptions. In practice, however, uncertainties in the material properties, in system input, and in geometry are unavoidable. These uncertainties can have a significant impact on the structural performance as well as on the optimized topology [5, 2], and their inclusion in the optimization process can lead to a significant decrease in the production cost and to a prolongated exploitation life of the designs.

The uncertainties in the geometry are modeled using probabilistic approach, i.e., with the help of random variables and random fields. These variations can be accounted for by two different formulations known as reliability based topology optimization (RBTO) and robust design optimization (RDO). The first formulation constraints the optimization with the small failure probabilities associated with a predefined failure subdomain. The second formulation has recently been studied extensively in the design of compliant mechanisms [3] with geometric uncertainties. It results in designs insensitive to the random parameters. The main burden in the proposed RDO approaches [3, 2] is the estimate of the response moments. The original formulation [3] is based on Monte Carlo simulations. Later, two alternatives using stochastic collocation [2] and stochastic perturbation [6] have been proposed as well. The first one can be applied only for moderate number of random parameters and the computational cost grows exponentially with increasing the details in the parametrization. The second approach based on stochastic perturbation is computationally inexpensive, however, it can be applied only for small variations of the response and the geometry. These limitations have to be fulfilled by

the optimized system, otherwise the design is tuned to be less sensitive only at a specific point in the design space and, in general, it lacks robustness. The inexpensive availability of the gradients with respect to the uncertain parameters can be utilized for estimation of the failure probabilities of the design, which makes RBTO approach a very attractive alternative to RDO. The system reliability is estimated using First Order Reliability Method (FORM) [1] and implementations of both, the reliability index (RIA) and the performance measure approach (PMA) formulations [4] for geometric imperfections are discussed. The reliability based optimized designs will be compared to the robust ones and a discussion with respect to computational cost, design performance and topological features will be presented in details.

Another important feature resulting from the introduction of the geometric variations is the appearance of length scale in the optimized designs [5]. The definition of length scale implicitly leads to reduction of the stress concentration - a fact which has only been discussed briefly in previous works. Close to the boundaries the length scale is affected by the imposed design boundary conditions which will be discussed in details. The optimization procedures are demonstrated and compared for both L-bracket and compliant mechanism designs.

References

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