Element connectivity parameterization method for the stress based topology optimization for geometrically nonlinear structure

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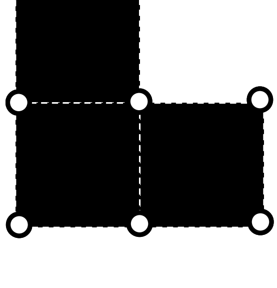
1. Abstract

This research develops a novel computational approach for the stress-based topology optimization method (STOM) minimizing volume subject to locally defined stress constraints of geometrically nonlinear structure in the framework of the element connectivity parameterization (ECP) method. It is a classical but difficult engineering problem to constraint the local stress constraints in topology optimization. In the density based topology optimization method, recently some successful optimization methods have been developed for linear elastic structure. Nevertheless being no research to consider static failure constraint in topology optimization for geometrically nonlinear structure, this research develops a novel computational approach for the STOM for geometrically nonlinear structure. In addition to the stress singularity issue, the many constraint issue and the highly nonlinear behavior issue of the local stress constraints, the so called unstable element issue should be properly addressed for geometrically nonlinear structure. To resolve this issue effectively, this research adopts the ECP method which does interpolate and optimize the connectivities among solid finite elements. It is also found that in addition to the unstable element, the stress singularity issue different to that of the density based TO arises in the ECP method. By investigating the singularity behavior in detail, a new qp-relaxation method suitable for the ECP method can be developed. To show the validity of the present ECP method with the modified qp-relaxation, some two dimensional TO problems are solved.

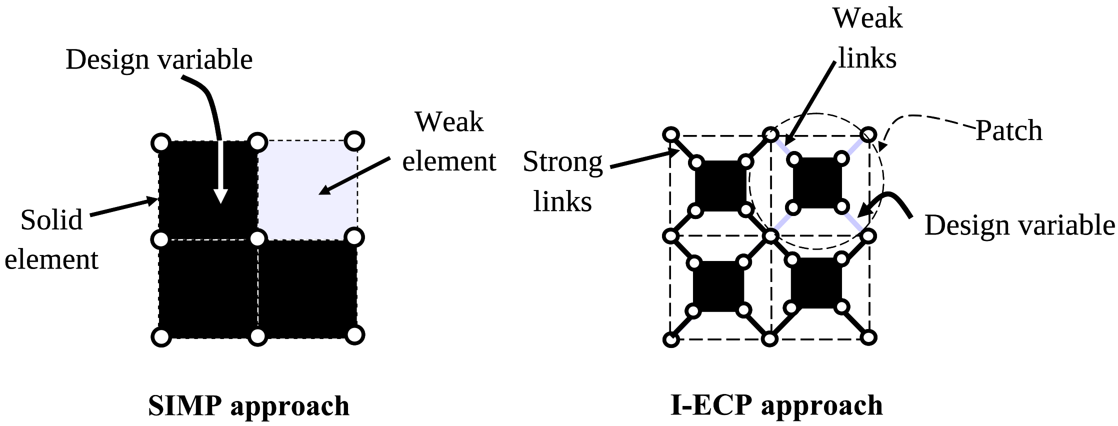
2. Keywords: stress-based topology optimization; geometrically nonlinear structure; element connectivity parameterization method

3. Introduction

This presentation develops a novel topology optimization (TO) method to address the classic, but still unsolved, challenging TO problem of minimizing volume subject to local stress constraints that are defined at the center of every finite element of a geometrically nonlinear structure[1,2,3,4,5,6]. Here it is named as stress-based topology optimization (STOM) for geometrically nonlinear structure and the details are reported in [1] (The most part of this paper is taken from the reference [1]). In this area of research, one of the most challenging TO problems continues to be to minimize volume subject to local stress constraints, to prevent static or dynamic failures [2,3,4,5,6]. For rigorous consideration of local stress constraints in TO, many studies have demonstrated that a number of theoretical and practical issues remain to be resolved, such as the singular behavior of stress with respect to the density design variables of TO, the highly nonlinear behavior of local stress constraints, and the trade-off between local and global stress constraints. In addition to these complex issues, from an engineering point of view the unstable element issue, i.e., flipped element and negative area element, which is often observed at finite elements with low stiffness values, should be properly addressed for TO problems for geometrically nonlinear structure[7]. To our best knowledge, there has been little relevant research related to the STOM problem for geometrically nonlinear structure before the present study. To rigorously conduct this study on the STOM without the unstable element issue, this research applies the element connectivity parameterization (ECP) method to parameterize connectivities among solid finite elements, as shown in Figure. 1.



(a)



(b) (c)

Figure 1: Comparison of the modeling approaches of the SIMP (solid isotropic material with penalization) approach and the internal ECP (element connectivity parameterization) method.

Compared with the density-based TO method, topological evolutions in the design domain are differently modeled and represented in the ECP method, as shown in Figure 1[7]. In the element density–based TO method, a design domain is usually discretized by the finite elements. And the elements' material properties are interpolated, such as Young's modulus, thermal conductivity, and electric conductivity, depending on the physics of interest, with respect to the density design variables of the SIMP method or with respect to the microstructure design variables of the homogenization method, as illustrated in Figure 1(b). By changing the material properties of the finite elements for either void or solid domains, topological modifications can be simulated without alterations of the manifold geometry of an FE model. By finding the material properties that minimize an objective subject to several constraints, the density-based TO method can provide innovative layouts for engineers. However, there is the critical complication of the unstable element in applying the density-based TO method for a geometrically nonlinear structure whose stress and strain measures are the 2nd Piola-Kirhoff stress and the Green-Lagrangian strain, with the Newton-Raphson method as a solution method. As larger displacements are allowed, unstable elements having negative areas mostly at void regions are inevitably observed. For these flipped elements with non-positive areas, the tangent stiffness matrix assembled with their stiffness matrices often becomes a non-positive matrix. To handle this complex issue, several numerical techniques and various optimization methods have been proposed in the framework of the density-based approach. Among the most recent contributions, a new interpolation method called the element connectivity parameterization (ECP) method, illustrated in Figure 1(c), was proposed. In the ECP method, the material properties of the finite elements are not interpolated, but their connections are interpolated, as shown in Figure 1(c). To account for the topological changes of the ECP method, zero-length links are used to connect the nodes of the discretizing finite elements. In short, because the ECP method can handle unstable elements effectively, this research adopts the internal ECP method for the STOM applied to geometrically nonlinear structure.

A new qp-relaxation technique for the ECP method

A new version of the qp-relaxation method is required for the STOM for geometrical nonlinear structure. Among many relaxation methods, the qp-relaxation method in which the penalty values of the constitutive matrices for forward analysis and sensitivity analysis are different has been widely and successfully used for TO of linear structures. In the qp-relaxation method, by employing different penalty values of the constitutive matrices for the analyses in the SIMP method, it seems that void regions having weak stiffness values and experiencing excessive distortions with non-zero stress values become non-favored, from an optimization point of view. The ECP method does not interpolate the material properties of the finite elements with respect to the design variables. Therefore, from the viewpoint of the SIMP method, it can be regarded that the penalization factor of the design variable of discretizing elements is set to 0. Although the ECP method does not have the singularity issue in the forward analysis, we should interpolate the constitutive matrix in the sensitivity analysis process with respect to the design variables to obtain physically acceptable layouts. In addition, we found that the qp-relaxation method of the density-based approach plays a partial role as a regulation method that prevents the stress-based topology optimization from having "no-structure." A main finding of this research is that the ECP method also requires some material interpolation in the sensitivity analysis for the STOM that minimizes volume subject to local stress constraints [1].

4. Stress-based topology optimization formulation

To deal with their being too many local constraints defined at every finite element, a regional scheme based on the sorting algorithm and the p-norm approximation of (3) are also employed for the ECP method, as follows:

$$
\begin{aligned}
&\underset{\gamma}{\text{Minimize}} \quad V(\tilde{\boldsymbol{\gamma}}) = \sum_{e=1}^{NE} \tilde{\gamma}_e v_e \quad (\tilde{\boldsymbol{\gamma}} : \text{Filtered density}) \\
&\text{subject to} \quad \langle \sigma_{\max} \rangle_1 \le \sigma^* \\
&\qquad \langle \sigma_{\max} \rangle_2 \le \sigma^* \\
&\qquad \vdots \\
&\qquad \langle \sigma_{\max} \rangle_{RN} \le \sigma^* \\
&\qquad \tilde{\boldsymbol{\gamma}} = \Xi(\boldsymbol{\gamma}) \text{ with the denstiy filter } \Xi
\end{aligned}
\tag{1}
$$

$$\langle \sigma_{\max} \rangle_k^{iter} \equiv c_k^{iter} \langle \sigma_{PN} \rangle_k^{iter} \tag{2}$$

$$\langle \sigma_{PN} \rangle_k^{iter} \equiv \left(\sum_e \left(\frac{\sigma_e}{\sigma^*}\right)^p \gamma_e \right)^{1/p} \quad (e \in \Omega_k) \tag{3}$$

$$c_k^{iter} = \alpha \frac{\sigma_{\max,k}^{iter-1} / \sigma^*}{\langle \sigma_{PN} \rangle_k^{iter-1}} + (1-\alpha)\, c_k^{iter-1} \tag{4}$$

$$\sigma_e = \frac{1}{\sqrt{2}} \begin{bmatrix} \left({}_0^{t+\Delta t}S_x - {}_0^{t+\Delta t}S_y \right)^2 + \left({}_0^{t+\Delta t}S_y - {}_0^{t+\Delta t}S_z \right)^2 + \\ \left({}_0^{t+\Delta t}S_z - {}_0^{t+\Delta t}S_x \right)^2 + 6\left({}_0^{t+\Delta t}S_{xy}^2 + {}_0^{t+\Delta t}S_{yz}^2 + {}_0^{t+\Delta t}S_{xz}^2 \right) \end{bmatrix}^{1/2} \tag{5}$$

where the volume of the e-th finite element is v_e. Conventional notations are used to indicate the nominal and shear stress components of the finite element in the above formulation. Here, the design variables assigned to the NE elements of the design domain are denoted as $\boldsymbol{\gamma}$ with which the link stiffness values are interpolated. It is worth noting that the design variables or the sensitivity values can be filtered as in the SIMP approach. To cope with the many constraints issue, the locally defined stress constraints are aggregated by the regional stress constraints $\langle \sigma_{\max} \rangle_k$ with the stress p-norm of the *k*-th region $\langle \sigma_{PN} \rangle_k^{iter}$. Because there are discrepancies between the *p*-norm value and the real maximum value, the correction factor c_k^{iter} was introduced, which is the ratio between the stress *p*-norm and the real maximum stress value $\sigma_{\max,k}^{iter-1}$ in the previous iter-1 optimization iteration at the *k*th region. Here, we should emphasize that the above updating of the correction factor, c_k^{iter}, is heuristic. In other words, the index order of finite elements for each region are being subject to change during an optimization, and the contributing elements for each region are also different, that is, a non-differentiable condition of an optimization. However as the optimization converges, for most cases, the index order for each region becomes almost stable and fixed. In the case of geometrical nonlinear analysis, this non-differentiable condition can cause a significant deterioration of an optimization history. Thus, we update the correction factor every few iterations, e.g., the 10th iteration. The maximum allowable von Mises stress value, σ^*, is provided by engineers to constrain the *e*-th stress value , σ^*.

From a great deal of relevant research, it is known that, for stable convergence in the element density method, the constitutive matrices for the static analysis and the sensitivity analysis are formulated differently, as follows:

The element density method (forward analysis): $\mathbf{C}_e = \gamma_e^n \mathbf{C}_0$ (6)

The element density method (sensitivity analysis): ${}_s\mathbf{C}_e = \gamma_e^{n_s} \mathbf{C}_0$ (7)

where the penalization factors for the forward analysis and sensitivity analysis are n and n_s, respectively, and many researchers suggest that their values should be between 3 to 5 and 0.5, respectively.

As explained before, a main idea of the ECP method is that the constitutive matrix of the discretizing elements remains a constant matrix.

$$\text{Forward analysis in the ECP method: } {}_{S}\mathbf{C}_{e} = \mathbf{C}_{0} \tag{8}$$

Because discretizing elements remain solid and the connectivities among finite elements are defined by the zero-length links, the side effects of the element density method can be resolved. And it is our concern that the constant constitutive matrix can still be used for a stable convergence of the original STOM for geometrical nonlinearity. However, in this research, our many numerical tests revealed that the original ECP method cannot overcome the singularity issue: The stress constraints should not be considered when the associated elements are modeled for the void region and the no-structure without any solid domain should be removed from the candidates of optimal solutions. Therefore, the present study has determined that the ECP method still needs the following interpolation of the constitutive matrix with respect to the design variable assigned to the *e*-th finite element, as follows:

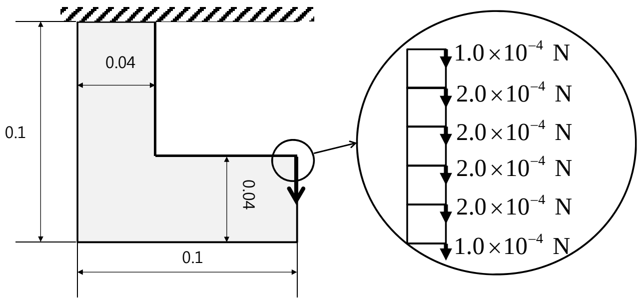
$$\text{Sensitivity analysis of the ECP method: } {}_{S\_ECP}\mathbf{C}_{e} = \gamma_{e}^{n_{ECP\_s}}\mathbf{C}_{0} \tag{9}$$

To denote that the above constitutive matrix and the penalization factor are used for the sensitivity analysis of the ECP method, "S_ECP" is inserted before the interpolated constitutive matrix, and the associated penalization factor is denoted by n_{ECP_s}. Then it becomes an important issue to determine how to choose this penalization factor for stable convergence in stress-based topology optimization for geometrically nonlinear structure.
After further analysis, this research determined the following relationship for the penalization factors of the element density-based approach and the ECP method for stable convergence.

$$\underbrace{n - n_s}_{\text{The element density based approach}} = \underbrace{0 - n_{ECP\_s}}_{\text{The ECP method}} \tag{10}$$

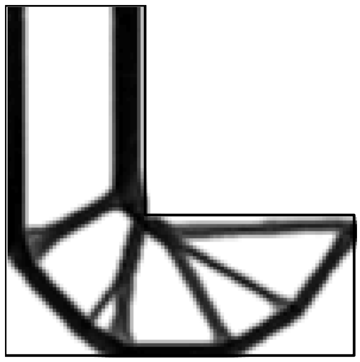
5. Optimization examples

For the topology optimization example, the well-studied L-shaped structure is considered. The design domain is 0.1 m by 0.1 m and is discretized by QUAD elements of 0.001 m by 0.001 m. To simulate the upper right hole, the corresponding design variables are set to the lower bound. The top line is clamped, and a downward structural force of 10^{-3} N and 40×10^{-3} N are applied at the 5 nodes to remove the stress concentration, as depicted in Figure 2(a) and (b), respectively. Then we minimize the mass usage, subject to the local von-Mises stress of the 2nd PK stress values with the different stress limits in Figure 2. As shown, depending on the stress limitations, the different designs can be obtained. With the larger force, the right vertical bars of the design become inclined due to the geometrical nonlinearity.

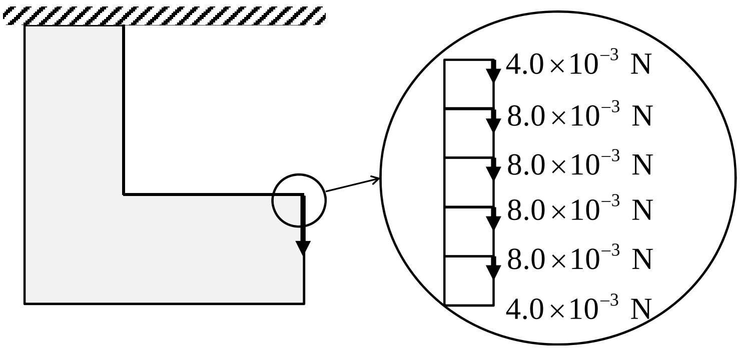


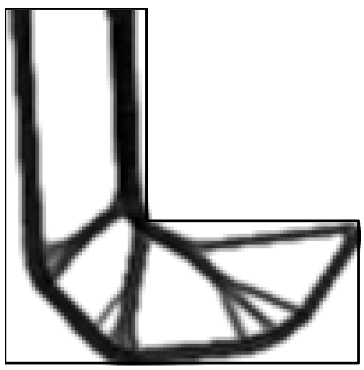
$E = 10^6\ \text{N/m}^2, \nu = 0.3$

100 by 100 QUAD elements



(a) σ^* =0.8 N/m^2 , Mass: 20.92 %





(b) $\sigma^* = 35$ N/m^2 , Mass: 19.03 %

Figure 2:. L-shaped structure and optimization results. (a) A problem definition (RN=8, n_{inter}=3, $l_{\max} / k_{diagonal}^{structure} = 10^9$, $l_{\min} / k_{diagonal}^{structure} = 10^{-9}$, radius of filter: 2 times the finite element), (b) an optimized design with 20.92% mass usage for the maximum stress limit σ^* =0.8 N/m^2 , and (c) an optimized design with 16.27% mass usage for the maximum stress limit σ^* =1 N/m^2 .

6. Conclusions

This paper develops the stress-based topology optimization (STOM) of minimizing volume subject to local von-Mises stress constraints of geometrically nonlinear structure using the element connectivity parameterization (ECP) approach. The ECP method is effective in solving TO of geometrically nonlinear structure because it parameterizes the link's stiffness values for the connectivity among finite elements(1). We found that the singularity issue, the highly nonlinear constraint issue, and the many constraints issue still needed to be solved. In particular, the singularity issue in the ECP method needed to be properly defined from a design variable parameterization point of view: we needed to determine whether interpolation of the constitutive matrix for the sensitivity analysis is required with respect to the design variables. By comparing the Cauchy's and the 2nd PK stress behaviors of some intuitive structures, this paper demonstrates that the ECP method additionally requires the material interpolation with the special penalization factor satisfying the derived condition, $-n_{ECP\_s} = n - n_s$, in the calculation of the stress for locally defined stress constraints.

7. References

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