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Construction of Parametrically-Robust Reduced-Order Models for PDE-Constrained Optimization

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1. Abstract

Optimization problems constrained by nonlinear Partial Differential Equations (PDE) arise in many engineering fields and contexts including inverse modeling, control, and shape optimization. Among the most computationally expensive and interesting PDE-constrained optimization problems is shape optimization. The high-dimensional nature of computational mechanics (CM) simulations makes shape optimization a very large-scale nonlinear program with constraints defined by the discretized continuum equations, which corresponds to tens of millions of nonlinear equations for practical simulations. The result is a large-scale non-convex, nonlinear program that is, in general, very difficult and computationally prohibitive to solve.

Despite the high-dimensional nature of many CM models, the solution trajectory is typically confined to a low-dimensional affine subspace. This is the inherent assumption in Model Order Reduction (MOR), whereby a Reduced Order Model (ROM) is constructed with many fewer degrees of freedom than the original High-Dimensional Model (HDM); typically several orders of magnitude fewer, without significant loss of fidelity. Despite the fewer degrees of freedom, an additional level of approximation, termed hyperreduction, is necessary to realize speedups for nonlinear problems. To address the computational cost of shape optimization, the HDM constraint is replaced with the hyperreduced ROM. The result is a search for an optimal shape by considering physics that lie in the constructed subspace.

The contribution of this work is a greedy, physics-based method for constructing parametrically-robust ROMs with a small number of queries to the HDM. A ROM is progressively built by training in regions of the parameter space where it lacks accuracy. Regions of high error in the reduced-order model are determined by the solution of a small nonlinear program. The method is designed in the scope of a strictly offline-online framework; the ROM is constructed offline and then exploited many times in the online phase. While the method is general in the sense that the constructed ROM can be used for any relevant application such as real-time analysis, uncertainty quantification, or PDE-constrained optimization, the emphasis of our work is PDE-constrained optimization. Despite the offline-online decomposition, the offline cost remains relatively low by only sampling the HDM a small number of times. The method is applied to shape optimization of a converging/diverging nozzle in which the flow is governed by the Euler equations, a standard nonlinear CFD optimization problem. The results show that the proposed method effectively generates a ROM with parametric robustness. Extension of proposed method to hyperreduction is the subject of ongoing research.