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Design of in-plane piezoelectric sensors for static response by simultaneously optimizing the host structure and the electrode profile

D. Ruiz · J.C. Bellido · A. Donoso · J.L. Sánchez-Rojas

D. Ruiz, J.C. Bellido, A. Donoso
Departamento de Matemáticas, ETSII
Universidad de Castilla - La Mancha
13071 Ciudad Real, Spain
David.Ruiz10@alu.uclm.es
JoseCarlos.Bellido@uclm.es
Alberto.Donoso@uclm.es
J.L. Sánchez-Rojas
Microsystems, actuators and sensors group, ETSII
Universidad de Castilla - La Mancha
13071 Ciudad Real, Spain
JoseLuis.Saldavero@uclm.es

1. Abstract

In this work, we present a method for designing piezoelectric transducers, in a static in-plane framework, by simultaneously optimizing the ground structure and the polarization profile of the piezoelectric layers. A new idea is required to avoid the appearance of gray areas for this particular problem. Our approach is finally illustrated with numerical simulations really close to 0-1 designs. This paper will be published in "Strucutal and Multidisciplinary Optimization", doi: 10.1007/s00158-013-0923-8.

2. Keywords: topology optimization · piezoelectric sensor · simultaneous design · electrode profile

3. Introduction

In Donoso and Bellido (2009), a systematic procedure, based on the topology optimization method, was developed to design piezoelectric modal sensors/actuators for plate-type structures; that is to say, piezoelectric transducers that isolate a single mode of a structure, but remain insensitive to the rest of modes. In such cases the host structure, surface bonded by a distributed piezoelectric material, was fixed. The design variable was the polarization profile of the piezoelectric material, a function taking on values -1 or 1, depending on the phase of the excitation voltage. This method works out, and it is capable to give optimal designs that are manufacturable as it only uses extreme values of the polarization function (this was actually explicitly shown in Donoso and Bellido 2009). Later, the investigation followed with the manufacturing of the designs at a micro scale, showing quite good performance of the manufactured devices in Sánchez-Rojas et al. (2010).

We consider that the aforementioned results can be improved if we do not restrict our attention to predetermined host structures, and contemplate this as an additional optimization variable, in such a way that the host structure is no longer fixed, and therefore both the material layout of the host structure and the electrode profile are simultaneously optimized.

In the last decade, many authors have applied topology optimization in piezoelectricity. A pioneering work in this field is Silva and Kikuchi (1999), where the material layout of a host structure is optimized, but keeping fixed the piezoelectric material. In Carbonari et al. (2007) and Luo et al. (2010), among others, the piezoelectric part is included in the optimization problem, obtaining interesting designs of multi-phase actuators. Other authors have considered the optimization of the piezoelectric part together with the polarization distribution such as Kgl and Silva (2005), and Kang and Tong (2008a), where the host structure is also included in the optimization process. Later, Kang and Tong (2008b) improved their approach (through an interpolation function in the actuation voltage), as the new optimized designs show in their recent works Kang and Tong (2011) and Kang et al. (2012).

In contrast to the works commented above, our approach assumes here that the piezoelectric material is surface bonded to the structure wherever it is, so the design variables are essentially two, the material layout of the whole structure-piezo and the electrode profile.The left and right margins should be 25mm, the top and bottom The emphasis of this note is placed on the interpolation scheme for the structure layout proposed in order to avoid gray areas. As we can see later, the designs obtained with our approach are really close to 0-1 designs in both variables, and therefore physically realizable. Regarding applications, many MEMS (microelectromechanical systems)-based actuators like surface probes, micro-grippers or micro-optical devices can be optimized following the procedure shown in this work.
The paper is organized as follows. In Section 4 the physics of the model is briefly described and the formulation of the problem is presented. In Section 5 the interpolation scheme is proposed together with several numerical examples. Finally, some conclusions and future work are commented in the last section.

4. Formulation of the design problem

We consider a two-dimensional structure clamped in its left side and subjected to an in-plane force F_{in} in the midpoint of its right side, as shown in Figure 1. The host structure is bonded to both the top

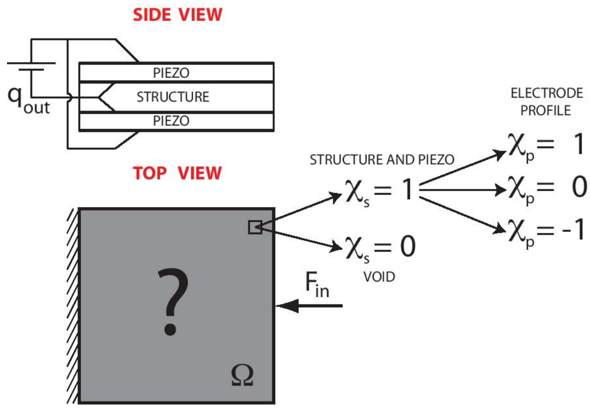


Figure 1: Design domain

and the bottom surfaces with two piezoelectric layers working as sensors. Imposing in-phase identical transducers, we can restrict ourselves to in-plane motion when working as actuators.

Assuming that the piezoelectric layers have negligible stiffness compared to the plate, the whole structure-piezo is deformed according to the equilibrium equation

$$-\text{div}(E_s \chi_s(x) \varepsilon(u, v)) = f, \tag{1}$$

where $x = (x_1, x_2)$ is the spatial variable, E_s is the stiffness tensor of the host structure, (u, v) the in-plane displacement field, ε the strain field and f the mechanical loading. $\chi_s(x)$ is a characteristic function that represents the material layout such that $\chi_s \in \{0, 1\}$ meaning that when $\chi_s = 1$ there is structure and piezo as well, and when $\chi_s = 0$ meaning that there is no structure and piezo neither.

As piezoelectric sensor layers, their deformations produce an electric signal, whose sensor output charge q_{out}, for in-plane motion, can be expressed as

$$q_{out} = \int_\Omega \chi_p(x) \left[e_{31} \frac{\partial u}{\partial x_1} + e_{32} \frac{\partial v}{\partial x_2} \right] dx \tag{2}$$

where e_{31} and e_{32} are the piezoelectric stress/charge constants in x_1 and x_2 directions, respectively. In the following, we will assume that $e_{31} = e_{32} = e$, as in e.g. hexagonal class 6 mm crystals. Here $\chi_p(x)$ is a trilevel function that represents the electrode profile such that $\chi_p \in \{-1, 0, 1\}$ (negative, none, or positive polarization). Notice that the role of the electrode is here crucial because only the area of the piezoelectric sensor covered by an electrode will be electrically affected. That leads us to expect that such a variable takes on extreme values only in order to maximize the sensor response (this is actually true for fixed host structures, as we have mentioned above).

It would be advisable to add a constraint on the maximum stress in the mechanism, but as far as the authors' knowledge, such a difficult issue has not been addressed in the literature in a systematic tractable way so far. Nevertheless, an efficient alternative in compliant mechanisms to indirectly control the maximum stress levels is by constraining the displacement at the input port (Sigmund 1997), and that is what we perform here.

Under all these considerations the design problem of determining simultaneously the material layout of the whole structure-piezo for a fixed volume fraction and the electrode profile in order to maximize the output of the sensor is mathematically formulated as

$$\max_{\chi_s,\chi_p} : q_{out}(\chi_s,\chi_p) \tag{3}$$

subject to

$$\begin{array}{ccc} -\mathrm{div}(E_s\chi_s(x)\varepsilon(u,v)) & = & f \\ u_{in} & \leq & U_{in}^{max} \\ \frac{1}{|\Omega|}\int_\Omega \chi_s \, dx & \leq & V_0 \\ \chi_s & \in & \{0,1\} \\ \chi_p & \in & \{-1,0,1\} \end{array} \tag{4}$$

where U_{in}^{max} and V_0 are the maximum allowed input displacement and volume fraction, respectively.

5. Numerical approach and examples.

The standard approach consists in discretizing the design domain in finite elements and letting each one of them has two variable densities as design variables, that is, ρ_s^e is the usual spatial material density and ρ_p^e the polarization density.

Having discretized the design domain in finite elements, the densities appear when we normalize the design variables in the following way: the SIMP method (Bendsoe and Sigmund 1999) proceeds replacing the binary function χ_s in (1) by the interpolation function $\boldsymbol{\rho_s^3}$, with $\mathbf{10^{-3}} \leq \boldsymbol{\rho_s} \leq \mathbf{1}$ to avoid singularities; and the electrode function χ_p in (2) is replaced by the continuos expression $(2\boldsymbol{\rho_p} - \mathbf{1})$, being now $\mathbf{0} \leq \boldsymbol{\rho_p} \leq \mathbf{1}$. In Kang an Tong (2008a) an efficient way to relax this variable is proposed. In our work, in line with Kgl and Silva (2005), we have checked that a penalty factor in ρ_p showed no significant effects.

After discretization, the topology optimization problem would be

$$\max_{\boldsymbol{\rho_s},\boldsymbol{\rho_p}} : q_{out} = e(2\boldsymbol{\rho_p} - \mathbf{1})\mathbf{L}^T\mathbf{U} \tag{5}$$

subject to

$$\begin{array}{ccc} \mathbf{K}(\boldsymbol{\rho_s})\mathbf{U} & = & \mathbf{F} \\ U_{in}(\boldsymbol{\rho_s}) & \leq & U_{in}^{max} \\ \mathbf{v}^T\boldsymbol{\rho_s} & \leq & V_0 \\ \mathbf{10^{-3}} \leq & \boldsymbol{\rho_s} & \leq \mathbf{1} \\ \mathbf{0} \leq & \boldsymbol{\rho_p} & \leq \mathbf{1} \end{array} \tag{6}$$

where $\mathbf{L}$ is the strain matrix, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{U}$ and $\mathbf{F}$ are the global displacements and force vectors, respectively, $\boldsymbol{\rho_s}$ and $\boldsymbol{\rho_p}$ are the vectors of both design variables, and $\mathbf{v}$ is a vector containing the volume of the elements.

The discrete problem is numerically solved by using MMA, a gradient-based optimization algorithm (Svanberg 1987). In this case, the sensitivity analysis does not entail any extra difficulties and that is the reason why it has not been included here.

On performing numerical simulations in the previous problem, it is observed that gray areas still appear concerning variable $\boldsymbol{\rho_s}$ in the final designs, though the well-known mesh-independent filter is used. The point is that these gray areas are contributing to the sensor output as being covered by an electrode. A way to avoid this is to modify the cost functional by multiplying it by a new term $R(\boldsymbol{\rho_s})$ that penalizes in a progressive way these gray areas. The heuristic scheme proposed R, depicted in Figure 2, is a piecewise linear function, with very small slope but positive in the first part, and much steeper in the second one. The parameters η and ζ are tunable, but we have kept them fixed in all the simulations because it was very effective in removing gray areas (we start taking $\eta = 0.8$ and $\zeta = 0.01$, and after some iterations these values eventually change to $\eta = 0.9$ and $\zeta = 0$). It is important to emphasize that the lack of differentiability in R does not affect the numerical results at all. However, if needed, it can

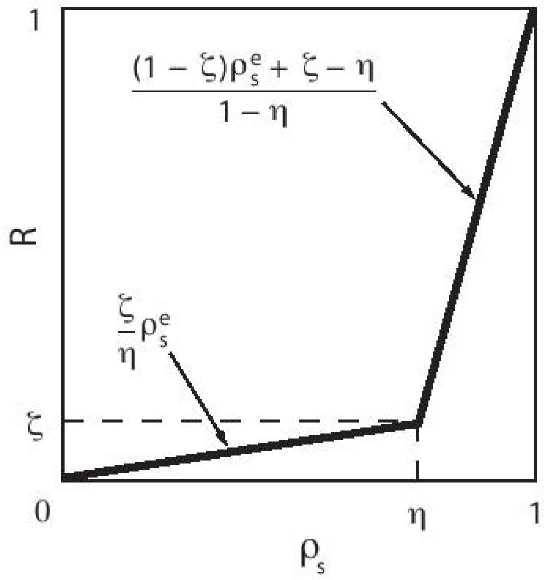
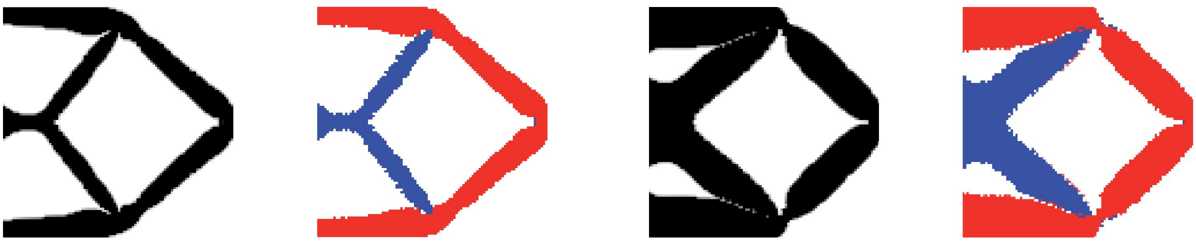


Figure 2: Penalization function $R(\boldsymbol{\rho_s})$.

be smoothed. This is the best and the less intrusive way we have found to help SIMP to get 0-1 designs. Hence, maximizing now $q_{out}R$ instead of q_{out} the problem is successfully solved because the gray areas almost completely disappear and the designs obtained are really close to 0-1 designs.

We illustrate our optimization approach through several numerical examples of interest in the area of MEMS because of the sizes used. We consider a square plate of length $L = 1000$ μm and thickness $h_s = 50$ μm. The material properties for the host structure are those corresponding to silicon, that is, Young's modulus $E = 165$ GPa and Poisson's ratio $\nu = 0.28$. As the piezoelectric properties only appear as a scale factor in the objective function, they do not affect the optimized designs. Due to symmetry, only a half of the design domain is discretized by using a mesh of 50×100 elements. Also, a continuation approach has been used.

In Figure 3, both the material layout and the electrode profile are shown for two different volume fractions and the same ratio $\frac{F_{in}}{U_{in}^{max}} = 2 \times 10^5$ N/m, which can be interpreted as the stiffness at the input port k_{in}. The material layout is practically black and white, and the electrode profile is nearly blue and red, meaning areas in tension and compression, respectively. These layouts makes us easy the fabrication process, which is a work in progress.

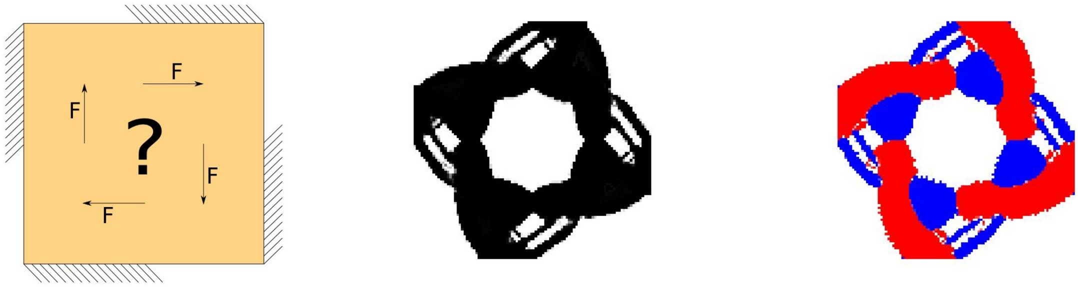


(a) $V_0 = 0.3$ (b) $V_0 = 0.5$

Figure 3: Material layout (left) and electrode profile (right) for different volume fractions: (a) $V_0 = 0.3$ and (b) $V_0 = 0.5$.

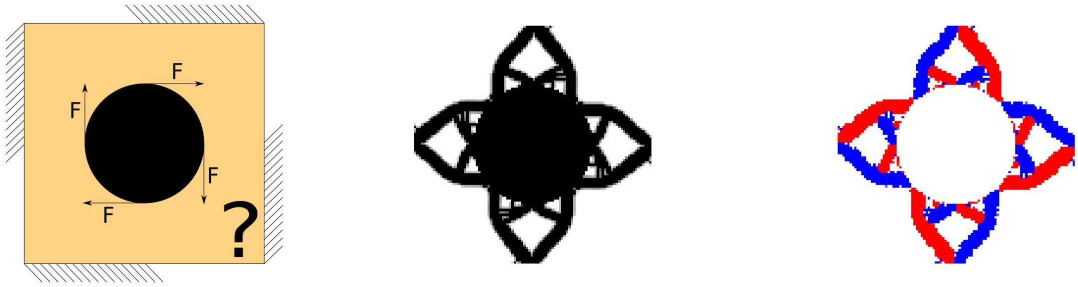
In Figure 4 and Figure 5 two more examples corresponding to the design of rotary piezoelectric actuators are shown. In the latter, a passive area is included in the design domain.

It is also interesting to see how the variation of the sensor output q_{out} is versus the volume fraction V_0. Here we are replacing the inequality volume constraint in the problem by the equality one. As we can see in Figure 6, there exists an optimal volume fraction, around 0.8. Below this value the maximum displacement constraint is saturated, and above it, it is not, since the structure becomes too rigid to attain such a displacement value.



(a) Boundary conditions (b) Material layout (c) Electrode profile

Figure 4: Example without passive zone $V_0 = 0.5$.



(a) Boundary conditions (b) Material layout (c) Electrode profile

Figure 5: Example with passive zone $V_0 = 0.4$.

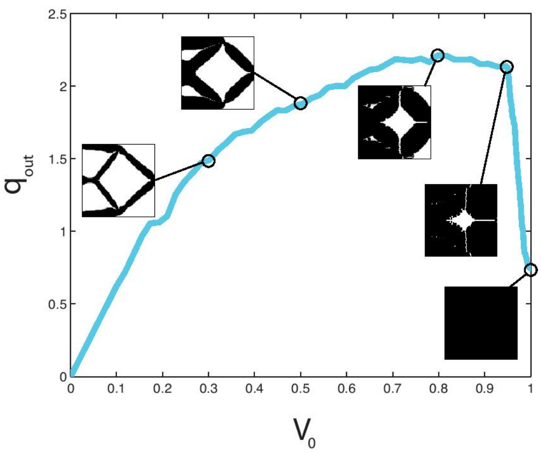


Figure 6: Sensor output q_{out} versus volume fraction V_0 for $k_{in} = 2 \times 10^5$ N/m.

6. Conclusions and future work.

In this note, we design in-plane piezoelectric sensors that maximize the static response. The design variables chosen are two: the whole structure-piezo layout and the polarization profile of the piezoelectric material. To successfully solve the simultaneous design problem is required a new particular interpolation scheme that avoids the appearance of gray areas. As seen in the examples, both layouts obtained are really close to 0-1 designs, what makes easy the fabrication process (a work in progress). Finally, it is important to point out that the situation treated here can be considered as the previous case of a more general and interesting problem in the context of modal filtering to be dealt with in the near future.

7. Acknowledgements

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