A prevalent problem in the field of topology optimization has been instabilities such as the appearance of checkerboard patterns when using low-order triangles and quads. It will be shown that discretizations based on polygonal finite elements naturally provide stable solutions. The better performance of polygonal discretizations is attributed to their enhanced approximation characteristics, which also alleviate shear locking in elasticity and lead to a stable low-order mixed variational formulation of incompressible Stokes flow. A simple but robust algorithm is provided, which utilizes centroidal Voronoi tessellations (CVTs) to generate convex polygonal meshes that possess enhanced regularity and isotropy. We will assess the performance of polygonal discretizations in elasticity and Stokes flow and discuss their applications to topology optimization problems in both solids and fluids.