Robust topology optimization of 2D and 3D continuum and truss structures using a spectral stochastic finite element method

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1 Introduction

This research focusses on a novel robust structural topology optimization [1] method for 2D and 3D continuum and truss problems. Structural optimization taking uncertainties into account is of significant importance to designers, since real-world structures require both efficient use of material and accurate modelling of material properties, manufacturing tolerances and loading of structures. When considering candidate designs, engineers are concerned with the sensitivity of the designs to small variations which can be quantified as uncertainties. Robust design optimization offers a framework for taking these uncertainties into account. While some work has very recently been done on robust shape and topology optimization of two dimensional structures [2] for mass minimization, 3D structures appear to have not been dealt with broadly [3], and, to the authors' knowledge, no common frameworks exist for robust optimization of both continuum and truss structures. The representation of uncertainties in robust optimization of truss structures has also been relatively neglected and investigations thus far have failed to take some key features of trusses, such as element length, into account.

2 A novel robust topology optimization method

In this investigation the material uncertainties are expressed in terms of a spatially varying random field, which is discretized using a Karhunen-Loève (KL) expansion. Random fields allow for expression of spatially correlated random quantities, while being general enough to model uncorrelated quantities too. KL discretization methods expand any realization of the original random field over a complete set of deterministic functions. Various discretization methods are available of which the KL is one of the most efficient in terms of the number of random variables required for a given accuracy, making it a good candidate for the computationally expensive task of design optimization. Spectral Stochastic Finite Element Method (SSFEM) [4] is used to derive the statistical measures of the response, allowing for a quantification of the terms of the objective function (a linear combination of the mean and standard deviation of the compliance), for a given volume fraction. Material models are generally expressed in terms of Gaussian or lognormal probability distributions, both of which can be taken into account in the SSFEM framework. In continuum structures the random field may be correlated over the entire domain, while in truss structures this is not the case. A novel analysis method for modelling the variation of material properties along the length of individual truss elements is developed, based on the SSFEM framework, and used for topology optimization of truss structures. Derivation of the objective function and the sensitivities necessary for the optimization procedure are demonstrated, making use of the response quantities.

3 Computational examples

The proposed method is demonstrated on both 2D and 3D continuum and truss problems. A 2D bridge problem is considered, in which the effects of the variation of the various material parameters are shown. The domain is discretized using 2D quad elements. Next a 3D bridge structure is considered, for which 8-node brick elements are used. Finally a truss problem demonstrates the approach to truss optimization. Results are shown for various values of the standard deviation and correlation length of the random field, as well as the additive coefficient in the objective function. Complete results will be available in the full paper and shown during the oral presentation.

References

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