On Numerical Instabilities of the Extended Finite Element Method for Topology Optimization

Kurt Maute and David Makhija
Department of Aerospace Engineering, University of Colorado Boulder

In the past decade, level-set methods have significantly gained in popularity for solving topology optimization problems. Most often an Ersatz material approach is used to map the geometry, defined by the level-set field, into a fixed-grid numerical mechanical model. While this approach confines the occurrence of intermediate densities to small regions in the vicinity of the structural boundaries and material interfaces, it leads to a blurred geometry representation in the mechanical model, may cause numerical artifacts, and complicates the application of boundary conditions. Alternatively, body-fitted and immersed boundary techniques (IBTs) can be used to directly represent the geometry in the mechanical model. While the generation of body-fitted meshes typically leads to robustness and efficiency issues, in particular in 3D, IBTs enjoy great popularity in broad range of related applications with emerging complex geometries, such as crack propagation in solids, multi-phase flows, solidification processes, etc. Among many IBTs, the extended finite element method (XFEM) is a promising candidate for predicting the physical behavior of designs defined by level-sets. For example, XFEM was used by Van Miegroet and Duysinx (2007) to optimize the shape of 2-D elastic structures, by Kreissl and Maute (2010) to find the optimum geometry in flow problems, and by Maute et al (2011) to determine the optimum layout of nano-structured thermal materials.

In this paper, we will carefully analyze the combination of level-set methods and XFEM, and show that standard XFEM may lead to numerical artifacts in the optimized geometries and suffers from ill-conditioning of the analysis problems. We will show that first problem can be cured by a generalized enrichment strategy and the second problem by a simple but efficient and robust nonlinear preconditioning scheme. Through numerical examples, we will demonstrate the applicability of the above remedies to a broad range of engineering design problems, including nonlinear structural mechanics, incompressible flows modeled by the Navier-Stokes equations,and diffusive and convective heat transfer.

References

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