

Estimating Material Cross-Property Bounds with Topology Optimization

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Abstract

Cross-property bounds describe the range of achievable properties for multiple phase materials, such as composites, as a function of individual constituent phase properties. Such bounds have been derived for many property combinations. Hashin-Shtrikman [1], for example, derived expressions relating the effective elastic moduli to the volume fraction of phases present in the material. Gibiansky-Torquato [2] derived bounds linking effective stiffness and effective conductivity given the stiffness and conductivity of the individual material phases. There are, however, combinations of properties for which theoretical cross-property bounds have not yet been developed.

This paper uses topology optimization to generate Pareto fronts that serve as estimates to the upper bounds of two such property combinations for porous materials: effective fluid permeability and porosity, and effective fluid permeability and effective stiffness. The “design” problem is posed as an inverse homogenization problem where the goal is to distribute material within the characteristic unit cell (at the microscale) such that effective properties of the bulk material (at the macroscale) are optimized. Finite element-based homogenization is used to estimate the effective properties from the unit cell topology and both level set and material distribution topology optimization methods are considered.

The topology optimization results represent first estimates to (a) effective permeability-porosity bounds and (b) effective permeability-effective stiffness bounds for a range of considered volume fractions. The corresponding unit cell topologies that achieve these bounds are novel and also presented, as well as a comparison to existing porous materials including foams and micro-lattices. The results are therefore interesting from a scientific perspective, but could also serve as a reliable guideline for material design with porous materials.

[1] Hashin, Z., Shtrikman, S., 1962. A Variational approach to the theory of the effective magnetic permeability of multiphase materials. *Journal of Applied Physics* 33, pp. 3125-3131.

[2] Gibiansky, L.V., Torquato, S., 1993. Link between the conductivity and elastic moduli of composite materials. *Physical Review Letters* 71, pp. 2927-2930.