10th World Congress on Structural and Multidisciplinary Optimization
May 19 -24, 2013, Orlando, Florida, USA

Optimization of Active Constrained Layer Damping Beam Performance Considering Operational Temperature Variability

Dooho Lee[1], Woo-Seok Hwang[2]

[1] Dongeui University, Busan, Republic of Korea, dooho@deu.ac.kr
[2] Daegu University, Daegu, Republic of Korea

1. Abstract
This study aims to investigate the dynamic characteristics of an active constrained layer damping (ACLD) beam due to operational temperature variability, and to optimize the ACLD beam performance with consideration of operational temperature variability. First, a finite element code is developed to simulate the active constrained layer damping beam using a 10 degree-of-freedom element. A constrained layer damping beam is introduced and modeled by the finite elements. Then, the effects of the operational condition variability on the control performance are simulated by the finite element model. To consider temperature variation, the shift factor concept of viscoelastic materials and the Arrhenius relation for the shift factor are also used in the formulation. The eigenvector dimension reduction (EDR) method is used with estimated temperature variability, material property uncertainties, and manufacturing uncertainties. A reliability-based design optimization (RBDO) formulation for the ACLD beam was proposed with a reliability constraint on the damping performance of the ACLD beam. A robust ACLD beam layout was obtained using a numerical optimizer under the consideration of operational temperature variability, damping material property uncertainty and manufacturing uncertainties.
2. Keywords: Active Constrained Layer Damping, Temperature Variability, Viscoelastic Material, Reliability-based Optimization

3. Introduction
Constrained layer damping is composed of a viscoelastic material layer and a metal constraining layer. The constrained layer damping treatment on structure can enhance the damping performance of viscoelastic material[1]. Further improving of the damping performance can be achieved using an active control strategy for the constraining layer [2-4]. However, the dynamic characteristics of the viscoelastic damping layer shows large variation with respect to environmental temperature as well as frequency. The material properties of the viscoelastic damping materials also have large uncertainty due to manufacturing uncertainty, experimental errors and numerical model errors[5, 6]. Thus, these variabilities in the active constrained layer damping (ACLD) should be considered in order to obtain robust damping performance.
In this paper, an ACLD beam is optimized using the reliability based design optimization (RBDO) method with consideration of the damping material variability due to environmental temperature variation. A finite element model is introduced to calculate the damping performance for the ACLD beam. The eigenvector dimension reduction (EDR) method is used to predict the probability density function (PDF) of the ACLD beam damping performance. Finally, the dimensions of the ACLD beam are optimized in order to minimize the weight of the ACLD beam with a reliability constraint on the damping performance of the ACLD beam.

4. Analysis of Damping Performance of ACLD Beam
In this study, an active constrained layer damping beam is considered as shown in Fig. 1. The ACLD beam consists of a piezoelectric actuator, a viscoelastic damping layer and a base beam.

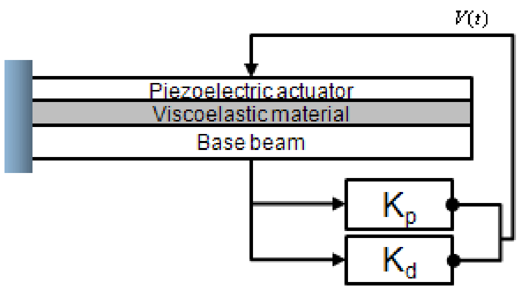

Figure 1: An active constrained layer damping beam

4.1. Finite Element Analysis of ACLD Beam

In this study, a 10-degree-of-freedom (DOF) beam element[1, 3] is used for the analysis of the ACLD beam. In the piezoelectric layer of the 10-DOF beam element, the constitutive relation can be described as follows.

$$\tau = E_p \left(\varepsilon - d_{31} E \right) \tag{1}$$

where τ is the stress, ε is the strain, E_p is Young's modulus of piezoelectric layer, E is the electric field, and d_{31} is the piezoelectric constant. In addition, assuming that the piezoelectric layer is very thin, the electric field can be written as :

$$E = \frac{V(t)}{h_p} \tag{2}$$

where V is the control voltage and h_p is the thickness of the piezoelectric layer. From the virtual work done by the piezoelectric actuator, we can obtain the element force vector as follows.

$$\{f\}_e = c_f \cdot [-1 \quad 0 \quad 1 \quad 0 \quad \frac{h_2}{2L} \quad 0 \quad -\frac{h_2}{2L} \quad h_v \quad 0 \quad -h_v]^T \tag{3}$$

where $c_f = bd_{31}E_PV$, $h_2 = h_b + 2h_v + h_p$, and b is the width of base beam. h_b, h_v and h_p are the thicknesses of the base beam, the viscoelastic layer, and the piezoelectric layer, respectively. Adopting a PD controller for the ACLD beam, the driving voltage applied to PZT layer can be written as follows:

$$V(t) = -k_p x - k_d \dot{x} \tag{4}$$

where k_p and k_d are the proportional coefficients of the controller.

Applying the virtual work principle and discretizing the resulting equation using the finite elements, it is possible to obtain the equations of motion as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F} \tag{5}$$

Here, $\boldsymbol{M}$ and $\boldsymbol{K}$ represent the global mass and stiffness matrices, respectively, and $\boldsymbol{x}$ and $\boldsymbol{F}$ are the generalized displacement and force vectors, respectively. A detailed finite element formulation and the element stiffness and mass matrices expressions are found in Ref. [3]. It should be noted that the stiffness matrix $\boldsymbol{K}$ becomes a complex-valued matrix due to the viscoelastic damping layer and the piezoelectric forces generated by the PD controller. Assuming a harmonic motion of the system, the corresponding eigenvalue problem can be written as:

$$\boldsymbol{K}\boldsymbol{y} = \lambda \boldsymbol{M}\boldsymbol{y} \tag{6}$$

where $\boldsymbol{y}$ is the complex eigenvector and λ is the complex eigenvalue. *i*-th eigenvector $\boldsymbol{y}_i$ satisfies the orthogonal conditions of the form:

$$\boldsymbol{y}_i^T \boldsymbol{M}\boldsymbol{y}_j = \delta_{ij} m_j \tag{7}$$

$$\boldsymbol{y}_i^T \boldsymbol{K}\boldsymbol{y}_j = \delta_{ij} k_j \tag{8}$$

where δ_{ij} is the Kronecker delta and m_i and k_i are the generalized modal mass and the stiffness of i-th mode, respectively. The normalization condition that sets m-th component of k-th eigenvector y_k^m equal to one should be imposed [1].

The natural frequency μ_k and the modal loss factor η_k of k-th mode are defined as:

$$\mu_k = \frac{\sqrt{Real(\lambda_k)}}{2\pi}, \quad \eta_k = \frac{Imag(\lambda_k)}{Real(\lambda_k)} \tag{9}$$

where *Real* and *Imag* refer to the real and the imaginary parts of the argument, respectively.

4.2. Fractional Derivative Model of Viscoelastic Material

The material properties of viscoelastic materials show highly frequency- and temperature-dependent characteristics. The dynamic responses of viscoelastic materials in frequency domain can be represented by complex modulus such as:

$$\tilde{\sigma} = E^* \tilde{\varepsilon} = (E' + iE'')\tilde{\varepsilon} \tag{10}$$

where ~ refers to the Fourier transform. E' and E'' are storage and loss moduli, respectively. From the temperature-frequency equivalence hypothesis, one can use the shift factor (α) to predict the complex modulus at any given temperature (T) by preparing the master curve at a reference temperature T_0. The shift factor and temperature can be related by the Arrhenius equation as:

$$\log[\alpha(T)] = d_1(1/T - 1/T_0) \tag{11}$$

The complex modulus following the fractional derivative model can be expressed as follows:

$$E^* = (E' + iE'') = \frac{a_0 + a_1[if\alpha(T)]^{\beta}}{1 + c_1[if\alpha(T)]^{\beta}} \quad (12)$$

where a_0, a_1, c_1 and β are the four parameters of the fractional derivative model.

5. Uncertainty Characterization of ACLD Beam

Damping performance of the ACLD beam varies according to the variation of the material properties and the control gains. Especially, damping material in the ACLD beam shows large variation in dynamic characteristics due to the material uncertainty and environmental temperature change[5]. Here, the uncertainty characterization method of the damping material in the previous study[5] is summarized for the completeness of the paper.

The variability in the dynamic material properties of viscoelastic material primarily results from the two sources: 1) operational temperature variation and 2) experiment/model errors associated with the viscoelastic damping material. To characterize the variability in the viscoelastic damping material properties, the complex modulus of the damping material can be expressed as[5, 6]:

$$E^*(f,T) = E_1^*(f,T) + \varepsilon_E^* \quad (13)$$

where $E^*(f,T)$ indicates the uncertain complex modulus of the viscoelastic damping material considering the operational temperature variability and experiment/model errors (or uncertainty). The uncertain complex modulus can be decomposed into two terms: the random complex modulus (E_1^*) and the error in the complex modulus (ε_E^*). The random complex modulus considers the operational temperature variability, whereas the error in the complex modulus is primarily due to experiment and model errors at a given temperature. The statistical information of two terms ($E_1^*(f,T)$ and $\varepsilon_E^*(f)$) in Eq. (13) must be sought to characterize the variability in the uncertain complex modulus, $E^*(f,T)$, which is propagated to variability in the uncertain dynamic responses in the ACLD system. The variability of E_1^* and ε_E^* is characterized based on the Arrhenius equation and the fractional derivative model described in the previous section. Figure 3 demonstrates the characterization method for the uncertainty of damping material.

In this study, a damping material, ISD-110, is used in the viscoelastic layer. The variability of the damping material due to operational temperature and experiment/model errors can be estimated from experiment data (storage shear moduli, loss factors and shift factors) and measured temperature history in terms of the material

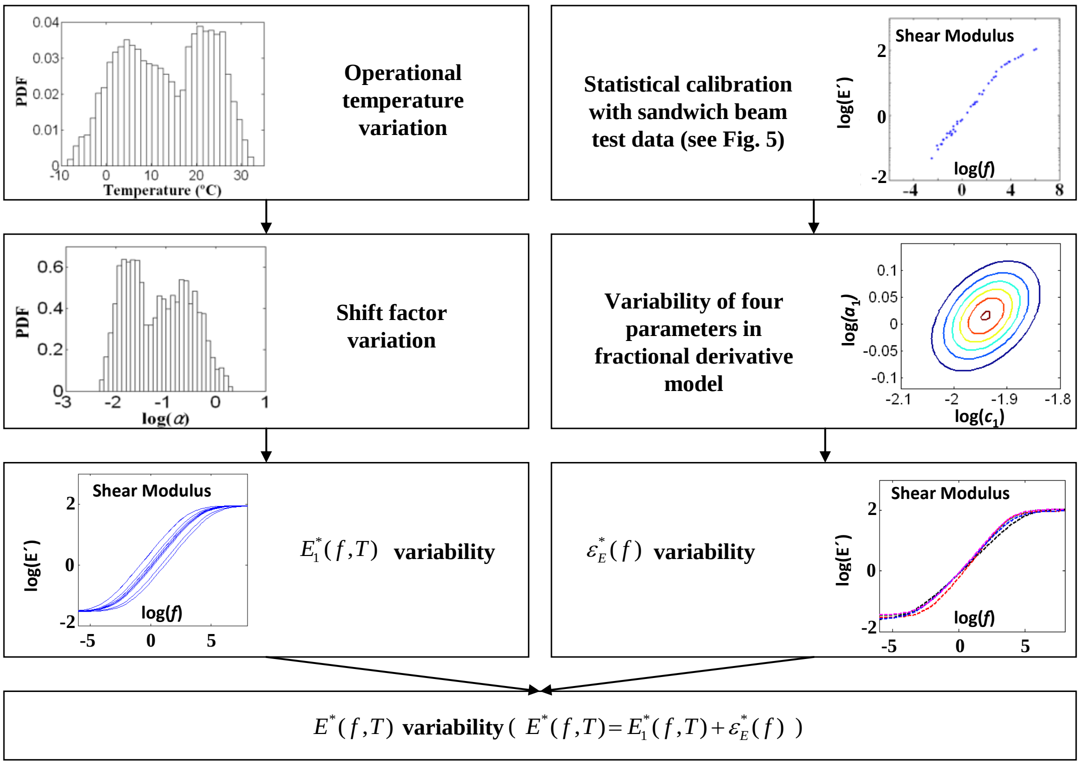

Figure 2: Uncertainty characterization of viscoelastic damping material

Table 1: Statistical properties of the damping material parameters (ISD-110)

Random Variable	Mean	Standard Deviation	Distribution Type
Temperature(℃)	13.28	9.79	Bimodal Data
d_1	5224	54	Normal
$log(a_0)$	log(0.0287)	0.06897	Normal
$log(a_1)$	log(1.0350)	0.05360 ($\gamma_{log(a1),log(c1)}$=0.4239)	Normal
$log(c_1)$	log(0.0115)	0.05360 ($\gamma_{log(a1),log(c1)}$=0.4239)	Normal
β	0.5	0.05574	Normal

parameters of complex modulus using the procedure described in Ref. [5]. Hourly temperature data measured for one year (2007) in Seoul was used as environmental temperature. Figure 3 shows the temperature histogram. The variability of the damping material estimated in terms of the fractional derivative model parameters is shown in Table 1.

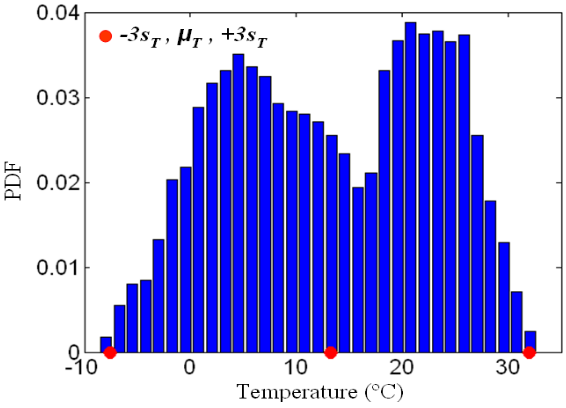

Figure 3: Temperature histogram of Seoul in 2007

5.2. Uncertainty Propagation Analysis

The eigenvector dimension reduction (EDR) method[7] is used in order to build probability distributions for the damping performances of the ACLD beam. The EDR method is an enhanced version of the univariate dimension reduction (DR) method. In the univariate DR method, statistical moments of responses, $Y(\mathbf{X})$, can be calculated as follows.

$$E\left\{Y^m(\mathbf{X})\right\} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} Y^m(\mathbf{x}) \cdot p_{\mathbf{X}}(\mathbf{x}) \cdot d\mathbf{x}, \quad m = 0,1,2\cdots \tag{14}$$

where E indicates an expectation operator and $p_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of random variable, $\mathbf{X}$. Multi-dimensional integration in Eq. (14) can be transformed into multiple one-dimensional integrations using an additive decomposition. Using the binomial formula, the multiple one-dimensional integrations can be solved recursively. To enhance both accuracy and efficiency in probability analysis, three technical elements were considered in the EDR method: 1) eigenvector sampling method to resolve correlated and asymmetric random input variables, 2) stepwise moving least squares (SMLS) method for one dimensional response approximation, and 3) a stabilized Pearson system for generating a PDF of a system response. Thus, for N number of random variables, the EDR method demands $2N$+1 or $4N$+1 eigenvector samples at which system responses are computed using simulations or experimental tests. The detail procedure of the EDR method is explained in Ref.[7]

6. Reliability-Based Design Optimization of ACLD Beam

5.1. RBDO Formulation of ACLD Beam

The design objective of the ACLD beam is to meet a reliability target on the damping performance while using a minimal amount of the damping layer. A fully-covered ACLD beam on a clamped aluminum beam was considered for demonstration purpose. The ISD-110 damping material was used for the damping layer of the structure. The thickness of the base beam was 2.286 *mm*. The beam structure has 2 design variables: the thicknesses of the damping layer (H_1) and the constraining layer (H_2). The thickness of the base beam remains constant during the optimization. The random variables for the ACLD beam problem in Table 1 were used in the RBDO formulation. The manufacturing variability of the design variables was also considered. As listed in Table 2, their coefficients of

Table 2: Design variable of ACLD beam

Design Variable	Initial Value	Lower Bound	Upper Bound	COV	Distribution Type
H_1 [mm]	0.25	0.1	3	10%	Normal
H_2 [mm]	0.762	0.762	3	5%	Normal

variations (COVs) were assumed to be 10%, and 5%, respectively.
RBDO problem can be formulated as:

$$\begin{aligned} &\text{Find } \mathbf{b}=\{H_1,\ H_2\} \text{ such that} \\ &\text{Minimize } \mu_\Phi + s_\Phi \qquad (15) \\ &\text{Subject to } P(G(\mathbf{b},\mathbf{X})<0) > R^t\ ,\ \ \mathbf{b}_L \le \mathbf{b} \le \mathbf{b}_U\ ,\ \mathbf{X}_L \le \mathbf{X} \le \mathbf{X}_U \end{aligned}$$

where **b** is the design variable vector; **X** the random variable vector; Φ the objective function; $P(\bullet)$ indicates probability; G the constraint function; and R^t the target reliability. The RBDO formulation minimizes the mean and standard deviation of the objective function for system robustness. The objective function was the weight in the ACLD beam as

$$\Phi(\mathbf{b})=2\cdot L\cdot(w_1\cdot H_1+w_2\cdot H_2)\cdot\xi \qquad (16)$$

where w_1 and w_2 are the densities of damping and constraining layers, respectively. ξ is width of the beam structure. The loss factor of the ACLD beam for the first mode was used to represent design requirements as

$$G=\eta_{\text{target}}-\eta_1 \qquad (17)$$

where η_{target} was set to 0.02. The reliability constraint is to enforce that the reliability of the first loss factor of the ACLD beam beyond the target loss factor is larger than the target reliability.

5.2. RBDO Results
The RBDO problem defined in the previous section was solved using “fmincon” function of MATLAB. The ACLD beam of the example was discretized with 20 10-DOF finite elements. The EDR method used $4N+1$ analysis calls for estimating the PDFs of the objective function and constraint. The target reliability was set to 3-sigma level (99.865%). The RBDO used the statistical information of the random variables shown in Tables 1and 2. The finite difference method was used for the sensitivity information of the RBDO problem. The optimizer gave the optimum solution of the RBDO problem after 16 iterations with 87 function evaluations. The optimization results are listed in Table 3.

Table 3: The RBDO optimization results for ACLD beam

		Initial	RBDO
Design Variable	H_1 [mm]	0.25	0.488
	H_2 [mm]	0.762	0.762
Object Function	kg	0.0812	0.0850
Reliability	EDR	68.262	99.865
	MCS	-	99.999

The weight performance (design objective) was increased to improve the reliability of the damping performance (design requirement) to 99.865% by 4.67%. In the RBDO, the damping performance (loss factor) became reliable by increasing the thickness of the viscoelastic material. Figure 4 shows the PDFs of the constraint at the initial and reliability-based optimum design points. The damping performance was considerably improved. In Fig. 5, the PDF of the constraint from the EDR method is compared with the histogram from MC simulation with 100,000 random samples. It is concluded that the EDR method is very accurate to predict the PDF and reliability.

7. Conclusions
In this paper, the statistical approach was proposed to predict variability in the damping performance of ACLD beam system. Then, an optimal robust layout of the ACLD beam amidst severe variability of operational temperature was obtained. Uncertainties in the viscoelastic damping material property due to operational temperature variability and experiment/model errors in the complex modulus were considered in the statistical approach. This approach was also applied to find an optimal reliability-based robust design of the active constrained layer damping layout while taking into account significant variability of operational condition and manufacturing uncertainties. The eigenvector dimension reduction (EDR) method was used for the probability analysis in the RBDO. The RBDO case was to find minimum weight layout of the ACLD beam under the reliability constraint on the damping performance. As a result, the weight (design objective) was increased to improve the reliability of the damping performance requirement to 99.865%. The numerical example shows that the RBDO gives more robust and reliable ACLD beam system designs amidst severe variability of viscoelastic damping material properties and operational temperature.

8. Acknowledgements
This work was supported by a National Research Foundation of Korea (NRF) grant funded by the Korean

government (MEST; Grant No. 2010-0023464).

9. References

[1] Lee, D.-H., Optimal Placement of Constrained-Layer Damping for Reduction of Interior Noise, *AIAA journal*, **46**(1), 75-83, (2008).
[2] Trindade, M.A. and A. Benjeddou, Hybrid Active-Passive Damping Treatments Using Viscoelastic and Piezoelectric Materials: Review and Assessment, *Journal of Vibration and Control*, **8**(6), 699-745, (2002).
[3] Lesieutre, G.A. and U. Lee, A finite element for beams having segmented active constrained layers with frequency-dependent viscoelastics, *Smart Materials and Structures*, **5**(5), 615, (1996).
[4] Baz, A. and J. Ro, Optimum Design and Control of Active Constrained Layer Damping, *Journal of Mechanical Design*, **117**(B), 135-144, (1995).
[5] Jung, B.C., et al., A statistical characterization method for damping material properties and its application to structural-acoustic system design, *The Journal of Mechanical Science and Technology*, **25**(8), 1893-1904, (2011).
[6] Jung, B.C., D. Lee, and B.D. Youn, Optimal Design of Constrained-Layer Damping Structures Considering Material and Operational Condition Variability, *AIAA Journal*, **47**(12), 2985-2995, (2009).
[7] Youn, B., Z. Xi, and P. Wang, Eigenvector dimension reduction (EDR) method for sensitivity-free probability analysis, *Structural and Multidisciplinary Optimization*, **37**(1), 13-28, (2008).

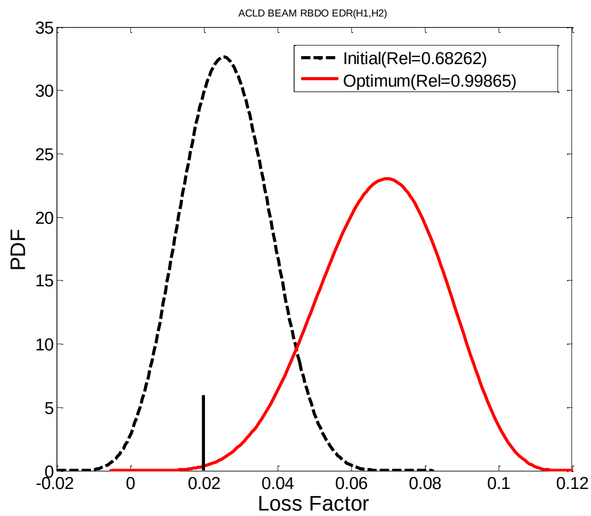

Figure 4: PDFs of the loss factor at the initial and optimum designs

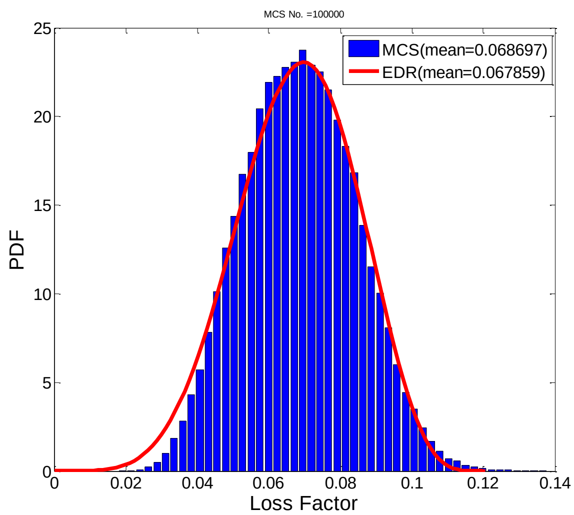

Figure 5: Comparison of PDF by the EDR method with histogram from 100,000 MC simulation at the optimum point