Reliability-Based Robust Design Optimization Using the Probabilistic Robustness Index and the Enhanced Single Loop Single Vector Approach

Seong-Beom Jeong[1], **Gyung-Jin Park**[2]

[1] Hanyang University, Seoul, Republic of Korea, sbjung80@hanyang.ac.kr
[2] Hanyang University, Ansan, Republic of Korea, gjpark@hanyang.ac.kr

1. Abstract

Robust design optimization (RDO) is an engineering methodology for optimal design of products and process conditions that are less sensitive to system variations. It pursues an insensitive and conservative design when there are variations on design variables and/or parameters. To accomplish the purpose, various robustness indices for the objective function and constraints have been developed. However, RDO application to practical engineering structures still has some issues such as the limitations of the robustness indices and high computing cost. In this research, a new robustness index for the robustness of the objective function is proposed based on the probabilistic theory. An investigation is performed to identify the characteristics and the drawbacks of the previous studies, and useful information about the probabilistic robustness index is described. Also, this research presents a new RDO algorithm to improve efficiency. The objective function in the algorithm is expressed by the proposed robustness index. The single loop single vector approach using the conjugate gradient method, which has been developed for reliability-based design optimization, is applied to realize the feasibility of the constraints in the RDO process. Consequently, the proposed RDO technique seeks the best compromise between an insensitive and a reliable design. The proposed RDO method is applied to numerical examples and structural applications. The results are compared to previous methods. It shows that the proposed method is accurate and more efficient than the other methods.

2. Keywords: robust design optimization; probabilistic robustness index; single loop single vector approach using the conjugate gradient method

3. Introduction

In recent years, several methodologies have been developed to consider the uncertainties in conventional design optimization. The robustness concept is added to conventional optimization. Therefore, conventional design optimization methodologies have been modified to improve the performance of products. Robust design optimization (RDO) is a representative theory for design under uncertainty [1-6]. It deals with both the robustness of the objective function and the feasibility of the constraints.

RDO is often implemented by diminishing performance variability. Therefore, the robustness of the objective function tends to be emphasized more than that of the constraints because robust design pursues an insensitive design of the objective function. The robustness of the objective function can be achieved by reducing the change of the objective function with respect to the changes of the design variables. Therefore, robustness indices of the objective function pursue an insensitive design when there are variations on design variables and/or parameters. To accomplish the purpose, various robustness indices on the objective function have been proposed [1, 5, 7-12].

Robustness of the constraints means that all of the constraints are satisfied within the range of the variations for the design variables and/or parameters. Typical approaches of previous studies can be classified into three types: the worst case method, the penalty method, and the probabilistic method [5, 13-20]. The worst case method is an approach that finds the worst case having the maximum value of each constraint and satisfying the constraint in spite of the uncertainties. The penalty method defines a new constraint having a penalty term related to the uncertainties. In the probabilistic method, the constraints with uncertainties are replaced by the probabilistic constraints. To estimate the violation probability of the constraints, various reliability assessment approaches can be utilized. This method requires more complicated formulation than the other methods. Generally, it is well known as reliability-based design optimization (RBDO). The reliability of the constraints in RBDO is similar to the robustness of the constraints in RDO. It is calculated exactly by a probability theory in RBDO, whereas it is approximately calculated in the worst case method and the penalty method. The reliability-based robust design optimization (RBRDO) is known as a method which treats the constraints with uncertainties by using RBDO methods in RDO [21-24].

In this study, the formulation and the numerical method for RDO are addressed. An RDO algorithm is made with the newly proposed robustness index and the enhanced single loop single vector approach. First, a probabilistic robustness index is proposed to deal with the robustness of the objective function. It is derived from the probabilistic theory. By using the robustness index, the mean value and the variance of the objective function are simultaneously controlled. Also, the index does not need any parameters determined by the designer's intuition. Second, an effective RBDO method is utilized to achieve the feasibility of the constraints. The feasibility robustness is exactly guaranteed by using the RBDO method. Among the RBDO methods, the single loop single vector approach using the conjugate gradient method (SLSVCG), which is recently proposed by the authors, is employed to treat the constraints efficiently [19, 20].

By using the new algorithm, the robustness of the objective function is considered in the optimization process while the feasibility of the constraints is guaranteed with the specified target reliability. Two case studies are solved to demonstrate the effectiveness of the proposed algorithm. An artificial mathematical problem is defined to show the distinct efficiency of the utilized methods, and a practical engineering problem is employed for assessment of the methods in the application viewpoint. The results of the proposed method are compared with those from combinations of other methods for RDO.

4. Robust design optimization

The formulation of robust design optimization is represented as follows:

$$\begin{aligned}
&\text{Find} && \mathbf{b} \in R^n \\
&\text{to minimize} && [\mu_f(\mathbf{b},\mathbf{p}), \sigma_f(\mathbf{b},\mathbf{p})] \\
&\text{subject to} && g_{\mathrm{i}}(\mathbf{b},\mathbf{p}) \le \mathbf{0}, \quad i = 1,\cdots,r \\
& && g_{\mathrm{j}}(\mathbf{b}+\mathbf{z}^{\mathbf{b}}, \mathbf{p}+\mathbf{z}^{\mathbf{p}}) \le \mathbf{0}, \quad j = r+1,\cdots,m \\
& && \mathbf{b}_L \le \mathbf{b} \le \mathbf{b}_U
\end{aligned} \tag{1}$$

where $\mathbf{b}$ is the design variable vector, $\mathbf{p}$ is the design parameter vector, and $\mathbf{b}_L$ and $\mathbf{b}_U$ are the lower bound and the upper bound of the design vector $\mathbf{b}$, respectively. f is the objective function of the deterministic optimization, and g_i is the i th constraint. μ_f, σ_f are the mean and the standard deviation of the objective function, respectively. The noises of design variables and design parameters are represented by $\mathbf{z}^{\mathbf{b}}$ and $\mathbf{z}^{\mathbf{p}}$, respectively.

Some or all constraints can be the constraints with uncertainties. Then, the constraints are divided into deterministic constraints and constraints with uncertainties. r is the number of deterministic constraints and m is the total number of constraints.

4.1. Robustness of the objective function

In order to obtain the robust optimum, a design formulation with a multi-objective function has been widely used due to simple and easy application. The most common approach in multi-objective optimization is the weighted sum method in Eq. (2) [5, 11].

$$F = w\frac{\mu_f}{\mu_f^*} + (1-w)\frac{\sigma_f}{\sigma_f^*}, \quad 0 \le w \le 1 \tag{2}$$

where μ_f^*, σ_f^* are the function values to normalize the mean and the standard deviation, respectively. μ_f^*, σ_f^* usually have the starting values of the optimization process. w is the value of the weighting factor which is determined depending on the importance of minimization of μ_f and robustness represented by σ_f.

The weighted sum method suffers from the difficulty in adjusting the weighting factor and obtaining the Pareto optimal solutions of the multi-objective function. w, μ_f^* and σ_f^* of Eq. (2) are determined subjectively by the designer's experiences. Therefore, the quality of the solution may depend on the designer. It should be noted that even though the defined weighting factor is used in the optimization process, the weighting factor may not be considered as much as the designer wants. Besides, various optimum values can be generated by changing the weighting factor. Therefore, as shown in Eq. (3), the virtual objective function can be used for minimizing the variation with a constraint on the mean of a supplementing constraint. It is called the constraint method [1, 8].

$$\begin{aligned}
&\text{Minimize} && \sigma_f \\
&\text{subject to} && \mu_f \le \mu_f^t
\end{aligned} \tag{3}$$

where μ_f^t is the limit value of the mean for the objective function. It is required to predetermine the limit value μ_f^t. However, it is difficult to determine the appropriate limit value.

From the above mentioned methods, two conditions for the robustness index can be obtained. First, the weighting factor should be removed to avoid a design made by the designer's intuition and the Pareto solutions. Second, the robustness index should simultaneously consider the mean and the variance in order to find the compromised solutions. Therefore, it is necessary to develop a new robustness index which satisfies the two requirements.

4.2. Robustness of the constraints

The robustness of the constraints is defined by the feasibility condition which indicates that the optimum value should always lie in the feasible region. Treatment of the constraints varies according to the detailed definition of Eq. (1). In order to consider the variation of the constraints, the penalty method and the probabilistic method have been widely used [5, 13]. The penalty method defines a new constraint $\mathrm{g}_{\mathrm{j_{new}}}$ having a penalty term related to the uncertainties caused by the variations of the design variables and design parameters. The constraint with uncertainties of Eq. (1) is redefined as follows:

$$\mathrm{g}_{\mathrm{j_{new}}} = g_j + k_j \sigma_{g_j} \le 0, \qquad j = 1, \cdots, m \tag{4}$$

where k_j is the penalty factor and σ_{g_j} is the standard deviation of the constraint g_j. The penalty factor controls the quality of the robustness in the constraint function. The larger the value of the factor becomes, the smaller the feasible area of the constraint becomes. However, the decision is up to the designer's intuition. Therefore, this approach can lead to over-designed results in many cases. σ_{g_j} is usually evaluated by an approximation method using the first-order Taylor series. Thus, second-order derivatives of the constraints are needed in the optimization process.

In the probabilistic method, the constraints with uncertainties are replaced by probabilistic constraints. The feasibility of the constraints can be written as

$$\Pr[g(\mathbf{b} + \mathbf{z^b}, \mathbf{p} + \mathbf{z^p}) \le 0] \ge P_0 \tag{5}$$

where $\Pr[\cdot]$ is the probability operator, and P_0 represents the probability of satisfaction for the constraints. To deal with the probabilistic constraints of Eq. (5), a high computational cost is required, especially when finite element analysis is performed in practical problems. Therefore, various numerical methods have been developed to reduce the cost for calculating the reliability. These methods are called reliability-based design optimization (RBDO) [14-20].

The penalty method of Eq. (4) cannot guarantee a reliable design, because it is not easy to determine the penalty factor. Therefore, this method may make an over-designed result. That is, the safety of a design is limited by the designer's intuition. On the other hand, RBDO provides a safer and more reliable design compared to the penalty method because it considers the uncertainty based on the probabilistic theory. However, even if the RBDO theories have been well established, the methods require more function evaluations than the equivalent deterministic optimization problem. Thus, the economical cost of RBDO is still an issue to be solved.

5. Proposed RBRDO method

5.1. Probabilistic robustness index

In this section, a new robustness index for the objective function is introduced. In general structural design optimization, most of the objective functions have a positive value. Suppose the mean of the objective function is always in the positive region and the objective function is normally distributed, then the distribution of the objective function is represented as illustrated in Fig. 1.

Suppose the positive region of the objective function is in the safe region, the probability of safety can be obtained as follows:

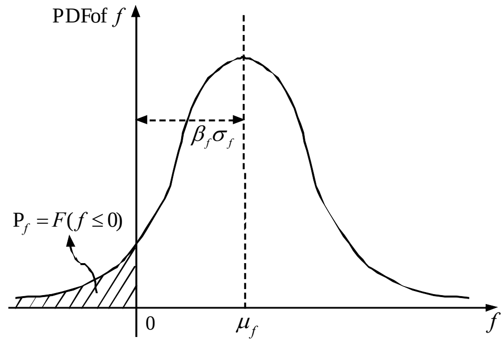

Figure 1: Probability density function of the objective function

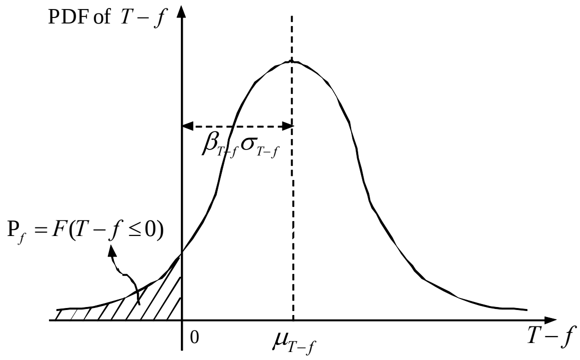

Figure 2: Probability density function of *(T-f)*

$$\Pr[f(\mathbf{b}) > 0] = \Pr\left[\frac{f-\mu_f}{\sigma_f} > \frac{0-\mu_f}{\sigma_f}\right] = \Phi\left(\frac{\mu_f}{\sigma_f}\right) = \Phi(\beta_f) \tag{6}$$

where $\Pr[\cdot]$ is the probability operator, $\Phi(\cdot)$ is the standard cumulative function of the normal distribution and β_f is the safety index of the objective function. If all b_i are statistically independent from each other, then the safety index can be expressed as follows:

$$\beta_f = \frac{\mu_f}{\sigma_f} \tag{7}$$

The safety index of Eq. (7) is reconstructed to define a robustness index for robust design optimization. Here, a new design parameter T is introduced. T is defined as an arbitrary constant which is larger than the maximum value of the objective function in the design range. The distribution of the objective function f is converted to that of $(T-f)$ as illustrated in Fig. 2 where μ_{T-f} and σ_{T-f} are the mean and the standard deviation of $(T-f)$, respectively.

β_{T-f} is the safety index of $(T-f)$. Eq. (7) can be rewritten as follows:

$$\beta_{T-f} = \frac{\mu_{T-f}}{\sigma_{T-f}} = \frac{T-\mu_f}{\sigma_f} \tag{8}$$

If Eq. (8) is maximized, the mean and the standard deviation can be simultaneously considered in the optimization process. In this case, both performance and robustness are considered and a compromise is found as a trade-off between minimizing the mean and the variance. In other words, the index of Eq. (8) can be used as a robustness index for robust design optimization. In this study, it is called as the probabilistic robustness index.

To calculate the probabilistic robustness index β_{T-f}, the values of μ_f and σ_f are needed. As mentioned earlier, various methods exist to evaluate the mean and standard deviation of the objective function. In this study, μ_f is defined as a function value at the current nominal design, and σ_f is computed by the first-order Taylor's series expansion. Thus, second-order derivatives of the objective function are required in the optimization process. The finite difference method (FDM) is used for the sensitivity calculation during the optimization process.

5.2. SLSV approach using the conjugate gradient method

In this research, the probability method is employed to treat the constraints with uncertainties in Eq. (1). Among the RBDO methods, the SLSVCG approach is utilized. The efficiency and convergence capability of this approach have been verified in previous research [19, 20]. This method includes robust and stable convergence regardless of the target reliability and the problem characteristics. In SLSVCG, the most probable point (MPP) $\mathbf{b}^{*(k)}$ of the k th cycle is calculated as follows:

$$\begin{aligned} \mathbf{b}^{*(k)} &= \boldsymbol{\mu}_{\mathbf{b}}^{(k)} + \beta^t \boldsymbol{\sigma}_{\mathbf{b}}^T \frac{\mathbf{d}^{(k-1)}}{\left\|\mathbf{d}^{(k-1)}\right\|} = \boldsymbol{\mu}_{\mathbf{b}}^{(k)} + \beta^t \boldsymbol{\sigma}_{\mathbf{b}}^T \frac{\boldsymbol{\sigma}_{\mathbf{b}}^T \nabla g^{(k-1)} + \gamma^{(k-2)} \mathbf{d}^{(k-2)}}{\left\|\mathbf{d}^{(k-1)}\right\|} \\ \gamma^{(k-2)} &= \frac{\left(\boldsymbol{\sigma}_{\mathbf{b}}^T \nabla g^{(k-1)}\right)^T \cdot \left(\boldsymbol{\sigma}_{\mathbf{b}}^T \nabla g^{(k-1)}\right)}{\left(\boldsymbol{\sigma}_{\mathbf{b}}^T \nabla g^{(k-2)}\right)^T \cdot \left(\boldsymbol{\sigma}_{\mathbf{b}}^T \nabla g^{(k-2)}\right)} \end{aligned} \tag{9}$$

where $\boldsymbol{\mu}_{\mathbf{b}}^{(k)}$, $\boldsymbol{\sigma}_{\mathbf{b}}$ and β^t are the mean value vector of the random variable $\mathbf{b}$ of the k th cycle, the standard deviation of the random variable $\mathbf{b}$ and the target reliability index of the j th probabilistic constraint, respectively. $\mathbf{b}^{*(k)}$ is the MPP of the random variable $\mathbf{b}$ of the k th cycle. $\mathbf{d}^{(k)}$ is the conjugate gradient of the k th cycle. The process continues until the entire process converges. The conjugate gradient is calculated after the second cycle. If the method converges at the second cycle, the results are equal to the original SLSV approach.

5.3. RBRDO algorithm

The algorithm for RBRDO is explained in this section. The proposed RBRDO algorithm is formulated by integrating the probabilistic robustness index and the SLSVCG approach in order to ensure the quality and reliability of a design. The generic formulation of RBRDO is as given below:

$$
\begin{aligned}
&\text{Find} && \mathbf{b} \in R^n \\
&\text{to minimize} && [\mu_f(\mathbf{b},\mathbf{p}), \sigma_f(\mathbf{b},\mathbf{p})] \\
&\text{subject to} && g_i(\mathbf{b},\mathbf{p}) \le 0, \quad i = 1,\ldots,r \\
& && \Pr[g_j(\mathbf{b},\mathbf{p}) > 0] \le \Phi(-\beta_j^t), \quad j = r+1,\ldots,m \\
& && \mathbf{b}^L \le \mathbf{b} \le \mathbf{b}^U
\end{aligned}
\tag{10}
$$

To deal with the probabilistic constraints of Eq. (10), the SLSVCG approach is employed. The probabilistic robustness index of Eq. (8) is used to achieve the robustness of the objective function. Therefore, the proposed RBRDO approach is aimed at seeking a compromise between robustness and reliability efficiently. The procedure of the proposed RBRDO is depicted in Fig. 3.

6. Examples

Two examples are solved to verify the proposed method. The suggested RBRDO method is applied to the examples. The results of the proposed method are compared with those from other combinations for RDO. In other combinations, the weighted sum method is used as a robustness index for the objective function, and the

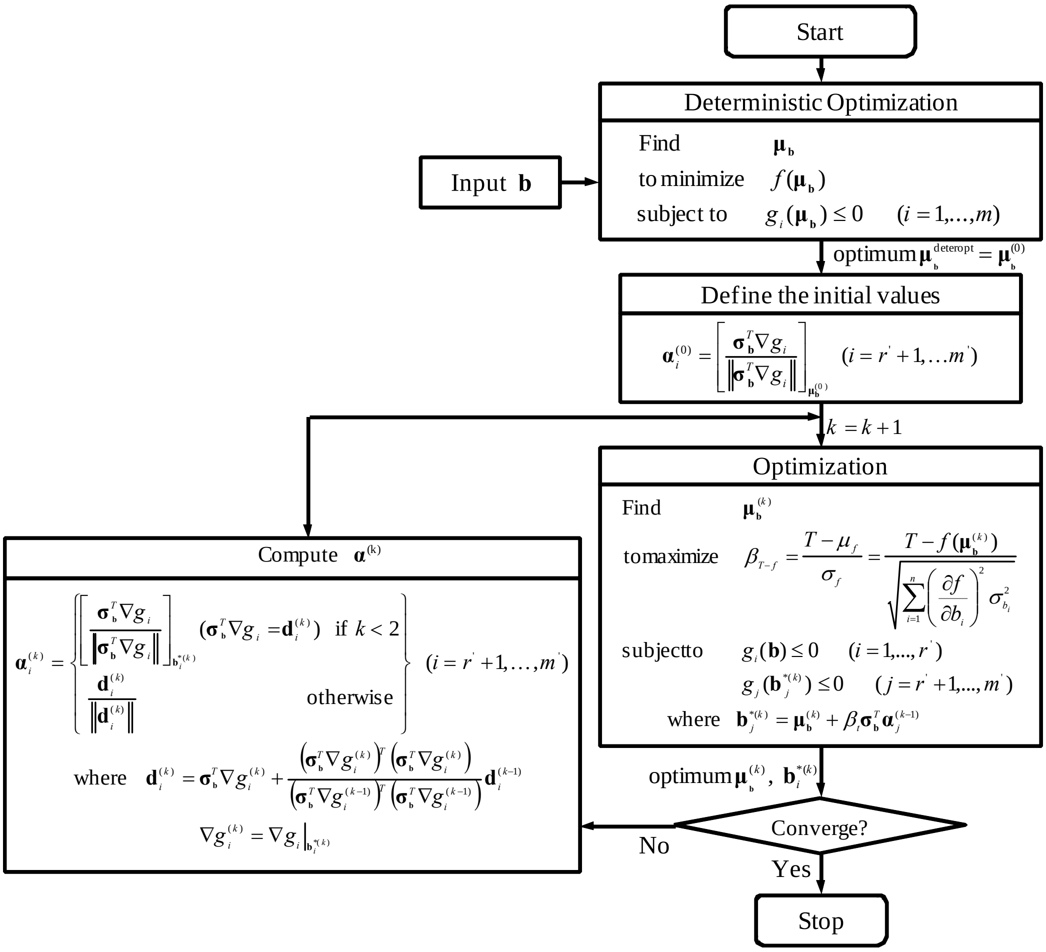

Figure 3: Flow of the proposed RBRDO method

constraints with uncertainties are treated by using the penalty method and the RBDO methods such as RIA, PMA, SORA and SLSV [14-20]. The RIA and PMA methods use the enhanced approach with analytic optimum sensitivities [15, 25]. All random variables are assumed to be normally distributed and statistically independent from each other. All the constraints of the examples are considered as the constraints with uncertainties. GENESIS 12.0 is utilized as a structural analysis tool and the applied methods are coded by using a FORTRAN environment which is interfaced with optimization software DOT 5.7 [26-28].

6.1. Example 1

The first example is a mathematical problem with a nonlinear objective function and three nonlinear constraints. The constraint $g_2(\mathbf{b})$ is a highly nonlinear performance function. The initial point is $\boldsymbol{\mu}_{\mathbf{b}}^{(0)} = [5.0\ 5.0]^T$ and the target reliability index is set to $\beta_j^t = 3.0\ (j = 1,2,3)$. The standard deviation of the random variable $\mathbf{b}$ is $\boldsymbol{\sigma}_{\mathbf{b}} = [0.3\ \ 0.3]^{\mathrm{T}}$. The design parameter T is 2.0 and the weighting factor w for the weighted sum method is 0.5. The deterministic optimization (DO) problem is formulated as follows:

$$
\begin{aligned}
&\text{Find} && b_1,\ b_2 \\
&\text{to minimize} && f(\mathbf{b}) = -\frac{(b_1 + b_2 - 10)^2}{30} - \frac{(b_1 - b_2 + 10)^2}{120} \\
&\text{subject to} && \mathrm{g}_1(\mathbf{b}) = 1 - \frac{b_1^2 b_2}{20} > 0 \\
& && \mathrm{g}_2(\mathbf{b}) = -1 + (Y-6)^2 + (Y-6)^3 - 0.6(Y-6)^4 + Z > 0 \\
& && \mathrm{g}_3(\mathbf{b}) = 1 - \frac{80}{(b_1^2 + 8b_2 + 5)} > 0 \\
& && Y = 0.9063b_1 + 0.4226b_2 \\
& && Z = 0.4226b_1 - 0.9063b_2 \\
& && 0 \le b_i \le 10 \quad \text{for } i = 1, 2
\end{aligned}
\tag{11}
$$

Tables 1 and 2 show the RDO results using the weighted sum method and the probabilistic robustness index, respectively. As shown in Tables 1 and 2, the results clearly present the difference between the DO and RDO solutions. In deterministic optimization, the mean is decreased by 172.5% from -0.833 to -2.292, while the standard deviation is increased by 117% from the initial value. Although the deterministic optimum shows improvement of the performance, it can be sensitive to the fluctuations of the design variables. On the other hand, all combinations for RDO show significant reduction of the standard deviation compared to the initial value. The mean of the robust optimum is increased. However, the standard deviation of the robust optimum is decreased by 50.7 % from the initial value, which means a considerable improvement of the robustness of the design. That is, the optimum evaluated from RDO is the one insensitive to the variations of design variables while the feasibility of the imposed constraints is satisfied.

In the penalty method, when the penalty factor is 1.0, over-designed results are obtained. If the penalty factor is 0.77, then the RDO results using the penalty method is similar to those of other combinations. However, it is very difficult to decide the value of the penalty factor before solving the problem.

In the weighted sum method of Table 1, the value of the weighting factor w is determined depending on the importance of minimization and robustness. When the weighting factor is increased, the mean is decreased while the standard deviation is increased. The deliberation of the weighting factor is difficult when the robust design is applied. On the other hand, the probabilistic robustness index does not require the designer's intuition such as the weighting factor.

In Tables 1 and 2, the combinations using RIA fail to converge while the other combinations converge to a similar optimum point. The RIA method does not converge in reliability analysis for the constraint g_2. The combinations using the PMA require more computational effort than the other methods. The number of function evaluation required by the proposed RDO method is (75+80), which is less than that required by the other combinations. The proposed method has converged efficiently. It is shown that the proposed method has the most efficient performance. The combinations using SLSV and SLSVCG show good numerical performances while the other methods are less efficient. The results of the proposed RDO method are equal to those of the RDO using the SLSV. The reason is that the proposed method converges at the second cycle and the optimum value is obtained before evaluating the conjugate gradient.

Table 1: RDO results using the weighted sum method for example 1

		Initial value	Deter. opt.	Penalty Method (k=1.0)	Penalty Method (k=0.77)	RIA	PMA	SLSV	SORA	SLSVCG
μ_{b_1}		5.0	5.196	3.025	2.590	-	2.540	2.545	2.545	2.545
μ_{b_2}		5.0	0.743	6.773	7.515		7.482	7.448	7.465	7.448
$w\frac{\mu_f}{\mu_f^*}+(1-w)\frac{\sigma_f}{\sigma_f^*}$		1.0		0.511	0.384	-	0.381	0.385	0.380	0.385
μ_f		-0.833	-2.292	-0.327	-0.215	-	-0.213	-0.216	-0.215	-0.216
σ_f		0.071	0.154	0.045	0.036		0.036	0.036	0.036	0.036
No. of function calls	f	-		84	96	-	81	123	153	123
	g	-		252	270	-	936	128	369	128
Remark		-		Converged	Converged	Failed	Converged	Converged	Converged	Converged

Table 2: RDO results using the proposed robustness index for example 1

		Initial value	Deter. opt.	Penalty Method (k=1.0)	Penalty Method (k=0.77)	RIA	PMA	SLSV	SORA	SLSVCG
μ_{b_1}		5.0	5.196	3.029	2.592	-	2.543	2.477	2.487	2.477
μ_{b_2}		5.0	0.743	6.772	7.513		7.481	7.562	7.572	7.562
β_{T-f}		40.07		52.16	61.48	-	61.79	63.32	63.28	63.32
μ_f		-0.833	-2.292	-0.328	-0.215	-	-0.204	-0.201	-0.201	-0.201
σ_f		0.071	0.154	0.045	0.036		0.036	0.035	0.035	0.035
No. of function calls	f	-		69	108	-	75	75	174	75
	g	-		207	324	-	1,245	80	328	80
Remark		-		Converged	Converged	Failed	Converged	Converged	Converged	Converged

6.2. Example 2

The second example is the 25-bar truss structure illustrated in Fig. 4. All elements are divided into 8 groups and each group is considered as a design variable. Each size variable controls the cross-sectional area of (1), (2~5), (6~9), (10,11), (12,13), (14~17), (18~21), and (22~25) members, respectively. The loads of 10,000 N in the y-direction and 5,000 N in the negative z-direction are applied at the two top nodes. The purpose of this example is to determine the cross-sectional area b_i of each group in the 25-bar truss structure.

Size optimization is conducted to maximize the first natural frequency of the structure while the mass, displacement and stress constraints are satisfied. The cross-sectional area b_i of each group is treated as a normal random variable, and its mean value is the design variable. The structural mass should not exceed 1000 lbs. The displacement of the top should be less than 0.35 inch. The stress of all elements should be less than the allowable stress of 5,000 psi. The problem has 8 random variables and 27 constraints. The initial value of all the cross-sectional areas is 2.0 inch^2 and the target reliability index of all constraints is set to $\beta^t = 3.0$. The standard deviation of the random variable $\mathbf{b}$ is $\boldsymbol{\sigma}_{\mathbf{b}} = [0.1 \ldots 0.1]^{\mathrm{T}}$. For the maximization problem, the robustness index of Eq. (7) is considered. The weighting factor w is 0.5. The deterministic optimization problem is formulated as follows:

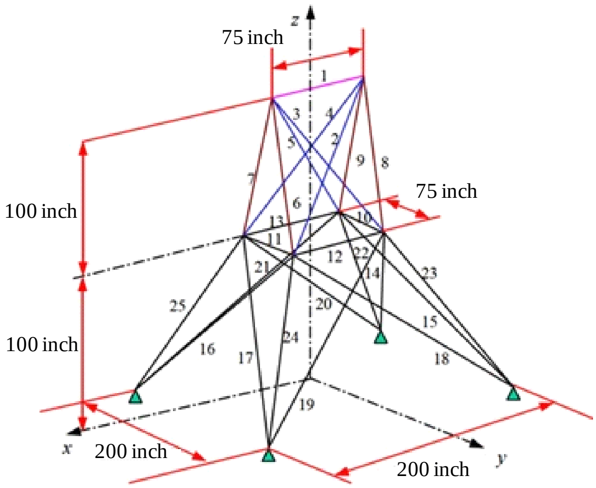

Figure 4: 25-bar truss structure

$$
\begin{array}{ll}
\text{Find} & b_i \quad (i=1,\ldots,8) \\
\text{to maximize} & \text{1st frequency} \\
\text{subject to} & \text{Mass} \le 1{,}000\ \text{lbs} \\
 & \delta_{\text{top}} \le 0.35\ \text{inch} \\
 & -5{,}000\ \text{psi} \le \sigma_{all} \le 5{,}000\ \text{psi} \\
 & 0.001 \le b_i \le 100.0\ \text{inch}^2
\end{array}
\qquad (12)
$$

The RDO results are shown in Tables 3 and 4 including the mean value and the standard deviation of the first natural frequency. As can be seen from Tables 3 and 4, the optimal solutions obtained for all the combinations are nearly the same. In both DO and RDO, the mean is increased and the standard deviation is decreased from the initial value. The deterministic optimum has the largest mean value, while the robust optimum shows the lowest standard deviation. The standard deviation of the deterministic optimum is decreased by 28.1 % from the initial value, while the standard deviation of the robust optimum is decreased by 54.7 % from 0.057 to 0.0258. That is, the RDO finds an insensitive optimum even though the optimum has a lower mean value than DO.

In the penalty method, when the penalty factor is 1.0, the over-designed results are obtained. It is not easy to find the value of the penalty factor which converges to the optimum of the probabilistic methods. The weighted sum method and the proposed robustness index show a similar solution. The RIA-based RDO method shows instability in solving the constraints with uncertainties. RIA fails to converge when the initial point is far from the optimum or the target reliability is increased. Other combinations except for the penalty method represent good stability and balance. Considering the number of function evaluations for all the approaches, the computational cost of the combinations using the PMA is extremely higher than those of the other combinations. The proposed RDO method shows the most economical solution while RDO using the SLSV requires more computational efforts. Among the

Table 3: RDO results using the weighted sum method for example 2

		Initial value	Deter. opt.	Penalty Method (k=1.0)	RIA	PMA	SLSV	SORA	SLSVCG
$w\dfrac{\mu_f}{\mu_f^*}+(1-w)\dfrac{\sigma_f}{\sigma_f^*}$		1.0		0.814	-	0.737	0.738	0.737	0.738
μ_f σ_f		3.512 0.057	4.974 0.041	4.154 0.032	-	3.610 0.026	3.609 0.026	3.611 0.026	3.609 0.026
No. of function calls	f	-		729	-	252	713	877	639
	g	-		3,326	-	12,465	1,765	3,123	1,384
Remark		-		Converged	Failed	Converged	Converged	Converged	Converged

Table 4: RDO results using the proposed robustness index for example 2

		Initial value	Deter. opt.	Penalty Method (k=1.0)	RIA	PMA	SLSV	SORA	SLSVCG
β_{T-f}		61.22		121.8	-	141.1	140.6	141.2	140.4
μ_f		3.512	4.974	4.141	-	3.668	3.669	3.668	3.670
σ_f		0.057	0.041	0.034		0.026	0.026	0.026	0.026
No. of function calls	f	-		376	-	396	581	315	450
	g	-		3,251	-	17,316	1,573	2,269	1,233
Remark		-		Converged	Failed	Converged	Converged	Converged	Converged

combinations, the efficiency of the proposed method is clearly demonstrated.

7. Conclusions

In RDO, the quality and reliability of a design are justified by sensitivity robustness and feasibility robustness. For these purposes, a new RDO algorithm using RBRDO has been proposed in this study.

First, a probabilistic robustness index is proposed to overcome the difficulties of the constraint method and the weighted sum method which are widely used in robust design optimization. In this approach, the mean and the standard deviation can be simultaneously considered in the optimization process. Also, the robustness index does not need to define the weighing factor assigned by the designer. Second, the RBRDO method is conducted by integrating the proposed robustness index and the SLSVCG approach. The method provides a robust and reliable design solution. The proposed robustness index is adopted to ensure the robustness of the objective function. The feasibility robustness is enhanced by using the SLSVCG approach. The method can efficiently reduce the design cost compared to other RBRDO methods. The proposed method has been examined by solving two examples and the results have been compared with those from previous methods and a deterministic case. The efficiency of the method has been demonstrated.

8. Acknowledgements

This research was supported by WCU (World Class University) program through the Korea Science and Engineering Foundation funded by the Ministry of Education, Science and Technology (No. R32-2009-000-10022-0).

9. References

[1] A. D. Belegundu and S. Zhang, Robustness of Design through Minimum Sensitivity, *Journal of Mechanical Design*, 114, 213–217, 1992.

[2] Ramakrishnan, B. and Rao, S., 1991, “A Robust Optimization Approach Using Taguchi’s Loss Function for Solving Nonlinear Optimization Problems,” Advances in Design Automation, Vol. 32-1, pp. 241–248.

[3] Sundaresan, S., Ishii, K., and Houser, D. R., 1995, “A Robust Optimization Procedure with Variations on Design Variables and Constraints,” Engineering Optimization, Vol. 24, No. 2, pp. 101–117.

[4] Chen, W., Allen, J. K., Mistree, F., and Tsui, K. L., 1995, “Integration of Response Surface Methods with the Compromise Decision Support Problem in Developing a General Robust Design Procedure,” ASME Design Automation Conference, DE-Vol. 82-2, American Society of Mechanical Engineers, New York, pp. 485–492.

[5] Lee, K. H. and Park, G. J., 2001, "Robust Optimization Considering Tolerance of Design Variables," Computers & Structures, Vol. 25, No. 2, pp. 99-116.

[6] Park, G. J., Lee, T. H., Lee, G. H. and Hwang, K. H., 2006, “Robust Design: An Overview,” AIAA Journal, Vol. 44, No. 1, pp. 181-191.

[7] Han, J. S. and Kwak, B. M., 2004, "Robust Optimization Using a Gradient Index: MEMS Applications," Structural Multidisciplinary Optimization, Vol. 27, No. 6, pp. 439~478.

[8] Ramakrishnan, B. and Rao, S. S., 1996, “A General Loss Function Based Optimization Procedure for Robust Design,” Engineering Optimization, Vol. 25, pp. 255-276.

[9] Parkinson, D. B., 1997, "Robust Design by Variability Optimization," Quality and Reliability Engineering International, Vol. 13, No. 2, pp. 97~102.

[10] Su, J. and Renaud, J. E., 1997, "Automatic Differentiation in Robust Optimization," AIAA Journal, Vol. 35,

No. 6, pp. 1072~1079.
[11] Doltsinis, I. and Kang, Z., 2004, "Robust Design of Structures Using Optimization Methods," Computer Methods in Applied Mechanics and Engineering, Vol. 193, No. 23-26, pp. 2221-2237.
[12] Jung, D. H. and Lee, B. C., 2002, "Development of a Simple and Efficient Method for Robust Optimization," International Journal for Numerical Methods in Engineering, Vol. 53, No. 9, pp. 2201-2215.
[13] Parkinson, A., Sorensen, C., and Pourhassan, N., 1993, "A General Approach for Robust Optimal Design," ASME Journal of Mechanical Design, Vol. 115, No. 1, pp. 74~80.
[14] Hasofer, A. M. and Lind, N. C., 1974, “Exact and Invariant Second Moment Code Format,” Journal of the Engineering Mechanics Division, Vol. 100, No. 1, pp. 111-121.
[15] Lee, T. W. and Kwak, B. M., 1987, “A Reliability-Based Optimal Design Using Advanced First Order Second Moment Method,” Mechanics of Structures and Machines, Vol. 15, No. 4, pp. 523-542.
[16] Tu, J. and Choi, K. K., 1999, “A New Study on Reliability-Based Design Optimization,” Journal of Mechanical Design, ASME, Vol. 121, No. 4, pp. 557-564.
[17] Chen, X., Hasselman, T. K. and Neill, D. J., 1997, “Reliability-Based Structural Design Optimization for Practical Applications,” AIAA-97-1403.
[18] Du, X. and Chen, W., 2002, “Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design,” DETC2002/ DAC34127, ASME Design Engineering Technical Conferences, Montreal, Canada.
[19] S. B. Jeong and G. J. Park, Preliminary Study on a Single Loop Single Vector Approach using the Conjugate Gradient in Reliability Based Design Optimization, *14th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2012.
[20] Jeong, S. B. and Park, G. J., 2013, “Convergence Improvement of a Single Loop Single Vector Approach Using the Conjugate Gradient in Reliability Based Design Optimization,” Reliability Engineering and System Safety, (submitted).
[21] Tang, Y., Chen, J. and Wei, J., 2012, “A Sequential Algorithm for Reliability-Based Robust Design Optimization under Epistemic Uncertainty,” Journal of Mechanical Design, Vol. 134, No. 1.
[22] Lagaros, N. D., Plevris, V. and Papadrakakis, M., 2007, “Reliability Based Robust Design Optimization of Steel Structures,” International Journal for Simulation and Multidisciplinary Design Optimization, Vol. 1, No. 1, pp. 19-29.
[23] Lee, I. J., Choi, K. K., Du, L. and Gorisich, D., 2007, “Dimension Reduction Method for Reliability-Based Robust Design Optimization,” Computers and Structures, Vol. 86, No. 13-14, pp. 1550-1562.
[24] Youn, B. D. and Xi, Z., 2009, “Reliability-Based Robust Design Optimization using the Eigenvector Dimension Reduction (EDR) Method,” Structural and Multidisciplinary Optimization, Vol. 37, No. 5, pp. 475-492.
[25] Bjerager, P. and Krenk, S., 1987, “Sensitivity Measures in Structural Reliability Analysis,” Proceedings of the First IFIP Working Conference on Reliability and Optimization of Structural Systems, Berlin: Springer-Verlag, pp. 459-470.
[26] GENESIS 12.0 User’s Manual (VR&D), 2010.
[27] DOT 5.7 User’s Manual (VR&D), 2001.
[28] Microsoft Visual FORTRAN (Microsoft Corp., Santa Ana, California), 1998.