

# Reliability-based Design Optimization applying Polynomial Chaos Expansion: Theory and Applications

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## Abstract

Reliability-based Design Optimization is achieving more and more agreement in the industrial design community. In fact, most of the industrial processes are permeated by uncertainties: the manufactured product is generally different, from a geometric point of view, from the product design because of the dimensional tolerances, and, more frequently, the working point is not fixed, but is characterized by some fluctuations in the operating variables.

This uncertainty is commonly transferred to the performance of the system, which cannot be determined with an exact and single value, but which is better described by a statistical distribution of results.

In this environment, a frequent industrial requirement is the satisfaction of constraints or limits, for which the percentage of solutions not satisfying these (failure probability) must be minimized as much as possible, to improve the reliability and quality of the product.

We propose in this paper a new methodology to deal efficiently with a reliability-based optimization problem of industrial relevance, which conjugates accuracy and small number of needed evaluations.

The methodology is derived from Robust Design Optimization implementing Polynomial Chaos, which estimates analytically therefore with high accuracy the main moments of the performance distribution. The polynomial coefficients can then be used to evaluate the complete cumulative distribution function of the performances of the design, and then to retrieve accurately the failure probability for the prescribed limits/constraints of the problem.

The final paper will contain all the theoretical details of the methodology proposed, and will propose a validation and an application case. An analytical problem (deflection of a cantilever beam) will be first proposed in order to describe the problem and to compare the results obtained by one of the traditional approach to solve this kind of problems (FORM) and by the new proposed approach here, highlighting the accurate convergence of the results in the two cases and the difference in terms of number of simulations required, in advantage of the new approach.

The new approach is therefore applied to a more challenging and not conventional application problem, which is the reliability-based optimization of a boomerang throw.

The boomerang trajectory is computed through the analytical integration of motion equations, using the boomerang aerodynamic coefficients varying at each time step and computed by a CFD code on a sampling set of flying conditions and extended on the whole domain through the usage of a Response Surface in modeFRONTIER. The optimization problem consists in the reliability optimization of the throw, subjected to many uncertainties like throw angle, velocity and spin, with the purpose of minimizing the percentage of throws which do not return accurately (arbitrary assumed as having an arrival distance higher than 1meter from the throwing point). The final paper will contain the description of the methodology adopted and the illustration of the results obtained.

Besides presenting a not conventional and challenging application case, the main purpose of the work is to stress the efficiency of the methodology that could be used in a wide range of industrial application cases.