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Topological Shape Optimization of Multiphysics Actuators using Level Set Method

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Abstract
This paper proposes a topological shape optimization method for the design of multi-physics piezoelectric actuators using a level set method of piecewise constants. A level set function taking level sets of piecewise constants is applied to implicitly represent design boundaries in a multiphase design domain, in which each constant level set denotes one material phase. As a result, only one level set function consisting of different constants is required to identify multiphase interfaces by making use of its discontinuities. In the design of smart actuators with in-plane motions, the optimization problem is defined to minimize a smooth energy functional under specific constraints. Thus, the design of smart actuators is transferred into a numerical iterative process to update the piecewise constants with a semi-implicit additive operator splitting (AOS) scheme. In such a way, multiple material phases are distributed simultaneously in the design domain until the compliant host structure and the equipped piezoelectric actuators are optimized, in which the compliant structure acts as a mechanical amplifier to enlarge the small strain stroke of the piezoelectric actuators. This method can avoid numerical difficulties in most conventional level set methods, such as the CFL condition, re-initializations and the non-differentiability of the Heaviside and Delta functions. One typical numerical example is used to demonstrate the effectiveness of the proposed topological shape optimization method.

Keywords: Topological shape optimization; Level set methods; Smart actuators; Multiphysics

1. Introduction
Micro-actuators are becoming increasingly popular due to their unique potential in a broad range of engineering applications. Amongst a variety of actuators, piezoelectric material has been regarded as one of the most appealing means as an actuation authority due to its favorable characteristics [1]. It is well known that the piezoelectric actuator takes advantage of the inverse piezoelectric effect to convert electric energy to mechanical actuation. This paper focuses on the design of piezoelectric actuators, which can produce in-plane motion. The devices with in-plane motion are more suitable for micro-mechanical systems [2] considering the microscale packaging.

Topology optimization is to find the optimal distribution of a given amount of material via an iterative numerical process in an extended fixed domain to extremize a specified objective function subject to design constraints [3]. The field of topology optimization has experienced considerable development over the past with several typical methods, e.g. the homogenization method [3], the SIMP method [3], the level-set method including both the standard level-set based methods [4] and alternative level-set methods [5]. Amongst many applications of the topology optimization, the design of piezoelectric actuators has been very promising [6-8]. However, most current studies are mainly concerned with single material design and the research on multi-material design is relatively rare. Hence, only a few available publications are on multi-material shape and topology optimization problems, which mostly employ material distribution methods to implement multiphase modeling [6-7]. However, the most effective level set method for the multiphase designs is still in demand.

This study will propose a topological shape optimization method for multi-physics piezoelectric actuators using PCLSM [9-11]. An indicator function consisting of several piecewise constants is applied to model different regions in the entire design domain for multi-material smart actuators. Each constant value function involved in the level set function is used to uniquely represent one material phase, and n different phases require n piecewise constant value functions. A minimization functional is defined as a smooth energy term to formulate the optimization problem with the specified constraints. In doing so, the design optimization is converted into a process of renewing all the constant value functions using a semi-implicit AOS algorithm [12] to descend the design sensitivity, until the shape and topology of the host structure and the locations, shapes and topologies of piezoelectric materials are optimized in the same design domain. The major characteristic of PCLSM is to remove the connection between the level set functions and re-initializations of the signed distance function, to relax the time step imposed by the CFL condition, and to avoid the non-differentiability related to the Heaviside and Delta functions [13,14]. It should be noted that the level set function in this study refers to an indicator function including piecewise constant functions that is different from that in the standard level set method.

2. Piezoelectric Finite Element Model

The constitutive equations describing the piezoelectric plate are based on the assumption that the total strain in the actuator is the sum of mechanical strain caused by the applied electric voltage. The equations for the linear piezoelectric material under quasi-static models can be written in tensor form as [15,16]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k \tag{1}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \lambda_{ik}E_k \tag{2}$$

where C_{ijkl} , ε_{kl} and σ_{ij} (for 3D $i, j = 1,2,3$, $k, l = 1,2,3$) represents mechanical elastic tensor, strain and stress tensor, respectively. e_{kij} , λ_{ik} , E_k and D_i are matrices of piezoelectric constant, permittivity constant of mechanical constant strain, electric field and electric displacement, respectively. It should be pointed out that Equation (1) denotes the direct piezoelectric effect while Equation (2) shows the converse piezoelectric effect.

As far as the piezoelectric actuator with in-plane motion is concerned, when a voltage is applied to one of the piezoelectric elements in the direction of polarization vector, the induced force due to coupling actuates the elastic structure and generates in-plane motions. According to the principle of virtual work of a deformed piezoelectric medium, the weak formulation of the equilibrium equations can be given as follow:

$$\int_\Omega \varepsilon_{ij}(u)C_{ijkl}\varepsilon_{kl}(v)d\Omega + \int_\Omega \varepsilon_{ij}(v)e_{kij}(\nabla\phi)_k d\Omega = \int_\Omega p_i v_i d\Omega + \int_S t_i v_i dS \tag{3}$$

$$\int_\Omega (\nabla\varphi)_k e_{kij}\varepsilon_{ij}(u)d\Omega - \int_\Omega (\nabla\varphi)_k \lambda_{ik}(\nabla\phi)_i d\Omega = \int_S \varphi_i q_i dS \tag{4}$$

where p and t represent body force and mechanical traction, and q denotes the electric body charge. v and φ are virtual displacement and virtual electric change, respectively.

Using finite element formulation, the following equilibrium equations for the piezoelectric medium is in matrix form can be obtained as

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{u\varphi}^T & \mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{\Phi} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{Q} \end{Bmatrix} \tag{5}$$

where $\mathbf{K}_{uu}$, $\mathbf{K}_{u\varphi}$ and $\mathbf{K}_{\varphi\varphi}$ are the stiffness, piezoelectric coupling and dielectric matrices, respectively, $\mathbf{U}$ and $\mathbf{\Phi}$ represent the vectors of nodal displacements and nodal electric potentials, and $\mathbf{F}$ and $\mathbf{Q}$ are the vectors of applied nodal forces and nodal electric charges. The electric potential is assumed to be linearly polarized from the top electrode of the plate to its bottom electrode. As a result, $\mathbf{\Phi}$ becomes a known vector for the actuator design. It is also noted that $\mathbf{Q}$ contains only internal electric charges. Thus, the mechanical and electric problems are decoupled and only the first set of equation need to be solved numerically, because in this case the dielectric properties involved in the second set of equations can be ignored.

$$\mathbf{K}_{uu}\mathbf{U} + \mathbf{K}_{u\varphi}\mathbf{\Phi} = \mathbf{F} \tag{6}$$

where the mechanical stiffness matrix and the piezoelectric coupling matrix can be written as

$$(\mathbf{K}_{uu})^e = \int_\Omega \mathbf{B}_u^{\mathrm{T}}\mathbf{C}\mathbf{B}_u d\Omega = \sum_{e=1}^{N}\left[\int_{\Omega^e}(\mathbf{B}_u^{\mathrm{T}})^e\mathbf{C}^e(\mathbf{B}_u)^e d\Omega\right],\ e = 1,2,\ldots N \tag{7}$$

$$(\mathbf{K}_{u\varphi})^e = \int_\Omega \mathbf{B}_u^{\mathrm{T}}\mathbf{e}^{\mathrm{T}}\mathbf{B}_\varphi d\Omega = \sum_{e=1}^{N}\left[\int_{\Omega^e}(\mathbf{B}_u^{\mathrm{T}})^e(\mathbf{e}^{\mathrm{T}})^e(\mathbf{B}_\varphi)^e d\Omega\right],\ e = 1,2,\ldots N \tag{8}$$

In the design of piezoelectric actuators with in-plane motion, it is a common practice to calculate the actuation force as follows if the voltage is applied as the input to the piezoelectric material

$$\mathbf{F}_{pzt}^e = -\left[\int_{\Omega^e}(\mathbf{B}_u^{\mathrm{T}})^e(\mathbf{e}^{\mathrm{T}})^e(\mathbf{B}_\varphi)^e d\Omega\right]\mathbf{\Phi}^e = \left[\int_{\Omega^e}(\mathbf{B}_u^{\mathrm{T}})^e(\mathbf{e}^{\mathrm{T}})^e d\Omega\right]\mathbf{E}^e \tag{9}$$

In this way, the global equilibrium equation is changed to the following form

$$\mathbf{K}_{uu}\mathbf{U} = \mathbf{F} + \mathbf{F}_{pzt}^e \tag{10}$$

Since the electric potential is supposed to distribute linearly along the thickness of the piezoelectric plate, the electric field vector for any element is given as

$$\mathbf{E}^e = \left[0 \ 0 \ v^e/t \right]^{\mathrm{T}} \tag{11}$$

where v^e is the applied related voltage and t is the thickness of the piezoelectric plate. φ is the electric potential, $\hat{\varphi}$ is the vector of nodal potentials and B_Φ is the strain matrix.

It is assumed that Ω is completely divided into a set of sub-regions $\{\Omega_i\}_{i=1}^n$. To identify the different sub-domains involved in Ω, it needs to find an indicator function Φ which takes the following constant values

$$\Phi(x) = i,\ x \in \Omega_i\ (i = 1, 2, \ldots, n) \tag{12}$$

A set of characteristic functions χ_i associated with the level set function Φ are then defined as follows for the sake of uniquely identifying the different sub-domains

$$\chi_i(\Phi) = \frac{1}{\alpha_i} \prod_{j=1, j\neq i}^{n} (\Phi - j) \ \text{and}\ \alpha_i = \prod_{k=1, k\neq i}^{n} (i - k) \tag{13}$$

Thus, any of a piecewise constant function ψ is defined in terms of the characteristic functions

$$\psi(\Phi) = \sum_{i=1}^{n} c_i \chi_i(\Phi) \tag{14}$$

The curves separating the regions can be described via the discontinuities of Φ taking a set of constant values.

3. Topology Optimization of Multi-Material Actuators

In this work, the PCLSM based topology optimization method is used to seek the optimum distribution of a given portion of materials by simultaneously determining the distributions of the elastic, piezoelectric and void phases under specified supports and applied loads.

3.1. Optimization formulation of multi-material actuators

To ensure that Equation (13) is a unique representation of ψ and also to ensure the indicator function Φ converges to piecewise constant values at convergence, the function Φ is required to satisfy

$$K(\Phi) = 0 \tag{15}$$

where

$$K(\Phi) = (\Phi - 1)(\Phi - 2)\ldots(\Phi - n) = \prod_{i=1}^{n} (\Phi - i) \tag{16}$$

In this way, the constraint in Equation (20) can be used to guarantee there is no vacuum and overlap between different material phases [23,24], compared with the conventional level set methods using other schemes to avoid vacuum and overlap [25,26]. The design of multi-material piezoelectric actuators is formulated as a constrained minimization problem with respect to the design variable Φ as follows:

$$\begin{cases} \underset{(\Phi)}{\underline{\text{Minimize}}} : J(u, \Phi) = \int_\Omega \psi(\Phi) f(u, v) d\Omega + \beta \int_\Omega |\nabla \Phi| \, d\Omega \\ \text{Subject to}: \begin{cases} G_0(\Phi) = K(\Phi) = 0, \\ G_1(\Phi) = \int_\Omega \psi(\Phi) d\Omega - V_1^* \leq 0, \\ G_2(\Phi) = \int_\Omega \psi(\Phi) d\Omega - V_2^* \leq 0, \\ \text{Equilibrium equation.} \end{cases} \end{cases} \tag{17}$$

In this work, only Φ is regarded as the design variable to be updated and c is a pre-determined value. V_1^* and V_2^* are the volume constraints related to elastic and piezoelectric materials, respectively. It is noted that the second term on the right-hand side of the objective formulation, the sum of the perimeters of the sub-domain boundaries, is used as a penalization to regularize the solution space because of the ill-posedness of the original problem. It is necessary to use the penalization parameter $\beta > 0$ to control the influence of the regularization term, and thus to control the jumps and the length of the sub-domain boundaries. The equilibrium equation is defined in terms of Equation (10) as

$$\bar{a}(u, v, \Phi) = \bar{l}\,(v, \phi, \Phi) \tag{18}$$

where (11) $\bar{a}(u,v,\Phi)=\int_{\Omega}\psi(\Phi)\varepsilon_{ij}(u)^{\mathrm{T}}C_{ijkl}\varepsilon_{kl}(v)d\Omega$ (19)

$$\bar{l}(v,\phi,\Phi)=\int_{\Omega}\psi'(\Phi)\,|\nabla\Phi|\,\tau v d\Omega+\int_{\Omega}\psi(\Phi)(\nabla\phi_k)^{\mathrm{T}}e_{kij}^{\mathrm{T}}\varepsilon_{ij}(v)d\Omega \quad (20)$$

where ∇ is the gradient operator, and v is the virtual displacement field. u and ϕ are the displacement field and electric potential vector, respectively. Φ is the indicator function including piecewise constants, and ψ represents any piecewise constant in Equation (14).

3.2 Design sensitivity analysis

Using the augmented Lagrangian function method, the constrained minimization problem is converted to the following unconstrained minimization problem

$$L(\Phi,\lambda_0,\mu_0,\lambda_1,\mu_1,\lambda_2,\mu_2)=J(u,\Phi)+\bar{a}(u,v,\Phi)-\bar{l}(v,\phi,\Phi)+\lambda_0\int_{\Omega}G_0(\Phi)d\Omega+\frac{1}{2\mu_0}\int_{\Omega}G_0(\Phi)^2d\Omega$$

$$+\lambda_1G_1(\Phi)+\frac{1}{2\mu_1}\left[G_1(\Phi)\right]^2+\lambda_2G_2(\Phi)+\frac{1}{2\mu_2}\left[G_2(\Phi)\right]^2 \quad (21)$$

where Ω denotes a smooth bounded open set. λ_0, λ_1 and λ_2 are the Lagrangian multipliers, and μ_0, μ_1 and μ_2 are the penalization parameters, respectively. The stationary point of the Lagrangian can be obtained by letting $\delta L=0$ with respect to the design variable Φ.

To ensure the decent of the objective function, a proper velocity field $d\Phi/dt=\mathrm{v}$ should be selected for the piecewise constant level set function. As indicated in the literature [4-5], the simplest way is to directly determine a steepest descent direction by letting

$$\mathrm{v}=\Phi'=-\vartheta(\Phi,\lambda_0,\mu_0,\lambda_1,\mu_1,\lambda_2,\mu_2) \quad (22)$$

The minimization problem can be solved by using the following scheme to update the level set function Φ [11]

$$\frac{\partial\Phi(x,t)}{\partial t}+\mathrm{v}(x)=0\Leftrightarrow\frac{\Phi^{n+1}-\Phi^{n}}{\Delta t}+\mathrm{v}(x)=0,\ \Phi(x,0)=\Phi_0(x) \quad (23)$$

where Δt represents a small positive number which can be determined by using trial and error [11].

4. Numerical Example

The design domain is shown in Figure 1 with fixed mechanical boundary conditions at left and right sides, and it is discretized with 150×50 isotropic plate finite elements with a uniform thickness 10μm. The piezoelectric field is displayed with an applied voltage 500V. The actual boundary lengths are 600×200μm. The volume constraints for elastic and piezoelectric material are specified as 15% and 10%, respectively. The constant values 1, 2 and 3 are used as different piecewise functions to represent the void, elastic and piezoelectric material, respectively.

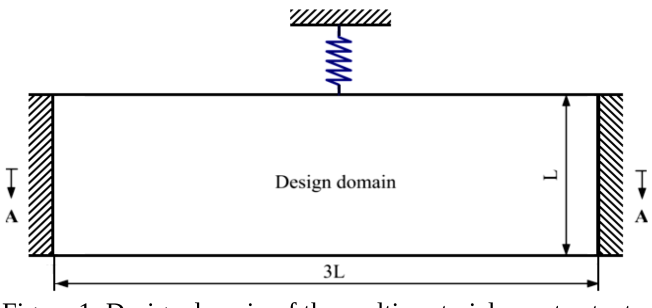
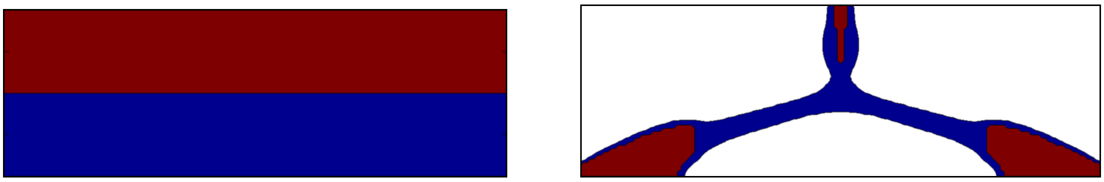


Figure 1: Design domain of the multi-material smart actuator

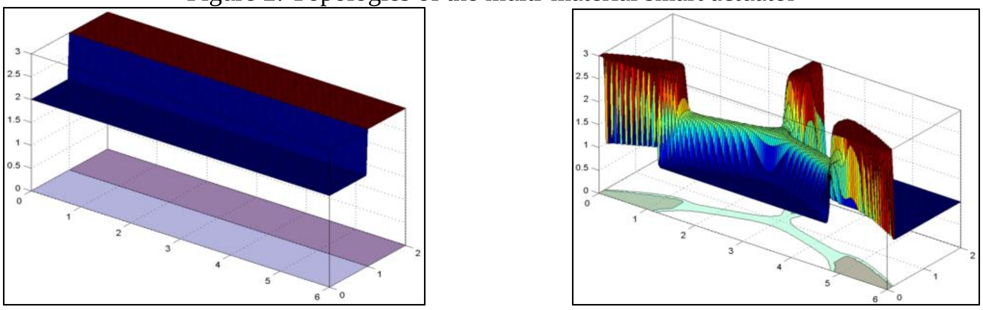
The evolvement process of the topologies and the corresponding level set surfaces are shown in Figures 2 and 3 respectively. The constant value function “1” represents the level set of the void region, “2” denotes the level set of the passive elastic material and “3” is the level set of the piezoelectric material. It can be seen that the proposed piecewise level set method is able to distribute the non-piezoelectric material and the piezoelectric material in the fixed extended design domain at the same time. As a result, the shape and topology of passive host compliant structure and placements, shapes and topologies of the PZT actuators can be optimized simultaneously. The passive structure can be regarded as a compliant mechanical multiplier to enlarge the small output stroke produced by the PZT material. Furthermore, we can find that the present piecewise constant level set method still retains the favorable characteristics of the implicit free boundary representation scheme. Figure 4 displays the convergent history of the design objective to have been maximized at the specified output position, in which the optimal value 9.87*um* is obtained after 200 iterations. The volume ratios in relation to the elastic and piezoelectric material are both conservative, where the elastic material converges from 0.5 to 0.15 and the lower is from 0.5 to 0.1.

Compared to the conventional level set method [30,31], the proposed level set method is free from the limitation of the CFL condition so that a large time step size is applied to speed up the convergence process. In addition, it is unnecessary to apply the re-initialization procedures [31] periodically to retain the regularization of the level set surface, and the velocity extension scheme is also avoided. In particular, the present method is able to generate new holes inside the design domain.



(a) Initial design (b) optimal design

Figure 2: Topologies of the multi-material smart actuator



(a) Initial design (b) optimal design

Figure 3: Level set surfaces of the multi-material smart actuator

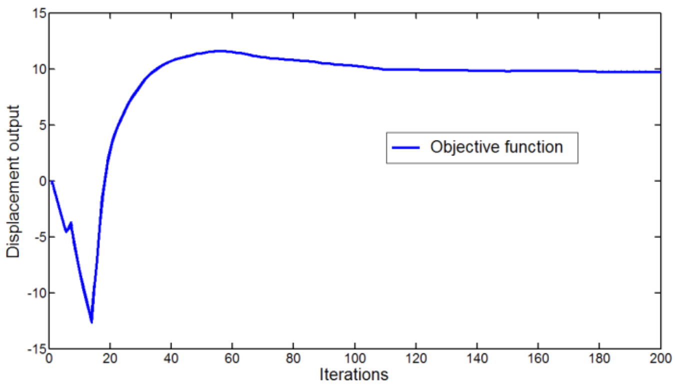


Figure 4: Iteration curve of the design objective function

5. Conclusions

This paper proposes a level set-based multiphase method for topological shape optimization of compliant piezoelectric actuators. An indicator function taking level sets of piecewise-constant values is used to implicitly represent and identify the different design boundaries related to the multiple material regions in the design domain. Only one indicator function rather than several level-set scalar functions is applied to identify all the boundaries between different phases via its discontinuities. The multi-material optimization problem is mathematically described as the minimization of a smooth energy functional under some specified constraints, to which the semi-implicit AOS scheme is applied. The numerical results show that the proposed level set method can remove several numerical difficulties associating with the solution of the Hamilton-Jacobi PDE in most conventional level set methods. The AOS scheme is stable for any practical time steps because the relaxation of the CFL condition in solving partial differential equations. The method in this study can also eliminate re-initializations in the conventional level set methods, and can create new holes inside the material domain. A widely-studied example is used to demonstrate the effectiveness of the present method. This method can be straightforwardly applied to optimization problems including more material phases.

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