A study on symmetry properties in structural optimization

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Abstract

Symmetry is a very important property for structure design owing to the convenience in theoretical analysis, numerical computation and manufacturability. So a question, which has received considerable attentions in structural optimization fields, emerges naturally: whether the optimal solution of a symmetric optimization problem is symmetric? In a review article, Prof. Rozvany (2011) presented a number of conjectures concerning the fundamental properties of exact optimal topologies and concluded that for a particular class of problems satisfied some specific restrictions, at least one optimal topology is symmetric if the external loads, the boundary of the design domain and the support conditions are all symmetric.

In the present work, symmetry properties of a class of optimization problems are revisited from a more general point of view. The main finding is that not the linearity but quasi-convexity is a key point to

guarantee the existence of symmetric solutions for symmetric optimization problems. Our analysis verdicts a conclusion that under some invariant assumptions, for a large class of structural optimization problems that can be formulated as quasi-convex programs, there is at least one symmetric global optimal solution if the prescribed loading and support conditions are symmetric. With this conclusion, the conjectures in Rozvany (2011) can be seen as the special cases and the existence of symmetric global optimum for a larger class of optimization problems can be proved. Some concrete examples with analytical solutions verify our proposition sufficiently. Moreover, through a practical structural optimization example that the global optimum of a symmetric optimization problem which is not quasi-convex may be highly asymmetric illustrates that the significance of quasi-convexity for guarantee the symmetry property of optimal solution. So heuristic symmetry reductions without serious theoretical analysis are inadvisable and may result in strongly suboptimal solutions. Furthermore, the symmetry analyses of robust and discrete structural optimization problems, which are seldom touched in the literature, are also discussed based on the former proposition. Besides, for some specific non-convex problems, based on the results of sequential convexification, it is proved that at least one of the K-K-T points of the optimization problem is symmetric as long as the optimization problem

is symmetric.

Keywords: Symmetry; Structural optimization; Quasi-convex programming; Robust/discrete optimization.

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