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Adaptive topology optimization based on fully error control for separated fields

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In this paper, we develop a new adaptive method for topology optimization by separately refining the analysis mesh and the density field according to different refinement criteria.

Topology optimization has been regarded as one of the most powerful tools to achieve the best designs of continuum structures. Among the existing topology optimization methods, the SIMP method may be one of the most popular approaches, due to its concise mathematical formulation and high computational efficiency. By taking constant element-wise densities as design variables, a power-law penalization model is then applied to compute the actual material properties. In order to prevent the checkerboard patterns and the mesh dependency, certain regularization is usually needed when performing the method, including the sensitivity filtering [1], the minimum size control [2], etc. With the purpose of improving boundary description qualities, nodal design variable-based method has recently attracted increasing attention [3, 4]. In special, Kang and Wang [5] proposed a finite element mesh-independent non-local density field interpolation method. The method successfully avoided the islanding phenomenon and seemed well-suited for topology optimization problems discretized with unstructured meshes.

In order to improve numerical efficiency and boundary description qualities, adaptive methods are introduced into the structural topology optimization [6]. The key concept of the adaptive topology optimization method is to implement refinement only when and where necessary. However, all the existing method either only implement refinement of the density field or employ a same refinement criterion for both the density and displacement fields. There is no reason to bound refinements of both fields together from the perspective of efficiency and effectiveness. Mathematical models and numerical techniques enabling separated refinements of the density field and the analysis mesh are thus highly desirable.

This paper develops a new adaptive method for topology optimization by separately refining displacement and density fields, according to different refinement criteria. Based on a non-local density interpolation model [5], the density value of point x is computed by

$$\rho(x) = \sum_{i \in S_x} \psi_i(x)\rho_i \tag{1}$$

where $\psi_i(x) = d(x - x_i) \Big/ \sum_{j \in S_x} d(x - x_j) \; (i \in S_x)$ is the interpolation function, S_x denotes the density points located within the influence domain of point x and $d(x - x_i) = \left\| x - x_i \right\|^{-2} \quad (i = 1, 2, ..., N^{\text{dens}})$.

Here, we define three indicators for refinements. A dimensionless energy error indicator for the displacement field analysis is defined on each element as

$$\xi_e^{\text{ele}} = \frac{\left\| \hat{\mathbf{e}} \right\|_e^2}{\left\| \mathbf{e} \right\|_{\text{tol}}^2} \quad (e = 1, 2, \ldots, N_e) \tag{2}$$

where energy error $\|\hat{\mathbf{e}}\|_e^2 = \int_{\Omega_e^{\mathrm{ele}}} \left(\boldsymbol{\varepsilon}^* - \tilde{\boldsymbol{\varepsilon}}\right)^{\mathrm{T}} \mathbf{D}\left(\rho(x)\right)\left(\boldsymbol{\varepsilon}^* - \tilde{\boldsymbol{\varepsilon}}\right) \mathrm{d}\Omega \quad \left(e = 1, 2, \ldots, N_e\right)$ and $\|\mathbf{e}\|_{\mathrm{tol}}$ is a given tolerance value of the energy norm $\|\hat{\mathbf{e}}\|_e$. Here, $\tilde{\boldsymbol{\varepsilon}}$ is the approximate solution of the strain field and $\boldsymbol{\varepsilon}^*$ is used to approximate the exact solution.

In order to identify the regions to be refined from the viewpoint of boundary description quality, a gray transitional region indicator (GTR indicator) is defined by

$$\begin{aligned} \eta_i^{\mathrm{cell}} &= \frac{1}{A_i^{\mathrm{cell}}} \int_{\Omega_i^{\mathrm{cell}}} \rho(x) \mathrm{d}\Omega \quad \left(i = 1, 2, \ldots, N_c\right) \\ \eta_e^{\mathrm{ele}} &= \frac{1}{A_e^{\mathrm{ele}}} \int_{\Omega_e^{\mathrm{ele}}} \rho(x) \mathrm{d}\Omega \quad \left(e = 1, 2, \ldots, N_e\right) \end{aligned} \tag{3}$$

for the density field and the displacement field, respectively.

Then, the refinements are separately implemented for the both fields. For the density field, a gray-scale criterion is applied to identify the regions in which the density points are to be refined. Here, the ith background cell is marked to be refined if $\gamma_{\mathrm{L}} \le \eta_i^{\mathrm{cell}} \le \gamma_{\mathrm{U}}$, with γ_{L} and γ_{U} are the lower and upper threshold values of the GTR indicators, respectively. For the displacement field, the eth finite element is marked to be refined if $\xi_e^{\mathrm{ele}} > 1$ or $\gamma_{\mathrm{L}} \le \eta_e^{\mathrm{ele}} \le \gamma_{\mathrm{U}}$.

One simple cantilever beam design problem is shown to illustrate the efficiency and effectiveness of the present method. The design domain with boundary conditions is shown in Figure 1. The volume fraction is set to be $f_{\mathrm{v}} = 0.5$.

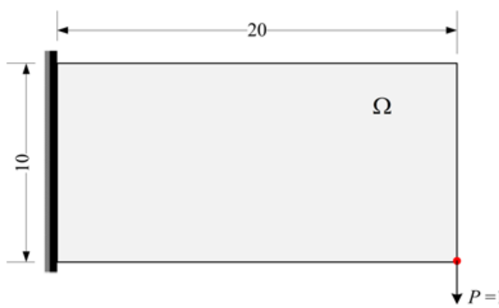
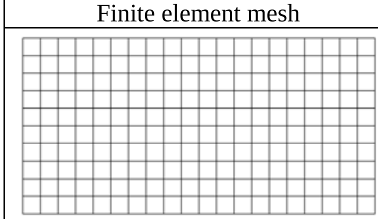
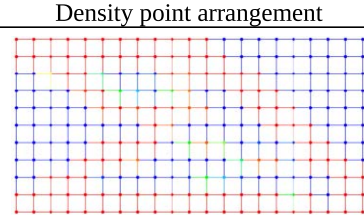
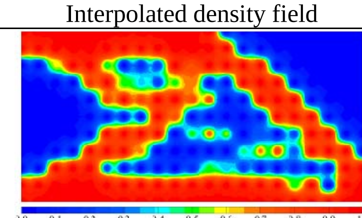


Figure 1: Design domain and boundary conditions for the cantilever beam problem

The optimal solutions under different refinement levels are plotted in Figure 2. It is found that refinements are carried out in different regions for the displacement field and the density field. The displacement field is refined to improve the numerical accuracy of local areas, while the density field is refined to improve the boundary qualities. When the procedure converges, the present method uses only 252.48 seconds, much less than the case with global fine densities and finite element mesh which is total 544.50 seconds. Apparently, the proposed adaptive method effectively improves the computational efficiency of the optimization process.

Finite element mesh	Density point arrangement	Interpolated density field
		

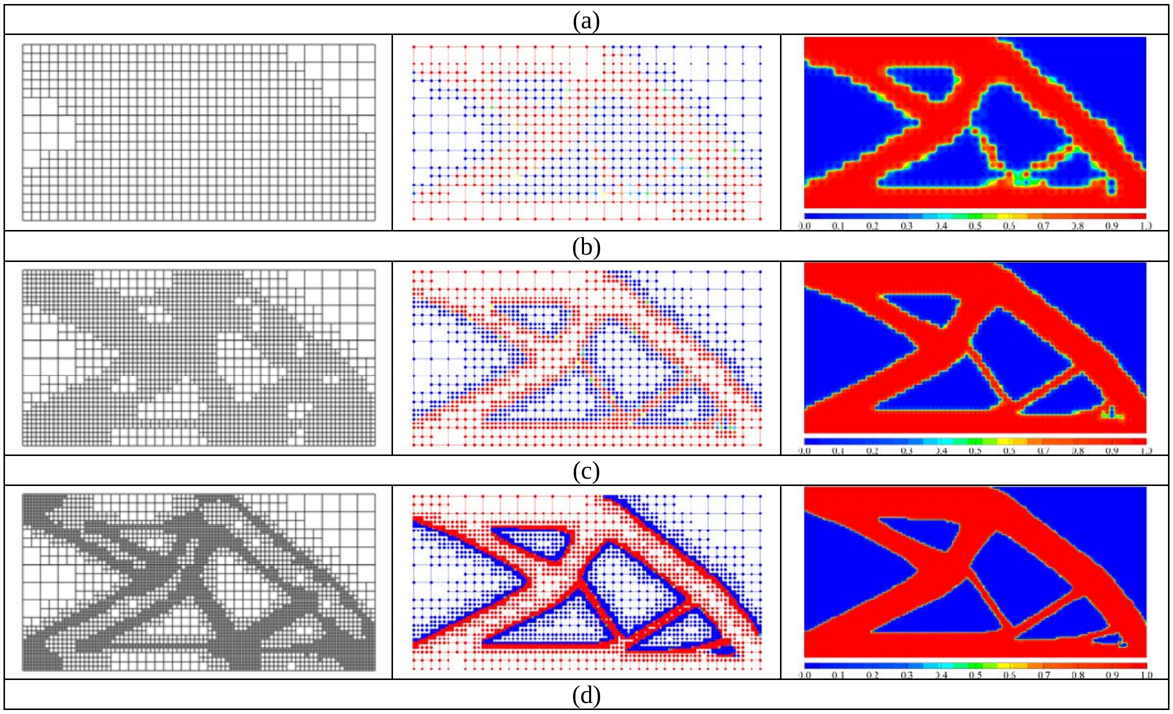


Figure 2: Optimization results under each refinement level

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