

## Isogeometric Shape Design Sensitivity Analysis of Curved Crack Problems in Mixed-mode Fracture Mechanics

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### Abstract

Isogeometric shape design sensitivity analysis of stress intensity factors for mixed-mode curved crack problems is developed. In an isogeometric approach, the NURBS basis function which is used to represent the geometric model in the CAD system is directly used in the response analysis, which enables the incorporation of exact geometry into the structural response. The NURBS basis function is calculated recursively from the given knot vector, and it is very simple to adjust the order of the basis function. Moreover, without changing the geometry, the response analysis results can be improved through h-refinement by knot insertion, p-refinement by degree elevation, and k-refinement combining both of knot insertion and degree elevation. Also, the NURBS basis function guarantees higher order continuity  $C^{p-k}$  where  $p$  is a degree of basis function and  $k$  is a knot multiplicity.

Interaction integral formulation using William's 2D solution for auxiliary fields is utilized for calculating stress intensity factor efficiently. Especially for curved crack problems, the line integral along the crack-face should be additionally considered in the standard interaction integral formulation due to the discontinuity of the strain energy density across the crack-face. For this line integral term, analytic shape design sensitivity is derived and utilized for calculating shape design sensitivity of curved crack problems of which design variable is a crack size.

Discontinuity is represented along patch interface and repeated knots whose multiplicity is same as a degree of basis function are used for locating crack-tip at a desired point. Two main features of isogeometric analysis method : exact geometry and higher order continuity are emphasized in calculating stress intensity factor and its shape design sensitivity. Firstly, the exact representation of crack geometry enables to obtain accurate selection of local Cartesian coordinate at the crack-tip, which is necessary for calculating auxiliary stress and displacement fields in interaction integral formulation. Also, the higher-order geometric information of crack like curvature and normal vectors in interaction integral and its material derivatives can be exactly obtained, which significantly improves the computation results, compared with finite element method utilizing approximated geometry. Secondly, higher-order continuity between elements in isogeometric analysis enables to obtain continuous stress and strain field, especially in the domain of interaction integral. There is still a limitation of  $C^0$  continuity across the patch interface and certain parametric positions whose knot is repeated. However, compared with  $C^0$  element based finite element method, enhanced stress and strain continuity in the domain of interaction integral gives better computation results of stress intensity factors and its shape design sensitivity.

Three numerical examples are presented for the verification of shape design sensitivity and the comparison with finite element method. Quadratic NURBS basis function is used in isogeometric analysis, and for comparison, linear and quadratic basis function is used in finite element analysis. Inclined straight crack, parabolic crack, circular arc crack problems which have analytic solution of stress intensity factors are dealt with in the numerical examples.

**KEY WORDS:** Isogeometric analysis, NURBS, Curved crack, Stress intensity factor, Shape design sensitivity