

## Maxwell's Reciprocal Diagrams and Discrete Michell Frames

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### 1. Abstract

Although Graphic Statics was a leading method of analyzing trusses in the late 19th and early 20th centuries, it has not been commonly employed in the 21st century due to the development of other mathematical approaches. This work uses Graphic Statics to unite two major legacies of James Clerk Maxwell's ground-breaking paper, *On Reciprocal Figures, Frames, and Diagrams of Forces* : (i) the fundamental theorem used by Michell to derive trusses of least weight and (ii) reciprocal frames. For a given connectivity of nodes, Graphic Statics provides all of the information needed to determine the total load path of the structure in the form and force diagrams. Because these diagrams are reciprocal, we discuss how in the course of finding one minimum load path structure, a second minimum load path structure is also found. Several examples are given to illustrate the notions of duality and self-reciprocity in these diagrams, with particular emphasis placed on discrete optimal benchmark structures. These observations between the corresponding form and force diagrams are also generalized for discrete cantilever Michell frames, and several comments on the extensions of this work are discussed.