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Design Optimization Using Hyper-Reduced Order Models

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1. Abstract

Partial Differential Equation (PDE)-constrained optimization requires the solution of large-scale systems of equations at each iteration of the design process. The size of such systems scales with the dimension of the discretized model associated with the PDE and incurs large CPU costs and memory requirements.

In the present work, projection-based reduced-order models (ROMs) [1] are constructed and used in the optimization process to accelerate the design process. Because such ROMs originate from the projection of the original large-scale equations onto a reduced basis, these ROMs have the capability, for a well-chosen basis, to be as accurate as the original model, unlike low-fidelity models. Two approaches will be considered in this work for the reduction of the equations: Galerkin projection and residual minimization. The first approach is recommended for equations with symmetric Jacobian matrices and the second for equations with non-symmetric Jacobians.

In the case of nonlinear systems, an additional level of approximation is required in both aforementioned approaches so that the cost associated with evaluating the equations of interest does not scale with the size of the large-scale system [2]. Such a hyper-reduction step is performed by evaluating the residual only on a subset of the computational domain [3]. In the present work, the cost associated with evaluating the objective and constraint functions will be also alleviated by approximation using radial basis functions evaluated on the aforementioned reduced domain only.

Several optimization strategies are considered and analyzed, namely Nested Analysis and Design (NAND) and full-space and reduced-space Simultaneous Analysis and Design (SAND). An advantage with using ROMs in a full-space SAND framework is that the linear systems that arise in the optimization process are of small-size and as such can be solved directly, unlike their high-dimensional counterparts that require iterative solvers and the construction of preconditioners. The structure of the KKT system is then identical with ROMs as with large-scale systems. The structure of the Lagrangian is also identical for Galerkin projection-based ROMs but not with residual minimization-based ROMs, impacting the choice of a merit function in a SAND framework.

An application to the inverse design of a rocket nozzle shape will be presented. The shape of the nozzle is parameterized using ten parameters. A reduced-order model is constructed in an offline phase and used to determine the parameters associated with a given target flow profile. Numerical experiments show that the proposed methodology based on hyper-reduced models can recover the target parameters at a fraction of the cost associated with an approach based on high-dimensional models. The effectivity of each optimization strategy will be also compared and discussed.

2. References

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[2] M. Rathiman and L. Petzold, A new look at proper orthogonal decomposition, *SIAM Journal on Numerical Analysis*, 41 (5), 1893-1925, 2004.

[3] K. Carlberg, C. Farhat, J. Cortial and D. Amsallem, The GNAT method for nonlinear model reduction: Effective implementation and application to computational fluid dynamics and turbulent flows, *Journal of Computational Physics*, 242, 623-647, 2013.