10th World Congress on Structural and Multidisciplinary Optimization
May 19-24, 2013, Orlando, FL, USA

HEMP'S ARCH IS AN OPTIMAL SOLUTION FOR SEVERAL UNIFORM LOAD PROBLEMS, JUST NOT THE ONE CONSIDERED BY HEMP

Aleksey V. Pichugin[1], Andrew Tyas[2], Matthew Gilbert[3],

[1] Brunel University, London, UK, aleksey.pichugin@brunel.ac.uk
[2] University of Sheffield, Sheffield, UK, a.tyas@sheffield.ac.uk
[3] University of Sheffield, Sheffield, UK, m.gilbert@sheffield.ac.uk

Abstract

In his groundbreaking 1974 paper W. S. Hemp analytically described an arch structure that can transmit the uniformly distributed horizontal load to two level pinned supports. The powerful semi-inverse method designed by Hemp for this purpose allowed him to construct full analytical solutions for both static and kinematic fields, which are coupled in this non-statically determinate problem. Unfortunately, Hemp's derivations also showed that his structure does not satisfy the requirements of the Michell criteria in a small part of the solution domain, hence, implying that Hemp's solution is not optimal. In 1975 H. S. Y. Chan showed that Hemp's solution may, in fact, be optimal if, instead of uniform loads, one considers certain configurations of distributed loads. Although both these results appear somewhat unsatisfactory, it is worth noting that the volume of Hemp's structure is still some 3.5% lower than the volume of equivalent parabolic arch with vertical hangers. The implication is that, despite its highly theoretical nature, Hemp's structure represents a non-trivial estimate of the theoretical minimum volume for the important practical problem.

The present paper complements Chan's result and shows that Hemp's structure can also be optimal for uniform loads, if one extends Hemp's solution to the case when the maximum allowable stresses are not equal to each other. Specifically, we demonstrate that if the ratio of allowable tensile stress to allowable compressive stress falls below a certain threshold, the Hemp's structure becomes an optimal solution for the upper half-plane. This result can also be reformulated for the lower half-plane Hemp's structures, which once again turn out to be optimal as long as the ratio of allowable compressive stress to allowable tensile stress falls below the same threshold. The case of full-plane optimization domains is also considered. The described results are illustrated by a range of numerical examples generated using the software for discrete layout optimization designed by the authors.