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Isogeometric Shape Sensitivity Analysis Based on Generalized Curvilinear Coordinate

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Abstract

Using an isogeometric approach, a geometric design is built and, rather than approximated into a finite element model, is exactly represented in computational framework by a direct utilization of the functions describing the geometry in analysis. Thus, the geometric properties of the design are embedded in the NURBS basis functions and control points without further parameterization. The parameterization of a design domain raises difficulties in shape design sensitivity analysis and shape optimization based on the finite element method.

In spite of the potential capability in both analysis and design, the conventional isogeometric analysis, in which the physical domain is represented in RCC, can lose its performance when there are distortions of the element geometry. Since higher order NURBS shape function is used in the isogeometric analysis, mesh distortion effects can be larger than that of the conventional FEM. Geometry distortions cause global co-ordinates to be no longer linear transformations of local co-ordinates in the geometry mapping, providing local co-ordinates in the geometry mapping, and local co-ordinates that are no more polynomial functions of the global co-ordinates. Therefore, conventional RCC based isogeometric analysis using Gaussian quadrature cannot accurately integrate the integrand which is no more a polynomial function. Therefore it requires as many numbers of degrees of freedom in the finite element model that would be excessively redundant for representing a geometry model.

To overcome the aforementioned difficulties, the introduction of generalized curvilinear coordinates (GCC) can be a remedy. When the physical domain is represented in GCC, unlike RCC, each basis vector at a point is defined differently in GCC. Moreover, the coordinate value on GCC systems can be linearly transformed to the parametric space. Therefore, GCC based isogeometric analysis using the Gaussian quadrature does not require as many numbers of degrees of freedom as the RCC based one to obtain an accurate solution. Also, representing the higher order geometric information, such as normal vector and curvature, into the solution domain yields the GCC-based isogeometric approach is the best way for analyzing responses and design sensitivities of an arbitrary curved geometry. In the GCC based finite element approach, it is almost impossible to obtain the position vector of the general surface except for some simple surfaces such as cylindrical surfaces or spherical surfaces. However, it is notable that NURBS has a two-parameter representation in the surfaces, and so arbitrary shaped geometry can be represented on the framework of GCC based isogeometric approach.

Through the numerical verifications of response analysis results for various curved beams, it is demonstrated that the GCC-based isogeometric approach provides superior results to both the conventional RCC-based isogeometric approach and GCC-based finite element approach. From the sensitivity analysis point of view, the domain for the shape sensitivity analysis is discretized in the early stage of geometric definition so that the normal vector and curvature can be calculated exactly. Also, shape sensitivity expressions include the shape dependences of higher order geometric information as well as exact responses. Moreover, the NURBS basis functions conveniently provide a smooth and non-local design velocity field whose computation is not easy in the finite element based sensitivity and optimization, but essential for the shape optimization. Through the demonstrative numerical examples, the GCC-based isogeometric shape sensitivity analysis provides superior results compared to the conventional isogeometric approach.

KEY WORDS: Isogeometric analysis, NURBS, Shape sensitivity, Generalized Coordinate Systems, Exact Geometry