

On the Implementation of an Advanced Interior Point Algorithm for Stochastic Structural Optimization

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1. Abstract

This paper focuses on the implementation of a particular feasible direction interior point algorithm for solving reliability-based optimization problems of high dimensional stochastic dynamical systems. The optimal design problem is formulated in terms of an inequality constrained non-linear optimization problem. A class of interior point algorithms based on the solution of the first-order optimality conditions is considered here. For this purpose, a quasi-Newton iteration is used to solve the corresponding nonlinear system of equations. One numerical example is presented to illustrate the potential of the proposed methodology.

2. Keywords: Advanced Simulation Methods, First-Order Schemes, Stochastic Excitation Model, Structural Optimization, Reliability-Based Design.

3. Introduction

Structural optimization by means of deterministic mathematical programming techniques has been widely accepted as a viable tool for engineering design [1]. However in many structural engineering applications response predictions are based on models whose parameters are uncertain. Despite of the fact that traditional approaches have been used successfully in many practical applications, a proper design procedure must explicitly consider the effects of uncertainties as they may cause significant changes in the global performance of final designs [2, 3]. Under uncertain conditions probabilistic approaches such as reliability-based formulations provide a realistic and rational framework for structural optimization which explicitly accounts for the uncertainties [4, 5].

In the present work structural design problems involving dynamical systems under stochastic loadings are analyzed. The optimization problem is formulated as the minimization of an objective function subject to multiple design requirements including standard deterministic constraints and reliability constraints. First excursion probabilities are used as measures of system reliability. The corresponding reliability problems are expressed as multidimensional probability integrals involving a large number of uncertain parameters. Such parameters describe the uncertainties in the structural properties and excitation. In the field of reliability-based optimization of stochastic dynamical systems several procedures have been recently developed allowing the solution of quite demanding problems [6, 7, 8]. However, there is still room for further developments in this area. It is the objective of this contribution to implement a class of interior point algorithms in the context of reliability-based optimization problems of high dimensional stochastic dynamical systems. In particular, an optimization scheme based on the solution of the first-order optimality conditions is considered here [9]. Based on this optimization scheme an application problem is presented to illustrate the potential of the design process in realistic engineering problems.

4. Optimal Design Problem

Consider the following inequality constrained non-linear optimization problem

$$\begin{aligned}
 & \text{Min}_{\mathbf{x}} \quad f(\mathbf{x}) \\
 & \text{s.t.} \quad g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, n_c \\
 & \quad \quad s_i(\mathbf{x}) = P_{F_i}(\mathbf{x}) - P_{F_i}^* \leq 0 \quad i = 1, \dots, n_r \\
 & \quad \quad \mathbf{x} \in X
 \end{aligned} \tag{1}$$

where $\mathbf{x}, x_i, i = 1, \dots, n_d$ is the vector of design variables with side constraints $x_i^l \leq x_i \leq x_i^u$, $f(\mathbf{x})$ is the objective function, $g_i(\mathbf{x}) \leq 0, i = 1, \dots, n_c$ are the standard deterministic constraints, $s_i(\mathbf{x}) \leq 0, i = 1, \dots, n_r$ are the reliability constraints, $P_{F_i}(\mathbf{x})$ is the probability function associated with the i^{th} reliability

constraint, and $P_{F_i}^*$ is the target failure probability for the i^{th} failure event. The failure event F_i is characterized as $F_i(\mathbf{x}, \mathbf{z}) = D_i(\mathbf{x}, \mathbf{z}) > 1$, where D_i is the so-called normalized demand function defined as

$$D_i(\mathbf{x}, \mathbf{z}) = \max_{j=1, \dots, l} \max_{t \in [0, T]} \frac{|h_j^i(t, \mathbf{x}, \mathbf{z})|}{h_j^{i*}} \quad (2)$$

where $\mathbf{z} \in \Omega_{\mathbf{z}} \subset R^{n_z}$ is the vector of uncertain variables involved in the problem, $h_j^i(t, \mathbf{x}, \mathbf{z})$, $j = 1, \dots, l$ are the response functions associated with the failure event F_i , and $h_j^{i*} > 0$ is the acceptable response level for the response h_j^i . The response functions h_j^i are obtained from the solution of the equation of motion that characterizes the structural model. The failure probability function $P_{F_i}(\mathbf{x})$ evaluated at the design \mathbf{x} can be written in terms of the multidimensional probability integral

$$P_{F_i}(\mathbf{x}) = \int_{D_i(\mathbf{x}, \mathbf{z}) > 1} p(\mathbf{z}) d\mathbf{z} \quad (3)$$

where $p(\mathbf{z})$ is the probability density function of the uncertain variables \mathbf{z} . The above multidimensional probability integral involves in general a large number of uncertain parameters (hundreds or thousands) in the context of dynamical systems under stochastic excitation [10, 11]. Therefore, the reliability estimation for a given design constitutes a high-dimensional problem which is extremely demanding from a numerical point of view.

5. Optimization Scheme

A first-order optimization scheme based on feasible directions is selected in the present implementation. In particular, a class of feasible direction algorithms based on the solution of the Karush-Kuhn-Tucker (KKT) first-order optimality conditions is considered here [12].

5.1. Optimality Conditions

The KKT first-order optimality conditions corresponding to the inequality constrained optimization problem (1) can be expressed as

$$\begin{aligned} \nabla f(\mathbf{x}) + \nabla \mathbf{g}(\mathbf{x}) \boldsymbol{\lambda}_g + \nabla \mathbf{s}(\mathbf{x}) \boldsymbol{\lambda}_s &= \mathbf{0} \\ \mathbf{G}(\mathbf{x}) \boldsymbol{\lambda}_g &= \mathbf{0}, \quad \mathbf{S}(\mathbf{x}) \boldsymbol{\lambda}_s = \mathbf{0} \\ g_i(\mathbf{x}) &\leq 0, \quad i = 1, \dots, n_c, \quad s_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, n_r \\ \boldsymbol{\lambda}_g &\geq \mathbf{0}, \quad \boldsymbol{\lambda}_s \geq \mathbf{0} \end{aligned} \quad (4)$$

where $\boldsymbol{\lambda}_g \in R^{n_c}$ and $\boldsymbol{\lambda}_s \in R^{n_r}$ are the vectors of dual variables, $\nabla \mathbf{g}(\mathbf{x}) \in R^{n_d \times n_c}$ and $\nabla \mathbf{s}(\mathbf{x}) \in R^{n_d \times n_r}$ are the matrices of derivatives of the standard and reliability constrain functions, respectively, and $\mathbf{G}(\mathbf{x})$ and $\mathbf{S}(\mathbf{x})$ are diagonal matrices such that $G_{ii}(\mathbf{x}) = g_i(\mathbf{x})$, $i = 1, \dots, n_c$, and $S_{ii}(\mathbf{x}) = s_i(\mathbf{x})$, $i = 1, \dots, n_r$. In order to solve the non-linear system of equations (4) for $(\mathbf{x}, \boldsymbol{\lambda}_g, \boldsymbol{\lambda}_s)$ a Newton-like iteration is considered [13]. The iteration is written as

$$\begin{bmatrix} \mathbf{B}^k & \nabla \mathbf{g}(\mathbf{x}^k) & \nabla \mathbf{s}(\mathbf{x}^k) \\ \boldsymbol{\Lambda}_g^k \nabla \mathbf{g}(\mathbf{x}^k)^T & \mathbf{G}(\mathbf{x}^k) & \mathbf{0} \\ \boldsymbol{\Lambda}_s^k \nabla \mathbf{s}(\mathbf{x}^k)^T & \mathbf{0} & \mathbf{S}(\mathbf{x}^k) \end{bmatrix} \begin{Bmatrix} \mathbf{d}_1^k \\ \boldsymbol{\lambda}_{g1}^{k+1} \\ \boldsymbol{\lambda}_{s1}^{k+1} \end{Bmatrix} = - \begin{Bmatrix} \nabla f(\mathbf{x}^k) \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad k = 0, 1, 2, \dots \quad (5)$$

where $(\mathbf{x}^k, \boldsymbol{\lambda}_g^k, \boldsymbol{\lambda}_s^k)$ is the starting point at the k^{th} iteration, \mathbf{d}_1^k is a direction in the design space, $\boldsymbol{\Lambda}_g^k$ and $\boldsymbol{\Lambda}_s^k$ are diagonal matrices with $\Lambda_{gii}^k = \lambda_{gi}^k$, $i = 1, \dots, n_c$ and $\Lambda_{sii}^k = \lambda_{si}^k$, $i = 1, \dots, n_r$, and \mathbf{B}^k is a symmetric matrix that represents an approximation of the Hessian matrix of the Lagrangian $L(\mathbf{x}^k, \boldsymbol{\lambda}_g^k, \boldsymbol{\lambda}_s^k) = f(\mathbf{x}^k) + \sum_{i=1}^{n_c} \lambda_{gi}^k g_i(\mathbf{x}^k) + \sum_{i=1}^{n_r} \lambda_{si}^k s_i(\mathbf{x}^k)$. Depending of the choice of \mathbf{B}^k the system of equations (5) may represent a second-order, a quasi-Newton or a first-order iteration. A quasi-Newton method is considered in the present implementation. In particular, a positive definite matrix \mathbf{B}^k is used as an estimate of the Hessian of the Lagrangian. This matrix is updated during the iterations by employing a BFGS-type of updating rule [14]. It can be shown that the vector \mathbf{d}_1^k is a descent direction of the objective function $f(\mathbf{x})$. However, this direction is not necessarily feasible since it is tangent to the active constraints and therefore is not useful as a search direction in the context of this algorithm.

5.2. Perturbed System

In order to obtain a feasible direction a new perturbed linear system is defined as

$$\begin{bmatrix} \mathbf{B}^k & \nabla \mathbf{g}(\mathbf{x}^k) & \nabla \mathbf{s}(\mathbf{x}^k) \\ \Lambda_g^k \nabla \mathbf{g}(\mathbf{x}^k)^T & \mathbf{G}(\mathbf{x}^k) & \mathbf{0} \\ \Lambda_s^k \nabla \mathbf{s}(\mathbf{x}^k)^T & \mathbf{0} & \mathbf{S}(\mathbf{x}^k) \end{bmatrix} \begin{Bmatrix} \mathbf{d}^k \\ \bar{\lambda}_g^{k+1} \\ \bar{\lambda}_s^{k+1} \end{Bmatrix} = - \begin{Bmatrix} \nabla f(\mathbf{x}^k) \\ \rho^k \lambda_g^k \\ \rho^k \lambda_s^k \end{Bmatrix} \quad k = 0, 1, 2, \dots \quad (6)$$

where ρ^k is a positive number. Note that by construction \mathbf{d}^k is a feasible direction at the active constraints. The inclusion of a negative number on the right hand side of Eq.(6) produces a deflection of \mathbf{d}_1^k towards the interior of the feasible design region. To ensure that \mathbf{d}^k is also a descent direction, an upper bound on ρ^k can be easily established by $\rho^k < \rho_{\text{limit}}^k = (\alpha - 1) \mathbf{d}_1^{kT} \nabla f(\mathbf{x}^k) / (\mathbf{d}_2^{kT} \nabla f(\mathbf{x}^k))$, where $\alpha \in (0, 1)$ and \mathbf{d}_2^k is the solution of the linear system

$$\begin{bmatrix} \mathbf{B}^k & \nabla \mathbf{g}(\mathbf{x}^k) & \nabla \mathbf{s}(\mathbf{x}^k) \\ \Lambda_g^k \nabla \mathbf{g}(\mathbf{x}^k)^T & \mathbf{G}(\mathbf{x}^k) & \mathbf{0} \\ \Lambda_s^k \nabla \mathbf{s}(\mathbf{x}^k)^T & \mathbf{0} & \mathbf{S}(\mathbf{x}^k) \end{bmatrix} \begin{Bmatrix} \mathbf{d}_2^k \\ \lambda_{g2}^{k+1} \\ \lambda_{s2}^{k+1} \end{Bmatrix} = - \begin{Bmatrix} \mathbf{0} \\ \lambda_g^k \\ \lambda_s^k \end{Bmatrix} \quad k = 0, 1, 2, \dots \quad (7)$$

and where the search direction \mathbf{d}^k has been written as $\mathbf{d}^k = \mathbf{d}_1^k + \rho^k \mathbf{d}_2^k$. Then, it is seen that the search direction \mathbf{d}^k is obtained by solving two linear systems with the same coefficient matrix (Eqs. (5) and (7)). Thus, the factorization phase for solving the primal-dual systems is done once per iteration only.

5.3. Implementation

The algorithm is implemented as follows:

- 1) Start from a design \mathbf{x}^k ($k = 0$) such that $g_i(\mathbf{x}^k) \leq 0$, $i = 1, \dots, n_c$ and $s_i(\mathbf{x}^k) \leq 0$, $i = 1, \dots, n_r$. The initial vectors of dual variables satisfy $\lambda_g^k \geq \mathbf{0}$ and $\lambda_s^k \geq \mathbf{0}$. Finally, define a symmetric positive definite matrix \mathbf{B}^k ($\mathbf{B}^k = \mathbf{I}$ at the initial step ($k = 0$)).
- 2) Solve the linear system (5) for $(\mathbf{d}_1^k, \lambda_{g1}^{k+1}, \lambda_{s1}^{k+1})$. If $\mathbf{d}_1^k = \mathbf{0}$ the algorithm stops, otherwise the linear system (7) is solved for $(\mathbf{d}_2^k, \lambda_{g2}^{k+1}, \lambda_{s2}^{k+1})$.
- 3) The search direction is then defined as $\mathbf{d}^k = \mathbf{d}_1^k + \rho^k \mathbf{d}_2^k$.
- 4) A step length τ is found satisfying a given constrained line search criterion on the objective function f and such that $g_i(\mathbf{x}^k + \tau \mathbf{d}^k) \leq 0$, $i = 1, \dots, n_c$, and $s_i(\mathbf{x}^k + \tau \mathbf{d}^k) \leq 0$, $i = 1, \dots, n_r$.
- 5) The new design is defined as $\mathbf{x}^{k+1} = \mathbf{x}^k + \tau \mathbf{d}^k$ and the dual variables are updated according to the rule suggested in [12].
- 6) The symmetric and positive definite matrix \mathbf{B}^k is updated by employing a BFGS-type of updating formula [14]. This type of updating rule assures the positiveness of the updated matrix, which is important for the global convergence of the procedure.
- 7) The process is repeated with a new design \mathbf{x}^{k+1} found after line search.

6. Numerical Considerations

6.1. Reliability and Sensitivity Estimation

The reliability constraints of the nonlinear constrained optimization problem (1) are defined in terms of the probability functions $P_{F_i}(\mathbf{x})$, $i = 1, \dots, n_r$. As previously pointed out, these reliability measures are given in terms of high-dimensional integrals. A generally applicable method, named Subset simulation is adopted in this formulation [10, 15, 16]. In the approach, the failure probabilities are expressed as a product of conditional probabilities of some chosen intermediate failure events, the evaluation of which only requires simulation of more frequent events. Therefore, a rare event simulation problem is converted into a sequence of more frequent event simulation problems. Details of this simulation procedure from the theoretical and numerical viewpoint can be found in the above references. On the other hand, the characterization of Eqs. (5) and (7) requires the sensitivity of the functions associated with the optimization problem with respect to the design variables. While it is assumed that the gradients of the standard deterministic functions involved in the problem are readily available, the estimation of the sensitivity of probability functions is not direct. The procedure to estimate the sensitivity of failure probability functions introduced in [7, 17] is implemented in the present formulation. Validation calculations have shown that the approach is quite effective for estimating reliability sensitivities for the class of problems considered in this work.

6.2. Approximate Line Search

As previously pointed out, once a search direction has been obtained, a new primal point is determined by a line search procedure. In particular an inexact search criteria based on the Armijo's and Wolfe's criteria for unconstrained optimization is considered here [12, 18]. First, an upper bound on the step length is defined by satisfying the following constrained line search criterion. The step length τ is defined as the first number of the sequence $\{1, \delta, \delta^2, \dots\}$ satisfying $f(\mathbf{x}^k + \tau \mathbf{d}^k) \leq f(\mathbf{x}^k) + \tau \eta_1 \nabla f^T(\mathbf{x}^k) \mathbf{d}^k$, $g_i(\mathbf{x}^k + \tau \mathbf{d}^k) \leq 0$, $i = 1, \dots, n_c$, and $s_i(\mathbf{x}^k + \tau \mathbf{d}^k) \leq 0$, $i = 1, \dots, n_r$ ($\delta \in (0, 1), \eta_1 \in (0, 0.5)$). Next, the step length satisfying the previous condition is tested according to the Wolfe's criterion, that is, if at least one of the following conditions hold: $\nabla f^T(\mathbf{x}^k + \tau \mathbf{d}^k) \mathbf{d}^k \geq \eta_2 \nabla f^T(\mathbf{x}^k) \mathbf{d}^k$, $g_i(\mathbf{x}^k + \tau \mathbf{d}^k) \geq \eta_2 g_i(\mathbf{x}^k)$, $i = 1, \dots, n_c$, or $s_i(\mathbf{x}^k + \tau \mathbf{d}^k) \geq \eta_2 s_i(\mathbf{x}^k)$, $i = 1, \dots, n_r$ ($\eta_2 \in (\eta_1, 1)$). The condition which is satisfied gives a lower bound on the step length. The actual implementation of the inexact line search technique is discussed in [19].

7. Example Problem

7.1. Structural Model

The finite element model shown in Fig. (1) is considered for analysis. The model consists of a non-linear 8-story building under stochastic earthquake excitation. The plan view of each floor is shown in Fig. (2). Each of the eight floors is supported by 76 columns of square cross section.

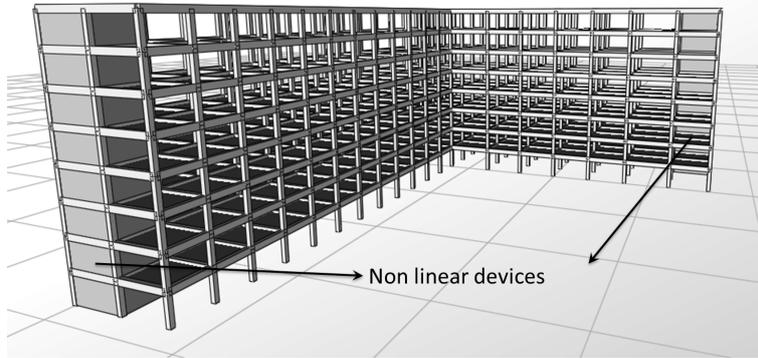


Figure 1: Isometric view of the finite element building model

All floors have a constant height equal to 3.5m, leading to a total height of 28m. Properties of the reinforced concrete have been assumed as follows: Young's modulus $E = 2.45 \times 10^{10}$ N/m², Poisson ratio $\mu = 0.3$, and mass density $\rho = 2500$ kg/m³. For the dynamic analysis it is assumed that each floor may be represented as rigid within the plane when compared with the flexibility of the other structural components. Then, the degrees of freedom of the finite element model are linked to three degrees of freedom per floor (two translational displacements and one rotational displacement) by using condensation techniques. A 5% of critical damping for the modal damping ratios is introduced in the model. The building is excited horizontally by a ground acceleration applied at 45° with respect to the axis x . The ground acceleration is modeled as a non-stationary stochastic process. In particular, a point-source model characterized by a series of seismological parameters is considered in this application [20, 21]. The model is a simple, yet a powerful means for simulating ground motions and it has been successfully applied in the context of earthquake engineering. The sampling interval used in the generation of the excitations is equal to $\Delta t = 0.01$ s and the duration of the excitation is $T = 20$ s. Based on this information and according to the excitation model the generation of each ground motion involves more than 2000 random variables. For an improved earthquake performance the structure is reinforced with friction hysteretic devices at each floor as shown in Fig (2). The devices follow the restoring force law

$$r(t) = k_d \left(\delta(t) - \gamma^1(t) + \gamma^2(t) \right), \quad (8)$$

where k_d denotes the stiffness of the device, $\delta(t)$ is the relative displacement between floors, and $\gamma^1(t)$ and $\gamma^2(t)$ denote the plastic elongations of the friction device. Using the supplementary variable $s(t) = \delta(t) - \gamma^1(t) + \gamma^2(t)$, the plastic elongations are specified by the nonlinear differential equations [22]

$$\begin{aligned}\dot{\gamma}^1(t) &= \dot{\delta}(t)H(\dot{\delta}(t))\left[H(s(t) - s_y)\frac{s(t) - s_y}{s_p - s_y}H(s_p - s(t)) + H(s(t) - s_p)\right], \\ \dot{\gamma}^2(t) &= -\dot{\delta}(t)H(-\dot{\delta}(t))\left[H(-s(t) - s_y)\frac{-s(t) - s_y}{s_p - s_y}H(s_p + s(t)) + H(-s(t) - s_p)\right]\end{aligned}\quad (9)$$

where $H(\cdot)$ denotes the Heaviside step function, s_y is a parameter specifying the onset of yielding, and $k_d s_p$ is the maximum restoring force of the friction device. The values $s_p = 0.006$ m and $s_y = 0.0042$ m are used in this case. Because of the yielding, energy dissipation due to hysteresis is introduced in the structural response.

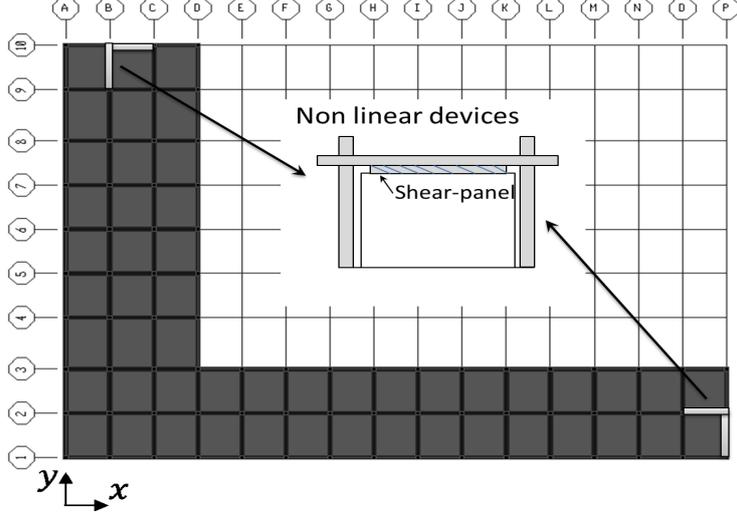


Figure 2: Floor plan of the building mode

7.2. Problem Formulation

The initial construction cost represented by the total weight W of the resistant elements is chosen as the objective function for the optimization problem. The design variables comprise the dimension of the resistant elements of the different floors. First, the dimension of the resistant elements are linked into two design variables ($x_i, i = 1, 2$). Design variable number one represents the columns dimension of floors 1 to 4, while the second design variable is associated with the columns dimension of floors 5 to 8. To control serviceability and minor damage the design criterion is defined in terms of the interstory drift ratios over all stories of the building. The failure event is defined as

$$F = \max_{i=1,\dots,8} \max_{t_k, k=1,\dots,2001} \frac{|\delta_i(t_k, \mathbf{x}, \mathbf{z})|}{\delta_i^*} > 1 \quad (10)$$

where $\delta_i(t, \mathbf{x}, \mathbf{z})$ is the relative displacement between the $(i - 1, i)$ th floors ($i = 1, \dots, 8$) evaluated at the design \mathbf{x} , and $\delta_i^*, i = 1, \dots, 8$ are the corresponding critical threshold levels. The threshold levels are equal to 0.2 % of the story height. The optimal design problem is formulated as

$$\begin{aligned}\text{Min}_{\mathbf{x}} \quad & W(\mathbf{x}) \\ \text{s.t.} \quad & P_F(\mathbf{x}) \leq 10^{-3} \\ & x_1 \leq x_2 \\ & 0.35\text{m} \leq x_i \leq 0.70\text{m} \quad i = 1, 2\end{aligned}\quad (11)$$

It is noted that the estimation of the probability of failure for a given design represents a high-dimensional reliability problem. In fact, as previously indicated, more than 2000 random variables are involved in the corresponding multidimensional probability integrals.

7.3. Results

The trajectory of the optimizer for two different initial feasible designs as well as some objective contours and iso-probability curves are shown in Figs. (3) and (4). Figure (3) corresponds to the linear model (without the hysteretic devices) while Fig. (4) considers the effect of the nonlinear devices. The failure probabilities are estimated using the advanced simulation technique introduced before.

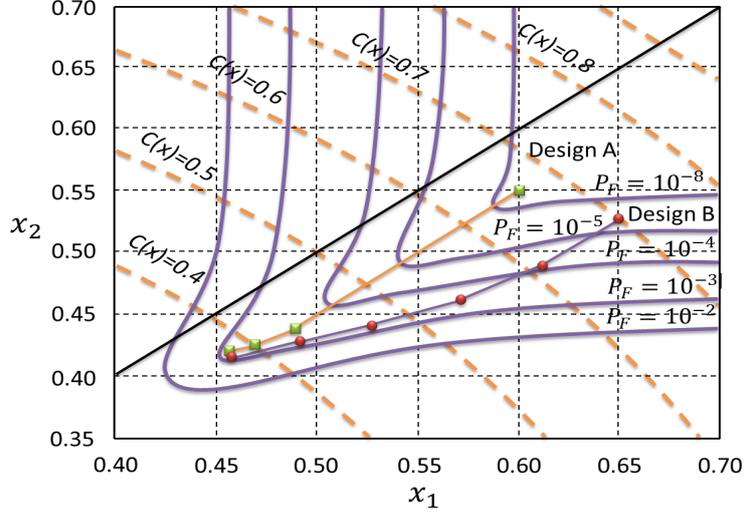


Figure 3: Trajectory of the optimizer and some objective contours and iso-probability curves. Linear model

It is observed that the iso-probability curves associated with the failure event show a strong interaction between the dimension of the resistant elements of the lower and upper floors for both models. This information gives a valuable insight into the interaction and effect of the design variables on the reliability of the final design. From the optimization point of view it is observed that the trajectory of the optimizer to the optimum is quite direct for all cases. The geometric and side constraints are inactive at the final design while the reliability constraint is active.

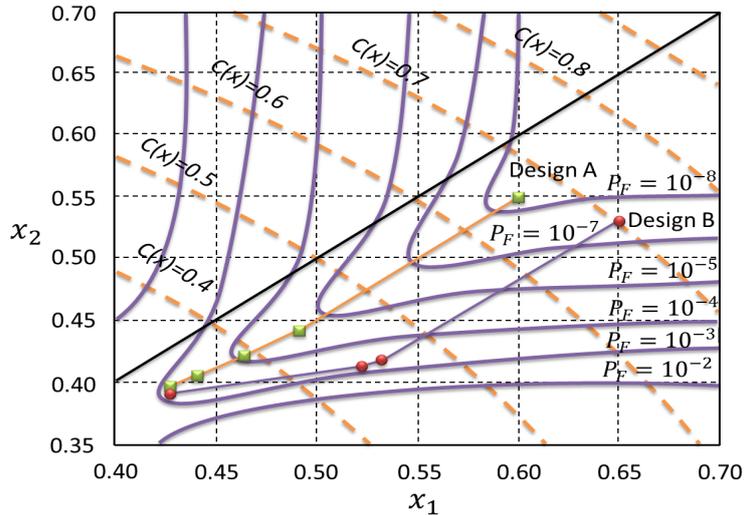


Figure 4: Trajectory of the optimizer and some objective contours and iso-probability curves. Non-Linear model

The corresponding final designs are presented in Table (1). It is seen that the dimensions of the structural components (columns) at the final design of the linear model are greater than the corresponding components of the nonlinear model. Thus, the nonlinear behavior of the hysteretic devices has a positive

Design variable	Initial design	Final design	
		Linear model	Non-linear model
x_1 (m)	0.60	0.46	0.42
x_2 (m)	0.55	0.42	0.40
Resistant elements total weight (kg)	1.76×10^6	1.02×10^6	9.06×10^5

Table 1: Final Design. Case of two linked design variables

impact in the overall performance of the final design. In fact, the hysteresis of the devices increases the energy dissipation reducing in this manner the response of the structural system. The total weight of the linear model increases more than 12% with respect to the weight of the nonlinear model at the final design.

Next, the dimension of the resistant elements are linked into four design variables ($x_i, i = 1, 2, 3, 4$). Each design variable corresponds to the columns dimension of two consecutive floors. The initial and final designs are presented in Table (2). Once again it is seen that the dimensions of the structural components (columns) at the final design of the linear model are greater than the corresponding components of the nonlinear model. In this case, the total weight of the linear model increases more than 20% with respect to the weight of the nonlinear model at the final design.

Design variable	Initial design	Final design	
		Linear model	Non-linear model
x_1 (m)	0.65	0.46	0.42
x_2 (m)	0.60	0.44	0.40
x_3 (m)	0.50	0.41	0.38
x_4 (m)	0.45	0.38	0.33
Resistant elements total weight (kg)	1.64×10^6	9.53×10^5	7.83×10^5

Table 2: Final Design. Case of four linked design variables

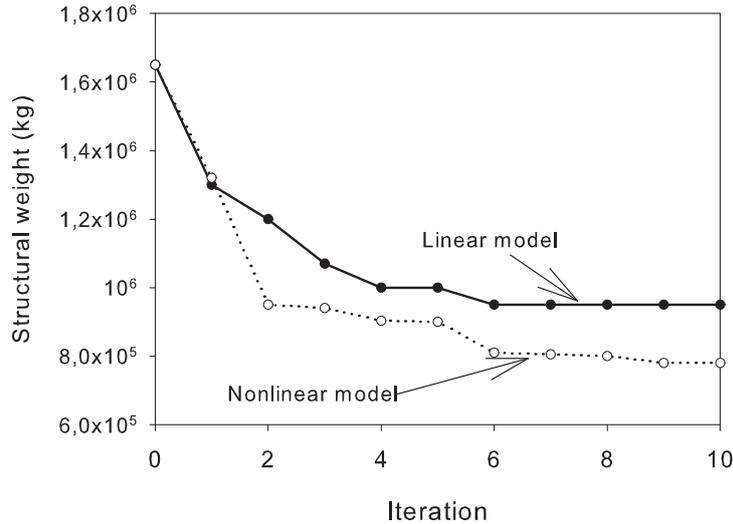


Figure 5: Iteration history of the optimization process in terms of the objective function for the linear and non-linear model

Finally, the iteration history of the design process in terms of the objective function is shown in Fig. (5) for the linear and nonlinear model. It is seen that the design process converges in few iterations for

both cases. In fact, most of the improvements of the objective function take place in the first optimization cycles. In addition, the optimization scheme generates a sequence of steadily improved feasible designs during the entire optimization process.

8. Conclusions

A first-order scheme for solving reliability-based optimization problems of high dimensional stochastic dynamical systems has been implemented. All iterations given by the algorithm strictly verified the inequality constraints and therefore the iterations can be stopped at any time still leading to better feasible designs than the initial design. This property is particularly important when dealing with involved problems such as reliability based optimization of high dimensional stochastic dynamical systems. In these problems each iteration of the optimization process is associated with high computational costs. Numerical results have shown that the algorithm converge in few optimization cycles. This in turn implies that a limited number of reliability estimates has to be performed during the optimization process. In conclusion, the different numerical validations considered in the context of this work have shown the potential of the proposed optimization scheme in solving complex stochastic structural optimization problems.

9. Acknowledgements

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