Hierarchical optimization of laminated fiber reinforced composites

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1. Abstract
The aim of this work is to perform hierarchical optimization [1] in laminated composite structures, considering simultaneously macroscopic and microscopic levels in the design of the structure and its material. In the macroscopic level, the optimization algorithm cares with orientations and fiber volume fractions of unidirectionally reinforced composite material layers. In the microscopic level, the goal is to define the microstructure of the layers by determining the cross-sectional size and shape of the reinforcement fibers. Both macro/micro scales are coupled by a resource constraint and interchange derivative information. The objective is to minimize compliance under a total fiber volume fraction constraint.

A previous work in this line [2] optimized a laminated composite representing it by a 3D finite element model, where each layer was treated as a group of such elements. In each of those layers, a unit cell of microstructure was defined by topology optimization in accordance with a hierarchical optimization approach. However, in the present case, this unit cell is considered simply as a portion of matrix reinforced by a piece of fiber whose cross-sectional dimensions will be defined by optimization. These fibers' cross sections are constrained to be elliptical, but with variable size and semi-axes aspect ratio. The present approach is more restrictive, since the general topology layout is predefined, but interesting in face of the cross-sectional shape of fibers more commonly available for composites fabrication. Moreover, the layered composites are here treated by finite elements based on laminated plate/shell theory.

The variation of the size and shape of the fibers is here considered by means of response surfaces for the constitutive parameters of an unidirectional composite lamina in terms of the fiber dimensions. Such surfaces are built upon function and derivative information [3] of constitutive parameters, evaluated from material microstructural models using asymptotic homogenization techniques [4]. The layers' orientations are chosen using the discrete material optimization (DMO) approach [5], where lists of candidate materials are interpolated by weighting functions, whose values are to be determined by optimization.

Results in laminated plates show the influence of the reinforcement fibers' shape and volume fraction in the global behavior of the test structures. It is shown that the present optimization procedure permits, in many cases, to improve the global behavior of structures when elliptical fibers are used to reinforce layers of a laminate. The optimal microstructures obtained are strongly influenced by the global loading conditions considered. In a final part, microstructural stresses are calculated in a laminated plate whose microstructure was already optimized, using results from asymptotic homogenization, in order to assess stress concentrations induced in the matrix by the fibers of distinct shapes.

2. Keywords: hierarchical optimization, laminated composites, microstructure of composites

3. Hierarchical Problem

This work has the goal of investigating by optimization the influence of microstructural details in the stiffness global behavior of laminated composites. The problem to solve is a compliance minimization (maximization of stiffness), which is stated in Eq.(1) and illustrated with the aid of Fig.1, using a hierarchical optimization [1] approach.

$$\min_{\substack{w_j(\mathbf{x}),\rho_i \\ \sum_i \rho_i V_i \leq V_f}} \quad \min_{\substack{a_i,b_i \\ \rho_i = \pi a_i b_i}} \quad c = \mathbf{f}^T \mathbf{u} \tag{1}$$

In Eq.(1), the outer minimization is the one in the laminate macroscopic level where interpolation weights $w_j(\mathbf{x})$ are related to choice of laminae orientations θ_i by the DMO method [5] and the ρ_i are reinforcement fiber volume fractions over the layers of the laminate. It is subjected to a resource constraint in the total fiber volume fraction of the laminate V_f. The inner minimization is at the microstructural

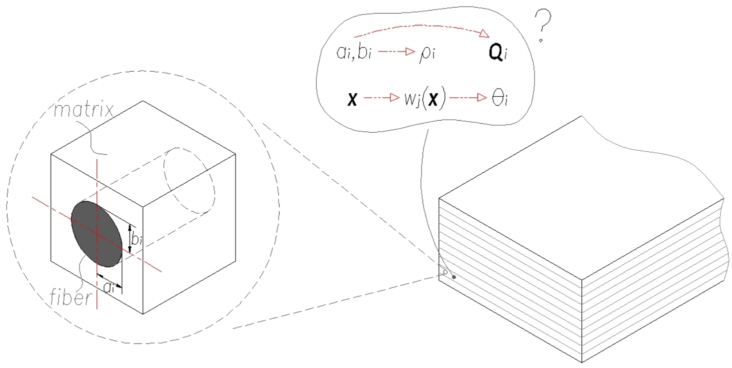

Figure 1: Microstructure unit cell of laminate layers. Parameters to be determined by the compliance minimization.

level of the laminate, where the parameters a_i,b_i define the cross section of the reinforcement fiber at the microstructure unit cell of Fig.1, for each of the layers of the laminate. It is subjected to equality constraints that establish the relation between the a_i,b_i and the ρ_i, therefore connecting both levels of the problem.

Furthermore, in the examples to be shown, the goal is to optimally characterize over the layers the following material interpolation:

$$\bar{\mathbf{Q}}_i(\mathbf{x}, a_i, b_i) = w_1(\mathbf{x})\bar{\mathbf{Q}}(a_i, b_i)_{-45} + w_2(\mathbf{x})\bar{\mathbf{Q}}(a_i, b_i)_0 + w_3(\mathbf{x})\bar{\mathbf{Q}}(a_i, b_i)_{45} + w_4(\mathbf{x})\bar{\mathbf{Q}}(a_i, b_i)_{90} \quad (2)$$

This is a DMO interpolation, where $\bar{\mathbf{Q}}_i$ is the resultant stiffness matrix of a lamina and the $\bar{\mathbf{Q}}_\theta$ are candidate materials, in this case the same material variable with the microstructural parameters a_i,b_i at the orientations $\theta = -45/0/45/90$ degrees.

The stiffness variation with the a_i,b_i is taken into account in the problem by appropriate response surfaces [3, 6] made upon asymptotic homogenization [4] data. An example is shown in Fig.2, counting with graphs of the response surface developed for the in-plane shear stiffness of an orthotropic single lamina, named here Q_{66}. All the other stiffnesses that characterize such lamina were modeled in the same way.

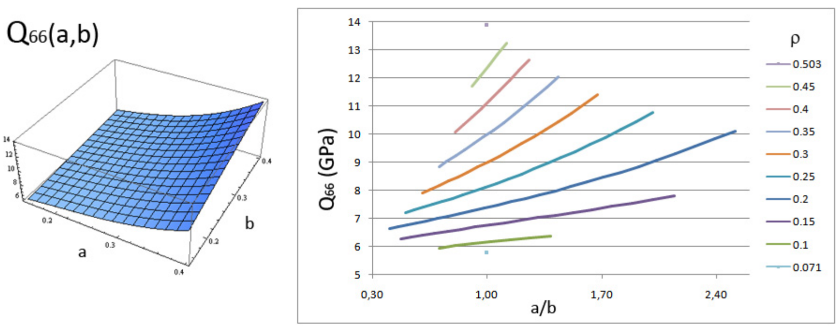

Figure 2: Plots for the response surface developed for Q_{66} in terms of a_i,b_i and a_i/b_i for constant volume fractions ρ_i.

4. Optimization Results

A hierarchical solution scheme [6] is employed in the solution of the optimization problem in Eq.(1), for laminated plates and several load cases and total fiber volume fractions $V_f = 0.15$ to 0.40. Some of the results obtained are shown in Fig.3 to 5. In such results, fibers of elliptical cross-sections were found in many cases, together with fibers of circular cross-sections.

The Fig.3 shows the results for a 7-layer rectangular plate under a traction-bending loading. Due to the loads applied, the neutral surface is expected to be shifted in relation to the plate mid-surface, somewhere on the upper half of the laminate, having the lower laminae more loaded. In terms of microstructure, these more loaded layers had the trend to assume as much fiber material as possible in accordance to V_f, also in all the cases. The cross-sections of the fibers ended up all circular. The orientation chosen for all the layers was $\theta_i = 0°$, in all the cases.

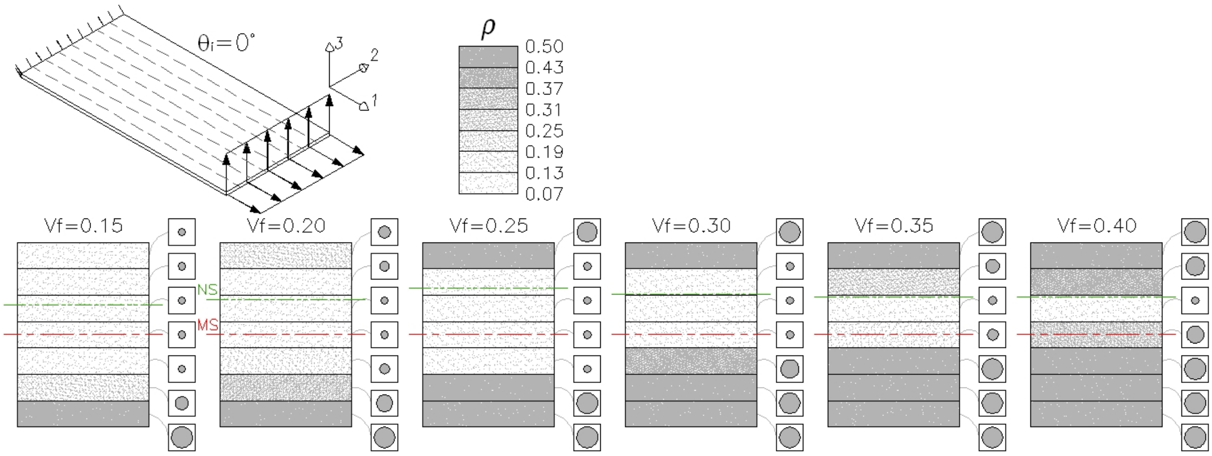

Figure 3: Rectangular plate traction-bending case for several volume fractions V_f. The neutral surface NS is shifted in relation to the plate mid-surface MS.

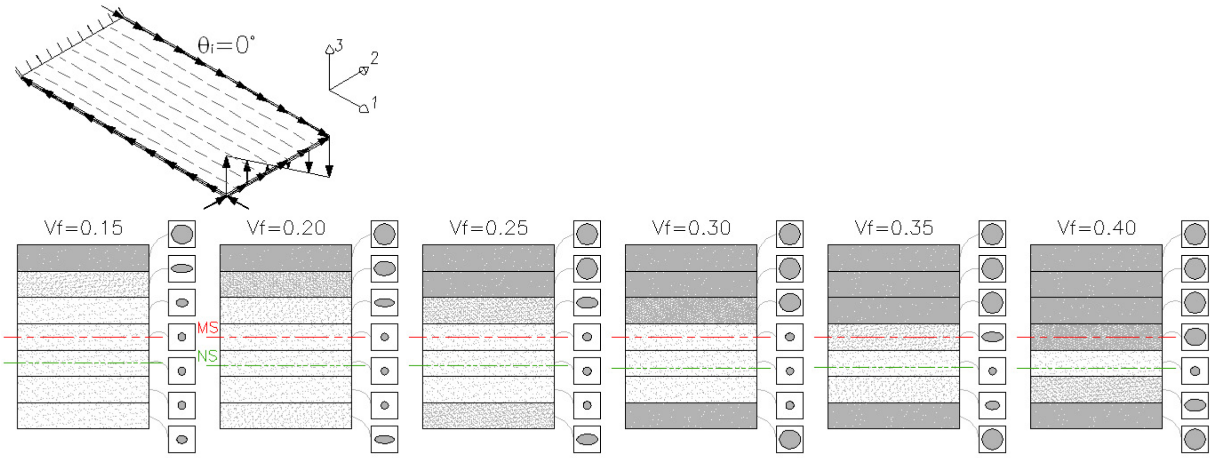

Figure 4: Rectangular plate shear-torsion case for several volume fractions V_f. The neutral surface NS is shifted in relation to the plate mid-surface MS.

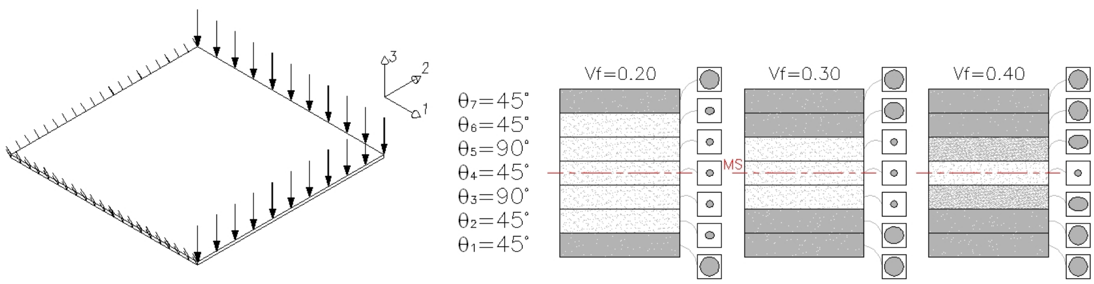

Figure 5: Square plate bending case for several volume fractions. MS is the plate mid-surface.

The Fig.4 shows the results for the same laminated plate under a shear-torsion loading. The loads applied also promote a shift in the neutral surface of the plate in relation to its mid-surface. The upper layers are the more loaded this time. In terms of microstructure, such layers assumed again as much fiber material as possible in accordance to V_f. Some of the fibers' cross-sections converged to circular shapes and other to elliptical. The orientation chosen for all the layers was $\theta_i = 0°$, in all the cases.

The Fig.5 shows the results for a 7-layer square laminated plate under bending loads. This time the neutral surface of the plate coincides with its mid-surface. Due to this, in terms of microstructure, the plate outer layers assumed as much fiber material as possible in accordance to V_f, and the fiber distribution is symmetrical with respect to the mid-surface. Some of the fiber cross-sections converged to circular shapes and other to elliptical. The orientations θ_i chosen for all the layers were this time distinct.

The obtainment of fibers of elliptical cross-sections at the results of Fig.4 and Fig.5 can be explained due to the fact that such fibers provide to the composite layers a higher in-plane shear stiffness, and also a higher stiffness in the direction perpendicular to the fibers [6]. These characteristics can be exemplified with the aid Fig.2, where it is shown the variation with a_i/b_i of the in-plane shear stiffness of an orthotropic single lamina, named there Q_{66}. It can be seen that fibers of higher a_i/b_i (elliptical) increase Q_{66} for the same fiber volume fraction ρ_i. A similar effect is observed to the lamina stiffness perpendicular to the fibers, not shown here for brevity.

5. Microstructural Stresses Evaluation

The optimization results shown in the last section were obtained by minimization of compliance. Now, one of these designs will be evaluated with the aim of assessing the influence of the elliptical fibers in the microstructural stresses at the composite layers' matrix. The design chosen to this evaluation is from the shear-torsion case with $V_f = 0.15$. It will be compared to a design with circular fibers and constant fiber volume fractions within the layers, but resulting in the same $V_f = 0.15$, which is a microstructural design representing the ones commonly found in practice. The optimized design is about 50% more stiff in comparison to the regular design. The parameters used in the comparison are the von-Mises failure criteria in the fiber and the Drucker-Prager failure criteria in the matrix, evaluated upon stresses within the microstructure calculated using results from asymptotic homogenization [4]. These parameters are plotted in a normalized form (in such a way that when reaching the value of 1 failure occurs), in the Fig.6 and 7.

It can be observed that the fiber is always far from failure in both the optimized and the uniform microstructure design. However, the matrix is the most loaded as could be expected, since it is less stiff. The maximum failure parameters observed layer to layer in the composites' matrix are highlighted in Tab.1. From this table, it can be seen that the optimized design performed better with respect to matrix failure with exception to the layer 6, where the elliptical fiber cross-section of high aspect ratio inserted stress concentrations in the matrix.

Table 1: Maximum Drucker-Prager failure indices in the matrix (from Fig.6/7).

layer	optimized design	uniform design	opt./unif. ratio
7	1.037×10^{-2}	7.186×10^{-2}	0.144
6	1.017×10^{-1}	5.708×10^{-2}	1.782
5	3.412×10^{-2}	4.232×10^{-2}	0.806
4	1.670×10^{-2}	2.755×10^{-2}	0.606
3	7.468×10^{-3}	1.283×10^{-2}	0.582
2	2.103×10^{-3}	2.347×10^{-3}	0.896
1	1.300×10^{-2}	1.644×10^{-2}	0.791

6. Conclusions

The hierarchical optimization approach here employed showed that working in the microstructural details of the laminate layers brought improvements to the composites studied, in terms of higher stiffness. This study opens space to the investigation of fibers of more varied cross-sections. Effects of microstructural stresses have to be more deeply investigated in such optimized designs and maybe be properly included in the optimization problem for the microstructure of the composites.

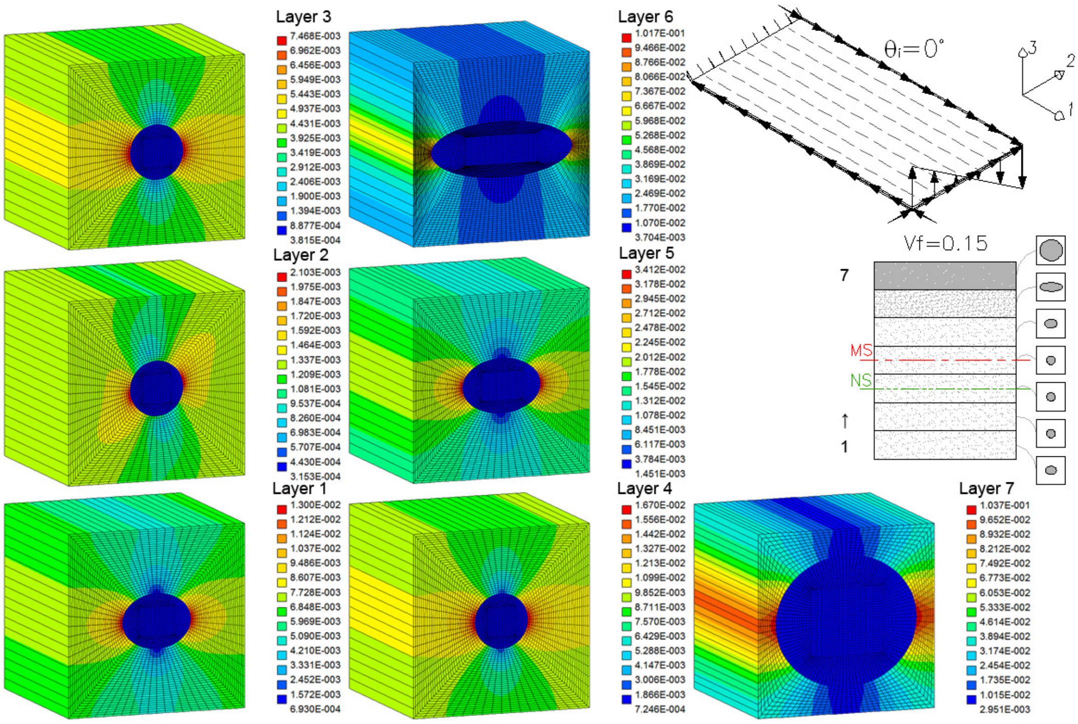

Figure 6: Failure parameters in the optimized design obtained for the rectangular plate in the shear-torsion case with $V_f = 0.15$.

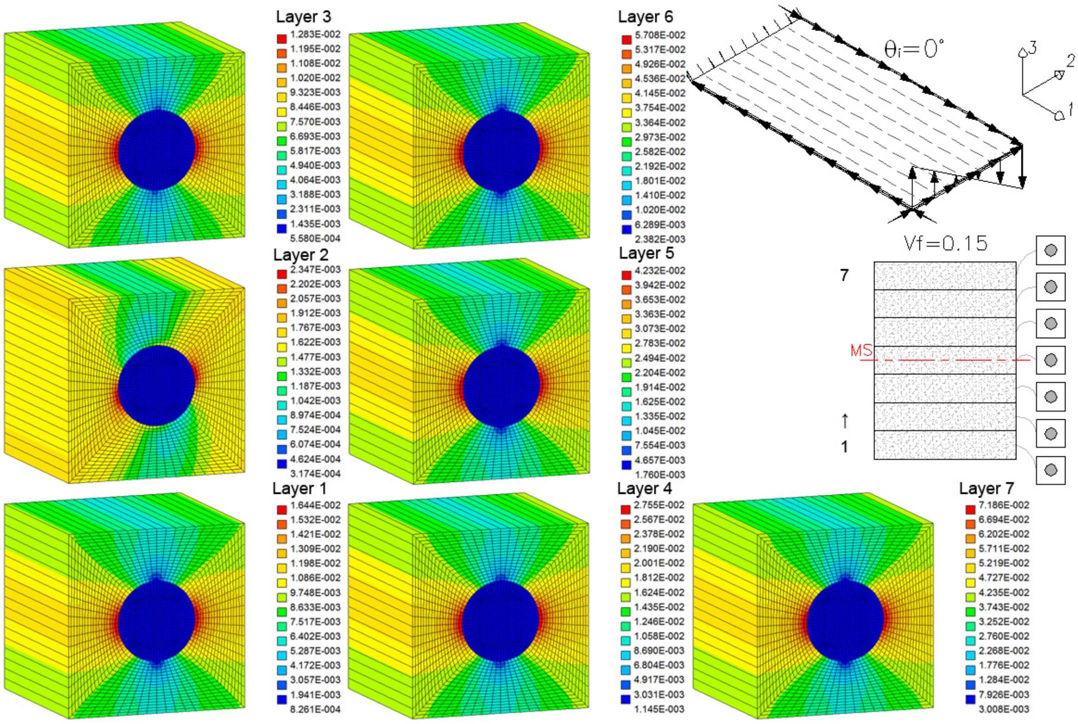

Figure 7: Failure parameters in a uniform design for the rectangular plate in the shear-torsion case with $V_f = 0.15$.

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