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An enhanced aggregation method for stress-constrained topology optimization problems

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1. Abstract
For the topology optimization with local stress constraints on all the elements, conventional "global constraint methods" may introduce high nonlinearity to the global constraint function and cannot adequately control the local stress state. This study aims to develop an enhanced aggregation method that not only effectively reduces the number of constraints in the search process, but also ensures sufficient accuracy of the solution for the SIMP-based topology optimization. By introducing a new reduction parameter into the Kreisselmeier-Steihauser (K-S) function, a new general K-S formulation is suggested for aggregating the local constraints that are active at the optimum. The approximation of the general K-S function to the feasible region restricted by active constraints is proved to be highly accurate even when the aggregation parameter takes a relatively small value. Numerical difficulties, such as high nonlinearity and serious violation of local constraints that may be exhibited by the original K-S function, are thus effectively alleviated. In the considered topology optimization problem, the material volume is to be minimized under local von Mises stress constraints imposed on all the finite elements. An enhanced aggregation algorithm based on the general K-S function, in conjunction with a "removal and re-generation" strategy for selecting the active constraints, is then proposed to treat the stress-constrained topology optimization problem. Several topology optimization examples with large number of local stress constraints are also presented for illustrating the efficiency of the enhanced aggregation method. It is shown that excellent solutions that are fully stressed without any violation on local stress constraints can be obtained at reasonable computational efforts.
2. Keywords: Topology optimization, aggregation method, stress constraint, general K-S function.

3. Introduction
Continuum topology optimization provides designers with more design freedom, and it has been greatly developed and applied to various engineering fields. An overwhelming majority of topology optimization studies focus on minimizing the structural compliance with a given amount of material, i.e. the compliance minimization problem. For an overview of this field, interested readers are referred to the monograph by Bendsøe and Sigmund [1], and to the review papers by Eschenauer and Olhoff [2] and Rozvany [3].
In the topology optimization of stress constrained structures, strength failure is required to be avoided at every material point. For a discrete finite element model, the number of stress constraints in the optimization problem becomes very large since one needs to impose local stress constraints on all the elements. To date, unfortunately, such a problem is still too expensive and time-consuming to be directly solved by existing gradient-based optimization algorithms, even for a medium-scale problem (e.g. containing 1000 nonlinear constraints). Therefore, the large-scale nature of local stress constraints presents a major obstacle to the practical application of stress constrained topology optimization.
For tackling this difficulty, two main strategies, namely the "active-set method" and the "global constraint method", for reducing the number of stress constraints have been adopted in existing studies. In the "active-set method", only the potentially critical stress constraints are taken into account at each iteration step [4]. By discarding a fraction of the stress constraints that are far from active, this method saves the computational cost to some extent. However, the selected active constraint set at the end of the optimization usually vary from 10%~40% of the total number of constraints and may still be large when a fine discretization is used. In this sense, the "active-set method" does not bring the required efficiency. By coupling a global compliance enforcement with local stress constraints, Bruggi and Duysinx [5] proposed a selection method leading to further reduction in the number of active constraints. The "global constraint method" is more efficient since it aggregates all the local stress constraints into one Kreisselmeier-Steihauser (K-S) or p-norm global constraint function. Such a strategy has been employed by many authors, for instance Yang and Chen [6], Duysinx and Sigmund [7], París et al. [8], Qiu and Li [9]. The main disadvantages of the global constraint method are the high nonlinearity of the global function and a weaker control on the stress level. The high nonlinearity may cause unstable convergence during the optimization procedure and the weaker control on the stress level may lead to a serious violation of some local stress constraints. The technique of "block aggregation" [10] by using more global constraints to reduce the number of constraints involved in a single global function would be helpful in avoiding these disadvantages, but

not completely. The adaptive normalization scheme proposed by Le et al. [11] works well in controlling the maximum stress. However, it is difficult to obtain the exact optimal solution since this scheme may prevent the local stresses of some elements to reach the permissible strength limit.

This paper aims to develop an enhanced aggregation method that not only effectively reduces the number of constraints in the search process, but also ensures sufficient accuracy of the solution for the SIMP-based topology optimization. As an adaptation of the conventional K-S function, a new general mathematical definition of the global function is suggested for aggregating the local constraints that are active at the optimum. Based on the SIMP model for linking the material properties and the relative density design variables, the topology optimization problem is typically formulated as to minimize the total material volume under von Mises stress constraints on all elements. Then, by using the "removal and re-generation" strategy for selecting the active constraints, an enhanced aggregation algorithm based on the general K-S function is proposed to solve the large-scale constrained optimization problem. The design sensitivity analysis is implemented with the adjoint variable method and the design variables are updated by the Method of Moving Asymptotes (MMA) [12]. Finally, the validity of the present method and the proposed optimization algorithm is demonstrated by several numerical examples.

4. Stress constrained topology optimization problem

In this study, we consider isotropic materials with equal tensile and compressive strength limits. The topology optimization problem is stated as to determine the optimal material distribution that uses the minimum material volume but can ensure von Mises stress constraints. Given a design domain Ω_{des}, the optimization problem is expressed by

$$\begin{aligned} \min_{c(x)} \quad & V = \int_{\Omega_{\text{des}}} c(x)\mathrm{d}\Omega \\ \text{s.t.} \quad & s^{\text{vM}}(x) \le s^{\text{lim}} \quad \text{if } c(x) = 1 \\ & c(x) \in \{0, 1\} \end{aligned} \tag{1}$$

where x is an arbitrary material point in the design domain, $s^{\text{vM}}(x)$ is the von Mises stress defined on the solid material point, and s^{lim} is the material strength limit. The discrete 0-1 valued topology variables $c(x)$ describe the material distribution, as

$$c(x) = \begin{cases} 1 & \text{if } x \in \Omega_{\text{solid}} \\ 0 & \text{if } x \in \Omega_{\text{des}} \setminus \Omega_{\text{solid}} \end{cases} \tag{2}$$

where $\Omega_{\text{solid}} \subseteq \Omega_{\text{des}}$ denotes the domain occupied by solid material.

In order to solve the discrete optimization problem (1) by commonly-used mathematical programming algorithms, the SIMP approach [13, 14] is adopted to replace the discrete valued function $c(x)$ with an continuous density function $r(x) \in [0,1]$. In the approach, a power-law relationship between the macroscopic elasticity tensor $\mathbf{D}(x)$ and the density function $r(x)$ is given by

$$\mathbf{D}(x) = r(x)^p \mathbf{D}_0 \tag{3}$$

where p is a prescribed penalization factor, $\mathbf{D}_0$ is the constitutive elasticity tensor of the solid material. In this study, the penalization factor is set to be a usually used value $p = 3$.

In topology optimization problems considering local stress constraints, it is important to define an appropriate interpolation scheme for the stress state of an arbitrary intermediate density material. For this purpose, a "local stress interpolation" that relates the micro-lever local von Mises stress to the macroscopic homogenized deformations was proposed by Duysinx and Bendsøe [4]. It is expressed by

$$\boldsymbol{\sigma}(x) = \frac{\mathbf{D}(x)\bar{\boldsymbol{\varepsilon}}(x)}{r(x)^q} = r(x)^{p-q}\mathbf{D}_0\bar{\boldsymbol{\varepsilon}}(x) \tag{4}$$

where $\boldsymbol{\sigma}(x)$ and $\bar{\boldsymbol{\varepsilon}}(x)$ represent the local stress and the macroscopic average strain of a material point, respectively. The exponent $q > 1$ is a real number. In our work we define $q = 2.5$, which is an appropriate value suggested by existing studies on stress constrained topology optimization.

In the implementation of the finite element method, the 2D design domain is discretizated into NE 4-node plane stress elements. For simplification purposes, we choose the strain in the element centroid as the elemental average strain $\bar{\boldsymbol{\varepsilon}}_e$, that is

$$\bar{\boldsymbol{\varepsilon}}_e = \mathbf{B}_e^c \mathbf{u}_e \tag{5}$$

where $\mathbf{B}_e^c$ is the strain-displacement matrix at the centroid of the eth element, and $\mathbf{u}_e$ is the elemental

displacement vector.

Taking Eqs.(4) and (5) into consideration, the equivalent von Mises stress s_e^{vM} for the *e*th element can be evaluated as

$$\begin{aligned} s_e^{\mathrm{vM}} &= \sqrt{\left(\boldsymbol{\sigma}_e\right)^{\mathrm{T}} \mathbf{V} \left(\boldsymbol{\sigma}_e\right)} = \sqrt{\left(r_e^{p-q} \mathbf{D}_0 \mathbf{B}_e^c \mathbf{u}_e\right)^{\mathrm{T}} \mathbf{V} \left(r_e^{p-q} \mathbf{D}_0 \mathbf{B}_e^c \mathbf{u}_e\right)} \\ &= r_e^{p-q} \sqrt{\mathbf{u}_e{}^{\mathrm{T}} \mathbf{M}_0 \mathbf{u}_e} = \sqrt{r_e \mathbf{u}_e{}^{\mathrm{T}} \mathbf{M}_0 \mathbf{u}_e} \end{aligned} \tag{6}$$

where $\mathbf{M}_0 = \left(\mathbf{D}_0 \mathbf{B}_e^c\right)^{\mathrm{T}} \mathbf{V} \mathbf{B}_e^c \mathbf{D}_0$. For a plane stress state, the constant matrix $\mathbf{V}$ is given by

$$\mathbf{V} = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \tag{7}$$

Therefore, the stress-based topology optimization problem (1) with the finite element model is expressed as

$$\begin{aligned} \min_{\boldsymbol{\rho}} \quad & V = \sum_{e=1}^{NE} r_e V_e \\ \text{s.t.} \quad & R_e\left(s_e^{\mathrm{vM}}\right) = \frac{\sqrt{r_e \mathbf{u}_e{}^{\mathrm{T}} \mathbf{M}_0 \mathbf{u}_e}}{s^{\lim}} - 1 \le 0 \qquad \left(e = 1, 2, \mathbf{K}, NE\right) \\ & r_{\min} \le r_e \le 1 \qquad \left(e = 1, 2, ..., NE\right) \end{aligned} \tag{8}$$

where $\boldsymbol{\rho} = \left\{r_1, r_2, \mathbf{K}, r_{NE}\right\}^{\mathrm{T}}$ is the vector of design variables; V_e is the element volume; $\mathbf{u}$ is the displacement vector; $g_e\left(s_e^{\mathrm{vM}}\right)$ is the dimensionless stress constraint imposed on the *e*th element; $r_{\min} = 10^{-2}$ is the lower limit of the design variables for avoiding singularity of the elemental stiffness matrices caused by zero density values.

5. General K-S function

Global constraint methods in structural topology optimization are usually based on an approximate strategy that constructs a differentiable global constraint equivalent to a set of "local" constraints. Without loss of generality, the optimization problem is stated in the "standard" form

$$\begin{aligned} \min_{\mathbf{x}} \quad & f\left(\mathbf{x}\right) \\ \text{s.t.} \quad & R_i\left(\mathbf{x}\right) \le 0, \quad i = 1, 2, \mathbf{L}, N_g \\ & \mathbf{x}^{\mathrm{L}} \le \mathbf{x} \le \mathbf{x}^{\mathrm{U}} \end{aligned} \tag{9}$$

where $\mathbf{x}$ is the vector of design variables, $f\left(\mathbf{x}\right)$ is the objective function to be minimized, N_g is the number of local constraints $R_i\left(\mathbf{x}\right)$, $\mathbf{x}^{\mathrm{L}}$ and $\mathbf{x}^{\mathrm{U}}$ are the lower and upper bounds of the design variables, respectively.

Typically, an aggregation formulation using the K-S function [15] is expressed as

$$G\left(\mathbf{x}\right) = \frac{1}{h} \ln\left\{\sum_{i=1}^{N_g} \exp\left[h R_i\left(\mathbf{x}\right)\right]\right\} - \frac{1}{h} \ln\left(N_g\right) \tag{10}$$

where $h > 0$ is the aggregation parameter and $\ln\left(\cdot\right)$ denotes the natural logarithm.

We note that a drawback of the above-mentioned global constraint method is the need to reduce the approximation error "$\ln\left(N_g\right)/h$". On the one hand, increasing h to seek a more accurate solution in the optimization problem would not be practically applicable in many cases, since a large value of h used to penalize the violation of local constraints introduces high non-linearity to the global constraint function. On the other hand, techniques for reducing the local constraint number in a global function based on the "block aggregation" [10] or the "regional stress measure" [11] may also be inefficient when the total number of local constraints is very large. It is therefore desirable to narrowing the gap between the maximum constraint function value and the aggregation function in the case of a finite aggregation parameter h and a large number N_g.

In order to make the problem (9) tractable, we first define an "optimal active set" $\mathrm{A}\left(\mathbf{x}^*\right)$ collecting the indices of the local constraints at which equality holds on the optimum $\mathbf{x}^*$, that is:

$$\mathrm{A}\left(\mathbf{x}^*\right) = \left\{i \in \left\{1, 2, \mathbf{L}, N_g\right\} \middle| R_i\left(\mathbf{x}^*\right) = 0\right\} \tag{11}$$

If $\mathrm{A}\left(\mathbf{x}^*\right)$ were known in advance, the optimum $\mathbf{x}^*$ could be obtained by solving the following problem

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & R_i(\mathbf{x}) = 0, \quad \text{if } i \in \mathrm{A}(\mathbf{x}^*) \\ & R_i(\mathbf{x}) < 0, \quad \text{if } i \notin \mathrm{A}(\mathbf{x}^*) \\ & \mathbf{x}^{\mathrm{L}} \le \mathbf{x} \le \mathbf{x}^{\mathrm{U}} \end{aligned} \tag{12}$$

It is crucial to identify the active constraints $R_i(\mathbf{x}) = 0$ $(i \in \mathrm{A})$ in the solution of the optimization problem (12). Inactive constraints $R_i(\mathbf{x}) < 0$ $(i \notin \mathrm{A})$ mainly affect the searching path of the optimization process. Therefore, it is beneficial to treat them differently in a global constraint method. Herein, a general formulation of the K-S function is formulated for the aggregation of active constraint functions as

$$G_{\mathrm{L}}(\mathbf{x}, b) = \frac{1}{h}\ln\left\{\sum_{i\in \mathrm{A}} \exp\left[h R_i(\mathbf{x})\right]\right\} - \frac{1}{h}\ln\left\{\sum_{i\in \mathrm{A}}\left[1 + b R_i(\mathbf{x})\right]\right\} \tag{13}$$

where $0 \le b < h$ is the reduction parameter which represents the reduction degree of the feasible region to $G(\mathbf{x})$. Note that Eq.(13) degenerates into Eq.(10) when we set $b = 0$.

The properties of the general K-S function can be illustrated from a simple example in the two-dimensional design space. As shown in Fig. 1, two constraints $R_1(\mathbf{x}) = 2 - x_1 - x_2 \le 0$ and $R_2(\mathbf{x}) = -x_1^3 + x_2 \le 0$ are aggregated by the general K-S function with different values of b. The aggregation parameter is set to be $h = 7$. When b is equal to zero, the global feasible region defined by the general K-S function approximately matches the original feasible region (denoted by the shade domain). When b tends to h, the global feasible region tends to the point $\mathbf{x}^*$, at which both constraints are active, *i.e.* $R_1(\mathbf{x}^*) = 0$ and $R_2(\mathbf{x}^*) = 0$.

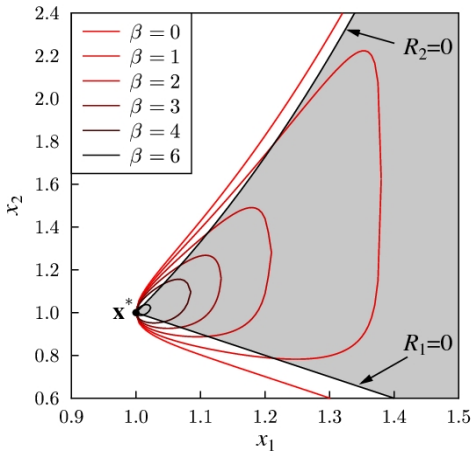


Fig. 1 Illustration of the general K-S function with different values of b

6. Enhanced aggregation method

The optimization problem defined by Eq.(8) is essentially a large-scale constrained one. The number of strength constraints is equal to the number of finite elements, which may be huge for most practical engineering structures. Since the direct handing of large-scale constraints by existing optimization algorithms may be very time-consuming, the "global constraint method", in which all or part of the strength constraints are approximately equivalent to a global constraint in the *Kreisselmeier-Steihauser* (K-S) or *p-norm* formulation, has gained considerable favour in recent years. However, the global constraint method will introduce high nonlinearity to the global function and cannot ensure sufficient approximation accuracy of constraints. Other alternative approaches such as the active-set method and the global compliance enforcement method, can avoid these disadvantages, but less efficiently in computational effort.

We hereby develop an enhanced aggregation method to efficiently solve this large-scale constrained problem. The method is based on the aggregation of active strength constraints into global ones by using a general K-S function, which is expressed by

$$G_{\mathrm{L}}(\mathbf{x}, b) = \frac{1}{h}\ln\left\{\sum_{e\in \mathrm{A}} \exp\left[h R_e(\mathbf{x})\right]\right\} - \frac{1}{h}\ln\left\{\sum_{e\in \mathrm{A}}\left[1 + b R_e(\mathbf{x})\right]\right\} \tag{14}$$

In conjunction with a "removal and re-generation" strategy for selecting the active constraints, the enhanced aggregation method allows the optimization solver to find the optimum without introducing the high nonlinearity to global functions. The algorithm consists of four main steps:

Step 1: Initialization and preparation

Start by defining initial design domain and generating mesh. Set the iteration step $k=0$. Input initial design variables $\boldsymbol{\rho}^{(0)}$, the relaxation parameter e, the aggregation parameter h, and the maximal number M of constraints involved in one general K-S function.

Step 2: Finite element analysis

Carry out the FE analysis. Evaluate objective function $V^{(k)}$, stress constraints $R_e^{(k)}$ on each element. Sort the total of NE strength constraints in descending order $R_1^{(k)} \geq R_2^{(k)} \geq \ldots \geq R_{NE}^{(k)}$. Evaluate the potentially optimal active set $\mathrm{A}^{(k)}$ by

$$\mathrm{A}^{(k)} = \left\{ i \,\middle|\, R_i^{(k)} \geq \Psi \text{ and } i \leq NE \right\} \tag{15}$$

where the boundary value Ψ is set as -0.3 and i is the strength constraint index after sorted.

Step 3: Aggregating strength constraints

If $R_i^{(k)} \geq 0.01$ and $i < 100$, keep the original strength constraint as it was.

Else if $i \in \mathrm{A}^{(k)}$, divide involved strength constraints into groups (denoted by $\mathrm{B}_1^{(k)}, \mathrm{B}_2^{(k)}, \ldots, \mathrm{B}_j^{(k)}, \ldots$) and aggregate each group (contains M constraints) by a general K-S function (14); Determine the reduction parameter b_j for the *j*-th aggregation function in terms of the following empirical procedure

$$b_0 = \left\{ \sum_{i \in \mathrm{B}_j^{(k)}} \left[\exp\left(h R_i^{(k)}\right) - 1 \right] - \max_{i \in \mathrm{B}_j^{(k)}} \left[\exp\left(h R_i^{(k)}\right) \right] + 1 \right\} \Bigg/ \left(\sum_{i \in \mathrm{B}_j^{(k)}} R_i^{(k)} - \max_{i \in \mathrm{B}_j^{(k)}} R_i^{(k)} \right) \tag{16}$$

$$b_j = \min\left(\max\left(b_0, 0\right), 0.99h\right)$$

Otherwise, divide involved strength constraints $R_i^{(k)}$ ($i \notin \mathrm{A}^{(k)}$) into groups. Set the reduction parameters to be zero and aggregate each group constraints by the general K-S function (14).

Step 4: Updating design variables

Perform the sensitivity analysis in next section. Update design variables by the Matlab implementation of the Method of Moving Asymptotes. For avoiding the checkerboard patterns, modify current design variables by the following density filter

$$r_{e,\mathrm{modify}} = \left(\sum_{m=1}^{N} r_m H_{e,m} \right) \Bigg/ \left(r_e \sum_{m=1}^{N} H_{e,m} \right) \quad (e = 1, 2, \mathbf{K}, N) \tag{17}$$

where $H_{e,m}$ is the weight factor defined by $H_{e,m} = \max\left(10^{-6}, r_{\min} - \mathrm{dist}\left(e,m\right)\right)$, the operator $\mathrm{dist}\left(e,m\right)$ is the distance between the centers of element e and element m, l_e is the element length, and $r_{\min}$ is the filter radius, which is chosen as 1.1 in our numerical examples.

Check the termination criterion. If it is satisfied, stop the algorithm process; otherwise, set $k = k+1$ and go to Step 2.

7. Examples

7.1. Desing of a L-shape structure

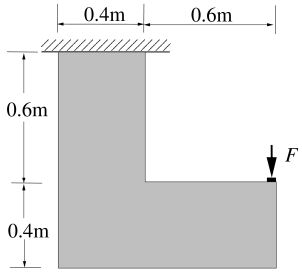
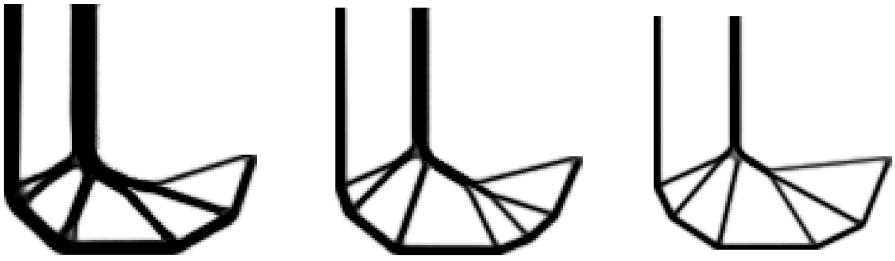


Fig. 2 Design domain of a L-shape structure.

The geometry and dimensions of the design domain are shown in Fig.2. The structure is clamped at the top edge,

and a downwards force of $F = 2.5\text{N}$ is applied to the upper tip of the right side. Using an element size of 10mm, the design domain is discretized by 6400 elements and 6601 nodes. The design objective is to minimize the material volume considering different limits of material strength. By using the proposed aggregation method, the constraint number in each aggregation function is set as $M = 50$.

The optimal solutions obtained by the proposed method with three different material strength limits, $s^{\lim} = 12,15$ and 20kPa, are shown in Fig.3(a-c), respectively. The resulting optimal solutions are all featured by a round inner corner, which plays a key role in eliminating the stress concentration. For a lower strength limit of material (see Fig.3(a)), the required material volume ratio and the length of the round inner corner are relatively larger so that the stress constraints are satisfied. As the material strength limit increases (see Fig.3(b, c)), not only less material is used, but also the topological details are different. It can be also seen from Table 1 that, by using the enhanced aggregation method, roughly equivalent computing times (about half an hour) for different cases are spent in obtaining the optimal design.



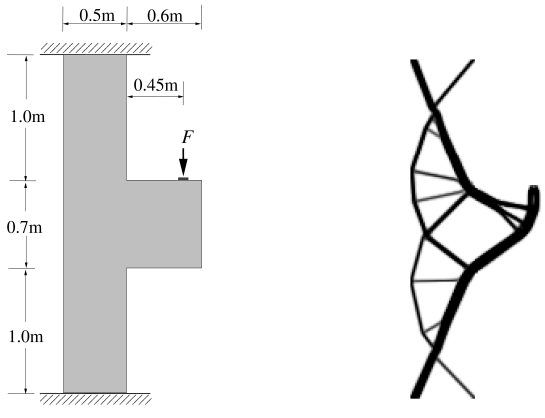
(a) (b) (c)

Fig. 3 Optimal topologies for the L-shape structure.
(a) $s^{\lim} = 12\text{kPa}$. (b) $s^{\lim} = 15\text{kPa}$. (c) $s^{\lim} = 20\text{kPa}$.

Table 1 Optimization results for the L-shape structure with different strength limits

Case	Material volume ratio (%)	Number of iterations	Total CPU time (h)	Maximum local constraint violation
$s^{\lim} = 12\text{kPa}$	41.30	104	0.51	0.0160
$s^{\lim} = 15\text{kPa}$	29.53	106	0.49	0.0105
$s^{\lim} = 20\text{kPa}$	21.00	117	0.48	0.0119

7.2. Design of a corbel structure



(a) (b)

Fig. 4 Design domain and optimal topologies for the corbel structure.
(a) Design domain. (b) Stress constrained optimization.

As shown in Fig.4(a), a corbel located at middle height of the column is considered as the initial design domain of the topology optimization. The column is clamped on the top and bottom side. A concentrated load of $F = 5\text{N}$ is applied on the upper edge of the corbel. The strength limit of solid material is $s^{\lim} = 10\text{kPa}$. Using the element size of 12.5mm, the design domain is meshed by 11328 4-node plane stress elements. The number of constraints in each aggregation function is $M = 50$.

For the stress constrained problem, it takes 113 iterations and only 1.51 hours (most of the time is spent in the finite element analysis and sensitivity analysis) to find the optimal topology by using the proposed method (as shown in Fig.4(b)). The material volume fraction ration at final step is 24.58%. This example shows that the proposed method is able to yield satisfying solutions when dealing with large-scale stress constrained problems (with over ten thousand local stress constraints).

8. Conclusions

This paper proposes an enhanced aggregation method for efficiently obtaining a relatively accurate solution to large-scale stress constrained topology optimization problems. The proposed method is based on aggregation of local stress constraints into global ones by a general formulation of K-S function. An interesting property of this general K-S function is that it provides a highly accurate approximation to the feasible region when all local constraints are active at the optimum. In conjunction with the "removal and re-generation" strategy for determining the optimal active set during iterations, this property allows the optimization algorithm to find a relative accurate optimal solution without using a large aggregation parameter as required by the conventional global approach.

The proposed method can be considered as a useful supplement to existing stress-constrained topology design methods. In addition, it may be useful to extend the proposed approach to more complex design problems, *e.g.* multi-material structures that contain more than one stress constraint for each element, and contact problems that require mesh refinement to capture localized displacement fields.

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