Recovery of two-phase microstructures of planar isotropic elastic composites

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Abstract: The isotropic elastic mixtures composed of two isotropic materials of the bulk moduli ($\kappa^2>\kappa^1$) and shear moduli ($\mu^2>\mu^1$) are characterized by the effective bulk and shear moduli κ^* and μ^*, respectively. In the planar problems the theoretically admissible pairs (κ^*, μ^*), for given volume fraction ρ of material (κ^2, μ^2), lie within a rectangular domain of vertices determined by the Hashin-Shtrikman numbers. The tightest bounds known up till now are due to Cherkaev and Gibiansky (1993). These bounds are described by a curvilinear rectangle of vertices A, B, C, D, the vertices A and C being Hashin-Shtrikman points, B being attributed to the Walpole result, see Fig.6 in Sigmund (2000). The microstructures corresponding to the interior of the rectangle ABCD can be of arbitrary rank, in the meaning of the hierarchical homogenization. In the present paper a family of composites is constructed of the underlying microstructures of rank 1 or with using higher rank microstructures. The consideration is confined to the microstructures possessing rotational symmetry of angle 120°. To find the effective moduli (κ^*, μ^*) the homogenization method is used: the local basic cell problems are set on a cell *Y* of the shape of a hexagonal domain. The periodicity conditions refer to the opposite sides of *Y*. Such a non-conventional basic cell choice generates automatically the family of isotropic mixtures. The subsequent points (κ^*, μ^*) are found by solving the inverse homogenization problems with the isoperimetric condition expressing the amounts of both the materials within the cell. The isotropy conditions, explicitly introduced by Sigmund (2000) into the inverse homogenization formulation, are not involved, as being fulfilled by the microstructure construction. The method put forward makes it possible to localize each admissible pair (κ^*, μ^*) within the Cherkaev-Gibiansky bounds by appropriate choice of the layout of both the constituents within the repetitive sub-domain of *Y*. The present paper proves that rank 1 microstructures cannot attain some regions close to the Cherkaev- Gibiansky bounds and close to the Walpole point. These regions can be encompassed by admitting the microstructures of rank higher than 1.

References:

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