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Topology optimization with a mixed u/p finite element formulation for acoustic-porous-structure interaction system

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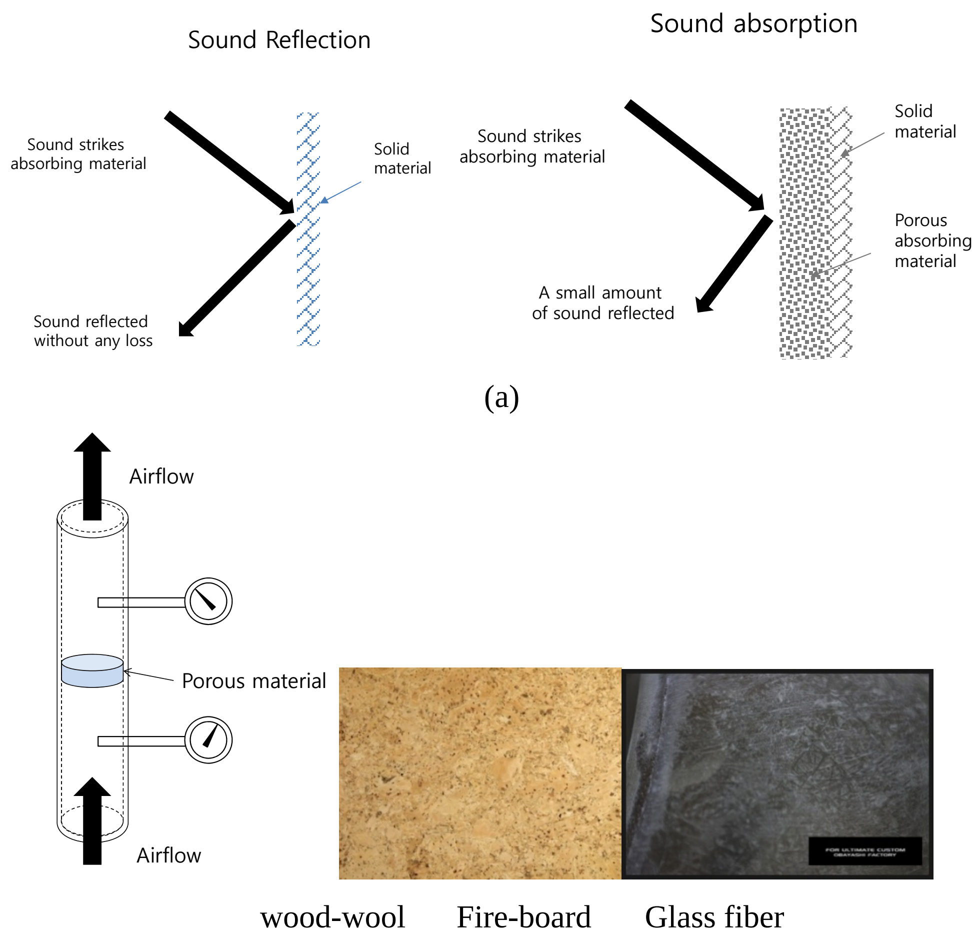
1. Abstract

This research aims to develop a new topology optimization (TO) approach for acoustic-porous-structure interaction system in the framework of a mixed u/p finite element (FE) formulation. Despite some relevant structural optimization research studies for various acoustic systems and phenomena, there is no research for TO for acoustic-porous-structure interaction system by the empirical material model for the modeling of porous media. It is one of the most challenging subjects of TO because the three different media with the different governing equations as well as the interaction boundary conditions for acoustic, porous and elastic structure should be alternated with respect to the spatially varying density design variables of TO. For the TO approach for this challenging multiphysics system, this research proposes to apply a mixed u/p formulation to consider the mutual coupling effects among acoustic medium, fibrous (porous) medium by the empirical material model and elastic structure medium. By combining the mixed FE formulation and the empirical Delany-Bazley model, we can consider the simulation of the sound propagation considering the coupling effects among the three media. For TO, the material properties of the mixed formulation, i.e., bulk modulus, shear modulus and density, are interpolated based on the polynomial SIMP (solid isotropic material with penalization) interpolation functions. To show the validity of the proposed approach, several analysis and topology design problems limited to a single frequency of interest are considered.

2. Keywords: acoustic topology optimization, acoustic-porous-structure interaction, Delany-Bazley model, acoustic analysis, porous material, empirical material model

3. Introduction

This research aims to develop a new topology optimization (TO) approach based on a mixed u/p formulation for acoustic multiphysics phenomena among fibrous medium with porosity close to 1, acoustic medium, and elastic structural medium in Figure 1. To reduce the adverse effect of loud noise or to improve static and dynamic characteristics of various engineering structures, structural optimization methods of size, shape and topology optimization have been developed and applied. Despite some researches relevant to the TO methods for acoustic, acoustic-structure, acoustic-porous-structure based on the Biot's theory (one of the phenomenological material models), TO for acoustic-porous-structure interaction system with the empirical material model for fibrous medium has not been considered yet. Indeed, this research presents a new analysis method based on a mixed u/p finite element formulation to consider acoustic-porous-structure interaction by the Delanzy-Bazley material model which is the representative empirical material models for the acoustic pressure attenuation inside fibrous medium. Compared with the other existing material formula for fibrous media, it is revealed that the present approach with the mixed u/p finite element formulation can consider the multiphysics coupling phenomena among the three media effectively and efficiently for TO by changing the involved material properties, i.e., Bulk and shear moduli and density. With the present unified mixed finite element analysis approach, it is possible to conduct TO for the improvement of pressure attenuation by distributing porous and linear elastic media simultaneously.



(b)

Figure 1: Examples and applications of absorptive porous materials. (a) A comparison of the reflected sounds by a solid (rigid) wall versus an absorptive wall, and (b) examples of porous materials and a drawing of the impedance tube method, used to measure the bulk acoustic properties of the porous materials

It becomes important to control and reduce the adverse effect of loud and irritating noise leading to adverse health issue in industry and daily life; for an instant, in a factory with machinery loud noise from machinery often causes ear health problem to its workers. Thus, with the help of the pressure attenuation of the coupling phenomena between structure and fibrous medium, many engineering approaches have been developed to reduce the adverse effects of loud and irritating noise. For examples, acoustic muffler type kind structures, i.e., expansion chamber, Helmholtz's resonator, fibrous texture attachment to wall, to obtain higher acoustic pressure attenuation or to shield some areas from incoming sound wave are often implemented in machine and architecture applications. Often some small vibrating structures whose resonance frequencies are matched with the frequencies of problematic noises and vibrations are implemented to absorb structural vibration energy and consequently to reduce propagating noises and vibrations in Figure 1. In addition, in order to choose one engineering approach among the above mentioned engineering methods, an awareness of the basic physical principle of dimensions of spaces of interest, wavelength, frequency and sound wave coupling between elastic structure and fibrous medium must be considered to clearly identity noise phenomena and to resolve them by means of acoustic engineering approaches. Particularly, when it is possible to neglect the couplings between acoustic domains and structures enveloping acoustic d omains, the Helmholtz's equation can be used to numerically estimate noise levels for some objective regions of interest. When the velocity of fluid plays an important role in acoustic phenomena, the direct analysis of Navier-Stoke's equations coupled or uncoupled with Helmholtz's equation can be solved. On the other hand, one of the i nteresting acoustic phenomena is the consideration of the coupling effects from elastic structure and fibrous med ia when the elastic structure is not considered to be rigid. For that vibro-acoustic system, the mutual couplings be tween acoustic and structure or among acoustic, structure and fibrous media should be considered. One of the po pular coupling theorems for this multiphysics may be the Biot's theory which models micro-scale interactions be tween structure and fluid.

This research contributes to this important acoustic problem by presenting a new mixed u/p finite element formulation. It was researched that topology optimization for acoustic-structure interaction can be conducted in the framework of a mixed u/p formulation [1,2,3]. This mixed u/p method has been addressed in many FEM books and research papers and has especially been used for incompressible or nearly incompressible elastic medium and vibroacoustic interaction problems [2,3]. In the framework of the mixed u/p formulation, pressure as well as displacements becomes a primal variable with the linearized Euler's equation and the equilibrium equation. Therefore it was shown that by changing the material properties of the mixed u/p formulation, the acoustic-structure interaction phenomena can be simulated [2,3]. In this research, the mutual couplings among fibrous, acoustic and structure are additionally considered by parameterizing the material properties of the mixed u/p formulation. In order to consider the pressure attenuation from fibrous material, the Delany-Bazley material model (one of the empirical material models relying on real experiments in the framework of the Helmhotlz's equation) is formulated in the framework of the present mixed u/p finite element formulation [4].

The three interaction phenomena among fiber, acoustic and structure are straightforwardly considered in the unified mixed u/p formulation. First of all, the acoustic-structure interaction phenomena, i.e., the continuity conditions of velocity and pressure, are satisfied in the mixed u/p formulation. Secondly the coupling conditions between acoustic and fibrous are satisfied by changing the material properties of the Helmholtz equation which is approximately simulated by the mixed u/p formulation. Before further the explanations about the satisfaction of these coupling conditions, it should be mentioned that one of the empirical material formulations called the Delany-Bazley material model is employed only applicable to fibrous media whose the porosities are close to 1 [4].. Nevertheless, this empirical material formulation has been widely used and has shown its validity in many acoustic engineering applications. One of the benefits of the Delany-Bazley model is that the pure Helmholt'z equation can be utilized for the computational calculation of the pressure propagation and the pressure attenuation due to a fibrous material. Finally the coupling conditions between fibrous and structure are satisfied as the fibrous material is treated as a special case of acoustic domain with complex density and bulk modulus. Indeed by changing the three material properties of the mixed u/p formulation, i.e., density, bulk modulus and shear modulus, it can be possible to simulate the mutual couplings among porous, acoustic and elastic structure that we consider for topology optimization.

4. Development of a mixed formulation for acoustic-fibrous-structure interaction

The classical Helmholtz equation is derived from the linearized Euler's equation for compressible media by neglecting dissipation of acoustic energy. However, often the acoustic energy dissipation from absorption and attenuation of sound pressure by fibrous medium should be considered for some practical acoustic engineering simulations and many theoretical and numerical models have been developed in order to simulate the dissipative energy loss from various fibrous materials. To our best knowledge, the developed methods can be categorized into several classes. One of the classes for fibrous material simulation is the phenomenological approach directly simulating the viscous and the thermal interactions between air and fibrous materials; one of the popular phenomenological approaches may be the Biot's theory. Despite some theoretical advantages, it requires several parameter values determined by the geometry and material properties of a fibrous material of interest. Another shortcoming may be that from a computational point of view it requires more degrees of freedom compared with that of the Helmholtz equation. In addition, often the geometric parameters of a fibrous material are too random and chaotic to be measured and be applied. Another class for the simulation of a fibrous material may be the empirical material formulation based on the Helmholtz equation with complex material properties. As stated above, many parameters are involved to use the phenomenological formulations for fibrous materials due to inhomogeneous and chaotic microstructures. As an alternative, some empirical material formulations only utilizing a few material properties tuned with real acoustic experiments have been developed and applied. From a computational point of view, unlike the phenomenological material formulation, the empirical material formulation can rely on the Helmholtz's equation with fewer parameters. To our best knowledge, depending on the characteristics of poroelastic and fibrous materials, many empirical material formulations have been proposed. Among many empirical formulations, this research considers the Delanzy-Bazley material formulation known as one of the most representative empirical material formulations for fibrous materials with porosities close to one formulated [4] as:

$$k_{\mathrm{c}} = k_{\mathrm{a}}\left(1 + 0.0978\left(\tfrac{\rho_{\mathrm{a}} f}{\sigma}\right)^{-0.7} - \mathrm{i}0.189\left(\tfrac{\rho_{\mathrm{a}} f}{\sigma}\right)^{-0.595}\right) \tag{1}$$

$$Z_{\mathrm{c}} = Z_{\mathrm{a}}\left(1 + 0.057\left(\tfrac{\rho_{\mathrm{a}} f}{\sigma}\right)^{-0.734} - \mathrm{i}0.087\left(\tfrac{\rho_{\mathrm{a}} f}{\sigma}\right)^{-0.732}\right) \tag{2}$$

$$c_{\mathrm{c}} = \frac{\omega}{k_{\mathrm{c}}}, \rho_{\mathrm{c}} = {k_{\mathrm{c}} Z_{\mathrm{c}}}\big/{\omega}, f = \frac{\omega}{2\pi},\ 0.01 < \left(\tfrac{\rho_{\mathrm{a}} f}{\sigma}\right) < 1.0 \tag{3}$$

where the wave number and the density of air without pressure attenuation are denoted by k_{a} and ρ_{a} , respectively. The complex wave number and impedance value of the Delany-Bazley empirical material model are k_{c} and Z_{c} , respectively. Note that the empirical material formula shown in Eqs. (1) – (3) are based only on measurements of the bulk airflow resistivity, σ , which is highly dependent on the chosen fibrous material. Because of its simplicity in numerical implementation, this Delany-Bazley empirical material model has long enjoyed a wide acceptance and works well for fibrous materials over a normalized frequency range of $\frac{\rho_{\mathrm{a}} f}{\sigma}$ from 0.01 to 1. From a finite-element (FE) method point of view, the formulations by Delany and Bazley have some benefits as the sound speed and the density are substituted with the complex sound speed and the complex density in the Helmhotlz's equation.

In the mixed u/p finite element formulation, the governing equations without body forces are:

$$\nabla \cdot \boldsymbol{\sigma} = -\omega^2 \rho \mathbf{u} \ \text{ on } \Omega \tag{4}$$

$$\boldsymbol{\sigma} = K\varepsilon_{\mathrm{v}}\,\boldsymbol{\delta} + 2G\,\mathbf{e} \ \text{ and } \ p = -K\varepsilon_{\mathrm{v}} \tag{5}$$

where K , G and ρ are the bulk modulus, the shear modulus and the mass density, respectively, and $\boldsymbol{\delta}$ is Kronecker's delta where the strain tensor is denoted by $\boldsymbol{\varepsilon}$. The above u/p finite element formulations of equations have been used for incompressible media and it has also been demonstrated that by varying the shear modulus G and the bulk modulus K, the acoustic domain and the structural domain can be described for the multiphysics simulation of acoustic and structure simultaneously.

5. Analysis Example

In order to show the validity of the present u/p mixed formulation for the coupled analysis of acoustic, fibrous , and structure, the following analysis example is considered here.

As a first analysis example, the complex impedance at the surface of a layer of fibrous material of 0.1 m thickness and of normal flow resistivity equal to 10000 $\mathrm{Nm^{-4}s}$ has been calculated. The analytical impedance at the surface of the layer fixed on a rigid wall in Figure 2 (a) can be derived as follows:

$$Z = -\mathrm{i} Z_c \coth(k_c d) \tag{6}$$

where the complex impedance (Z_c) and the complex wavenumber (k_c) are determined by the equations (1) and (2) with the flow resistivity value, $\sigma =$ 10000 $\mathrm{Nm^{-4}s}$ and the thickness of d equal to 0.1 m. Note that although the above simple equation is derived by the one dimensional acoustic model assumption, a two dimensional finite element model is constructed in Figure 2 with the present mixed FE formulation to calculate the impedance at the surface of the layer of the fibrous material. In the left acoustic domain, the bulk modulus and the density of the air are assigned with zero shear modulus that transforms the above mixed formulation to the Helmholtz equation. In the right fibrous layer domain, the complex bulk modulus and the complex density of the fibrous material are assigned. With the present monolithic mixed FE formulation, the coupling boundary conditions between the acoustic domain and the fibrous domain are satisfied in the unified FE formulation. To simulate the sound source, the normal incident pressure input is applied by the Sommerfed boundary condition imposed at the most left side as shown.

Sommerfed boundary condition: $\mathbf{n} \cdot \nabla p + \mathrm{i} \cdot k \cdot p = 2\mathrm{i} \cdot k \cdot p_{\mathrm{in}}$ (7)

where $\mathbf{n}$ and p_{in} are the outward unit normal to the acoustic domain and the pressure amplitude of the incoming wave, respectively in Figure 2 (b). The impedance being defined as the ratio of the pressure to the velocity, the pressure values at the mid top surface of the layer of the fibrous material, marked by A in Figure 2 (a), are calculated by varying the exciting frequency values and the following simple impedance calculations are performed at the point A in Figure 2 (a).

$$Z = \frac{p}{du/dt} = \frac{p}{i\omega u} \tag{8}$$

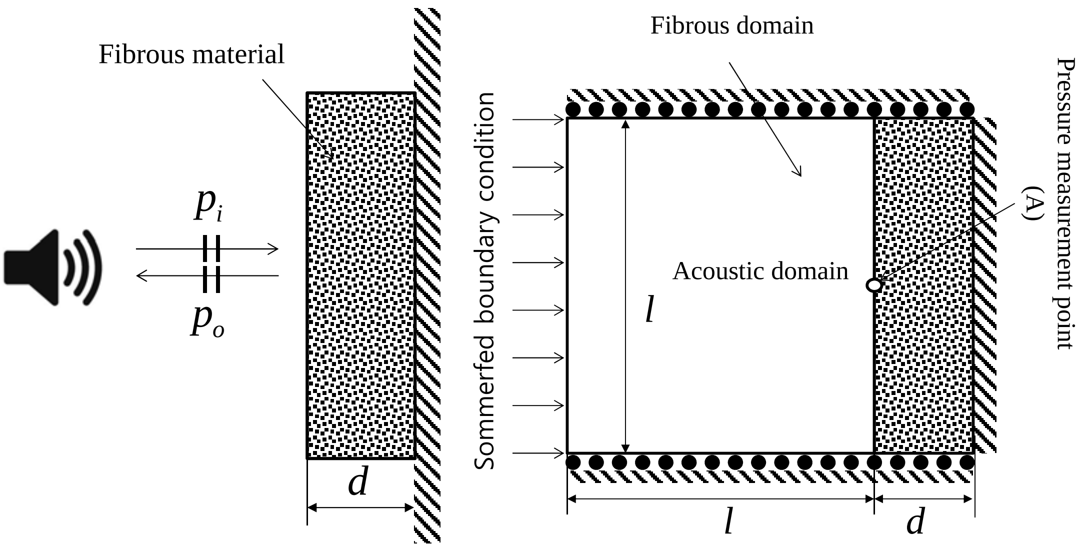
where the x-displacement value is denoted by u which is one of the components of u. Therefore, the time derivative of the harmonic varying x-displacement, i.e., the x-velocity, becomes $i\omega u$. Figure 2(c) shows the curves of the analytical and the calculated impedance values. As illustrated, the present mixed FE formulation can predict the one dimensional analytic impedance very accurately.

Furthermore the Reflectivity value, R, is calculated from its definition as follows:

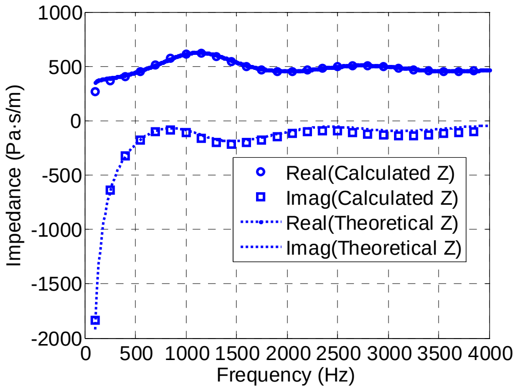
$$R = \frac{p_o - p_i}{p_i} = \frac{Z\text{-}Z_{\mathrm{a}}}{Z+Z_{\mathrm{a}}} = 1\text{-}|A|^2 \tag{9}$$

$$p_i = p_{in}\mathrm{e}^{-ik_a x}, \text{ where } x=L \tag{10}$$

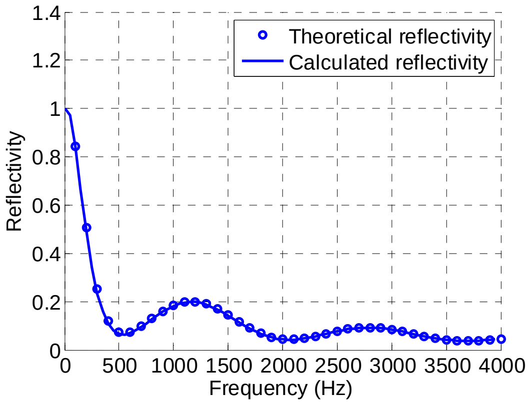
where the pressure at x=L and its magnitude are p_i and p_{in} , respectively and the absorptivity is denoted by A. The acoustic impedance and the wavenumber of air are $Z_{\mathrm{a}} = \rho_a c_a$ and ka, respectively. Figure 2(d) shows the analytical and calculated reflectivity values. As shown, the present monolithic mixed FE formulation can simulate the absorptive phenomena of the fibrous layer very accurately.



(a) (b)



(c)



(d)

Figure 2: Pressure attenuation of a layer of a fibrous material of thickness d with the normal flow resistivity equal to 10000 Nm^{-4}s . (a) A schematic layout to measure the absorptivity and the impedance, (b) the employed finite element model and the sommerfed boundary condition for the sound source, (c) the complex impedance Z calculated by (6) and the present approach and (d) the normal incidence reflectivity from the uniform fibrous layer.

6. Conclusions

This study has developed a new monolithic mixed u/p formulation for TO of acoustic-porous-structure interaction system. Compared with the existing staggered or the monolithic approaches, the present monolithic analysis and optimization approach do not require explicit boundary curves among the three media. The present finite simulation with the mixed u/p formulation is found to provide a means for rapid and easy consideration of the effects of changes of the shape and the topology of acoustic domain. To model the pressure attenuation of fibrous material, one of the empirical material models called the Delanzy-Bazley material model is implemented rather than the phenomenological model. The main benefit of employing the empirical model is that complicated pressure propagation behavior of fibrous material is easily considered with few parameters of fibrous material. The empirical material model is in general employed and formulated in the acoustic equation, i.e., the Helmholtz's equation but this research reveals that it is also possible to model the empirical material model inside the mixed u/p formulation. In future, we would present some research for topology optimization for acoustic-fibrous-structure interaction problem.

7. References

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