Multi-Disciplinary Analysis under Uncertainty

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Abstract

Uncertainty quantification and design under uncertainty of realistic structures requires multiple functional evaluations in the presence of natural variability, data uncertainty and model uncertainty. When the structural system requires high-dimensional coupled multi-disciplinary analysis between several disciplines, the computational effort multiplies tremendously. This paper develops efficient uncertainty propagation and sampling techniques to overcome such challenges.

A new method is developed in this paper for including data uncertainty and model error quantification in multi-disciplinary system analysis that requires iterative analyses with large-dimensional coupling quantities. This methodology estimates the probability distributions of the coupling variables based on computing the probability of satisfying the inter-disciplinary compatibility equations. The estimation of the coupling variables is equivalent to likelihood-based parameter estimation in statistics. This idea is used to develop a likelihood-based approach for multidisciplinary analysis (LAMDA). Using the distributions of the feedback variables, the bi-directional coupling can be reduced into unidirectional coupling without loss of generality, while still preserving the mathematical relationship between the coupling variables. The calculation of the probability distributions of the coupling variables is theoretically exact and does not require a fully coupled system analysis.

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To quantify and include the model error within the LAMDA method, an auxiliary variable method is exploited. The auxiliary variable is defined to explicitly represent the variability in the stochastic model prediction even for deterministic input (due to model error). With the help of the auxiliary variable, the variability of the model prediction is explicitly represented by another random variable with a [0,1] uniform distribution. This representation is superior to a sampling-based strategy in terms of computational effort, especially when the model errors are themselves represented as random variables.

In practice, the amount of data exchange between two coupled analyses can be very large, e.g. coupling of computational fluid dynamics (CFD) and finite element structural analysis (FEA) in the aero-elastic analysis of a wing. The feedback variables are nodal forces and nodal displacements with 6 degrees of freedom at all the nodes. This high dimension poses challenges for the LAMDA method. Principal component analysis is adopted to decrease the number of coupling quantities and to retain only the most influential elements of the original data; hence lightening the burden of LAMDA.

In the LAMDA method, either Monte Carlo sampling or the First-order Reliability Method (FORM) is used to calculate the joint CDF of the coupling variables, and the Finite Difference Method is used to retrieve the joint PDF. However, as dimensions of the problem increase, the number of function evaluations required at each point grows in geometric progression. Therefore a Bayesian network approach, along with copula-based representation and sampling of the joint PDF, is employed to achieve tremendous computational efficiency. The proposed methods are illustrated using an aero-elasticity analysis of a 3-D aircraft wing.