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Hierarchical topology and shape optimization of crash-loaded profile structures

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1. Abstract

In many areas of the structural design methods for topology optimization are applied very successfully but up to now the topology synthesis of crash-loaded structures is difficult. Responsible for this are among others the nonlinearities occurring during a crashworthiness problem such as large displacements and rotations, contact and nonlinear, velocity-dependent material behavior taking into account failure models. Furthermore the huge number of design variables, the existence of bifurcation points and the costly determination of sensitivity information cause problems. Usually crash simulations are nonlinear dynamic finite element simulations which use explicit time integration.

One approach to overcome these problems for the optimization of profile cross-sections is the method of Graph and Heuristic Based Topology Optimization (GHT), which separates the optimization problem into an outer optimization loop for the topology optimization and an inner optimization loop for the shape and sizing optimization. In the outer optimization loop heuristics (rules) derived from expert knowledge modify the topology and the shape of the structure based on the results of finite element crash simulations. In the inner optimization loop the automatically generated shape and sizing parameters of the structure are optimized with mathematical optimization methods.

During the complete optimization procedure the three-dimensional structure is abstracted by a planar mathematical graph and algorithms of the graph theory are used to modify the structure and to check the adherence of manufacturing constraints.

2. Keywords: Topology optimization - Crashworthiness - Heuristic approach - Graph theory - Optimization in nonlinear dynamics

3. Introduction

In many areas of the structural design methods for topology optimization are applied very successfully but up to now the topology synthesis of crash-loaded structures is difficult. Responsible for this are among others the nonlinearities occurring during a crashworthiness problem such as large displacements and rotations, contact and nonlinear, velocity-dependent material behavior taking into account failure models. Furthermore the huge number of design variables, the existence of bifurcation points and the costly determination of sensitivity information cause problems. The design space has a large number of local optima which makes the determination of the global optimum even more difficult. Usually crash simulations are nonlinear dynamic finite element simulations which use explicit time integration.

In the recent years extensions to already existing optimization methods or complete new optimization methods have been developed to enhance the possibilities of optimization in nonlinear dynamic structural problems. Pedersen [1] developed a method which uses co-rotational beam elements with a plastic zone model to mesh the design space. By omitting the contact between the beam elements and by using numerical damping, the numerical stability of the simulation is enhanced and implicit time integration can be used to solve the problem. Therefore sensitivity information can be obtained analytically. The geometrical properties of the beam elements are used as design variables and the differences between the acceleration-time-curve of the structure and a prescribed acceleration-time-curve are minimized.

The Hybrid Cellular Automaton (HCA) method [2] divides the design space into cells which have an artificial density. The material properties of these cells are coupled to the artificial densities, which are used as design variables. The design variables are modified by a heuristic update scheme which is based on the optimality criterion of a homogenous distribution of the internal energy and needs no sensitivity information. Because of the heuristic update scheme the objective function is fixed and cannot be modified.

The combination of a nonlinear dynamic analysis domain and a linear static optimization domain is used in the Equivalent Static Loads Method (ESLM) [3]. Every iteration consists of a nonlinear dynamic analysis and a linear static optimization, whose result is used as the initial design of the next iteration. Equivalent static loads are calculated for several time steps of the nonlinear dynamic analysis. The loads are calculated such, that they cause the same structural response field in the initial design of the linear static optimization as the structure has in the nonlinear dynamic simulation at the specific time. Despite the chosen structural response (e.g. displacement field) the structural responses of the nonlinear dynamic analysis and the initial design of the linear static optimization are different. The equivalent static loads of different time steps are used in multiple loading conditions in the optimization. The definition of typical crashworthiness optimization functions like energy absorption or plastic strain may be difficult because of the linear static mechanical definition of the optimization problem.

An approach for the combined topology, shape and sizing optimization of profile cross-sections is the method of Graph and Heuristic Based Topology Optimization (GHT) [4], which separates the optimization problem into an outer optimization loop for the topology modification and an inner optimization loop for the shape and sizing optimization. In the outer optimization loop heuristics (rules) derived from expert knowledge modify the topology and the shape of the structure based on the results of finite element crash simulations. In the inner optimization loop the automatically generated shape and sizing parameters of the structure are optimized with mathematical optimization methods. The two optimization loops are hierarchically nested in each other following the concept of the Bubble Method [5].

The complex process of developing heuristics for the topological modifications of the structure is discussed in another contribution of the WCSMO-10 [6].

During the complete optimization procedure the three-dimensional structure is abstracted by a planar mathematical graph. Hereby the geometric optimization problem is reduced to a two-dimensional one, although the structure itself and all performed finite element simulations are three-dimensional. All changes of the structure are performed to the graph representation and the graph is the basis for the automatic generation of finite element models for crash simulations. Algorithms of the mathematical graph theory are used to modify the graph representation of the structure and to monitor the adherence of manufacturing constraints.

This contribution differs from previous work [4] in the controlling of the heuristics. For the new control mechanism an application example is provided.

4. Abstraction of mechanical structures by mathematical graphs

For the abstraction and description of mechanical structures a special graph syntax has been developed based on the work of [7]. The profile cross-section of the structure and additional information like the extrusion length and the density of the material are stored within the graph. An overview of the different kinds of vertices used in this graph syntax is given in Fig. 1. The graphs are simple, undirected and planar and can only describe structures whose profile cross-section is topological constant over the complete length of the structure.

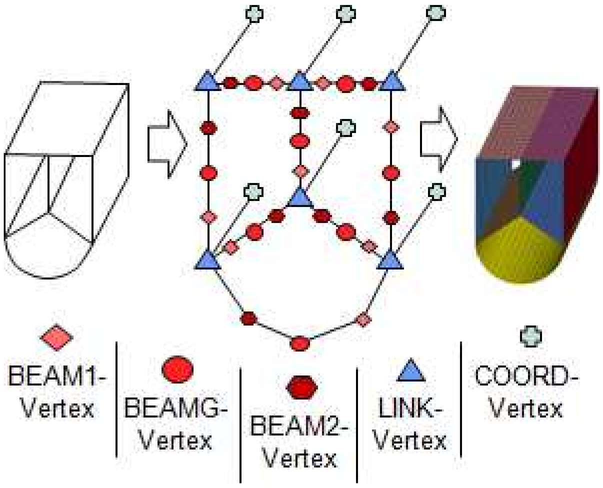


Figure 1: Abstraction of a mechanical structure by a graph [4]

A wall of the profile cross-section of the structure is described by a graph beam element which consists of a BEAM1-Vertex, a BEAM2-Vertex and a BEAMG-Vertex. The first two vertices are used to define the wall’s orientation while the BEAMG-Vertex stores information about the thickness and the curvature of the wall. The kind of connection between the walls (e.g. a straight connection or the usage of a radius) is defined by the LINK-Vertex. These vertices are connected with the COORD-Vertices, which contain Cartesian coordinates and position the walls within the profile cross-section. The connections of the vertices are realized by using undirected edges. The PARAM-Vertex (not part of Fig. 1) is not connected to any other vertex and stores general information like the extrusion length of the structure or the material density.

There are two main advantages of using this graph syntax for the abstraction of mechanical structures. The first one is the access to a wide range of algorithms home to the mathematical graph theory. The second advantage is the direct control and easy manipulation of the structure. Even complex geometrical operations like topology changes can be performed easily with the ASCII-based graph.

One area of application of the mentioned graph based algorithms is the check of manufacturing constraints like minimum and maximum wall thicknesses, minimum connection angles or maximum number of walls connected to each other. At the moment the implemented manufacturing constraints are based on the manufacturing processes of aluminum extrusion profiles, which have been the first application area of the presented optimization method.

An automatic finite element model creation from a mathematical graph with the mentioned graph syntax is needed for the optimization process. For this purpose a JAVA-based program has been developed, the GRAMB (Graph Based Mechanics Builder). In addition to the information of the graph a spline for the extrusion of the profile cross-section can be defined, non-design spaces and local modifications of the structure like wholes for screws. GRAMB translates all this information into program execution commands for one of the following CAE (computer aided engineering) systems: Altair HyperMesh®, Dassault CATIA® or SFE CONCEPT®. The CAE system is started and controlled by GRAMB and creates the finite element mesh.

5. Procedure of the Graph and Heuristic Based Topology Optimization

The basic principle of this optimization approach is the separation of the optimization problem into two different optimization loops as described in the introduction of this contribution. Two types of heuristics are used within the outer optimization loop: concurrent and non-concurrent heuristics. Concurrent heuristics want to perform a topology modification of the structure and because there is only one topology modification allowed in each iteration, these heuristics are concurrent to each other. The heuristics are briefly introduced in chapter 6 of this contribution. Non-concurrent heuristics only want to modify the shape or sizing parameters of the structure.

The basic idea of the sequence of this optimization procedure is illustrated in Fig. 2 and can be summarized in detail for one iteration as:

Outer optimization loop

1. Extraction of the finite element simulation results (e.g. node velocities or element strains) of the result of the last iteration
2. Activation of the concurrent heuristics with this information
3. Determination of the heuristics' proposals of topology change
4. Evaluation of the mechanical performance (objective function value and constraint violation) of these proposals with a single function call (finite element simulations of all load cases) for each heuristic
5. Check for the fulfilment of the stop criteria
6. Modification of the topology of the structure according to the heuristic with the best performance
7. Activation of the non-concurrent heuristics based on the new structural design
8. Modification of the shape and sizing parameters of the structure according to all non-concurrent heuristics

Inner optimization loop

1. Determination of the design variables and their borders for the current structure (determination of the borders with an algorithm which simulates crystal growth)
2. Combination of the information of the design variables with the definition of the optimization problem (objective function, constraints and load case information) to an optimization model
3. Calculation of the maximum number of function calls of this iteration based on the number of design variables
4. Start of a shape and sizing optimization
5. Determination of the result of the shape and sizing optimization and execution of copy and data cleanup operations

Almost every step is influenced by the settings of the method like the maximum number of function calls in the inner optimization loop or the parameters of the heuristics. Changes to the structure are always done with respect to the manufacturing constraints, therefore the current design is always feasible.

For the execution of this optimization process the JAVA-based program TOC (Topology Optimizer for Crash Loaded Structures) has been developed. TOC has an interface to the general purpose optimization software LS-OPT® and uses this software for the shape and sizing optimization of the inner optimization loop. The finite element simulations (explicit time integration) are performed with LS-DYNA®. The ASCII-based result files of LS-DYNA® are used for the extraction of the simulation results.

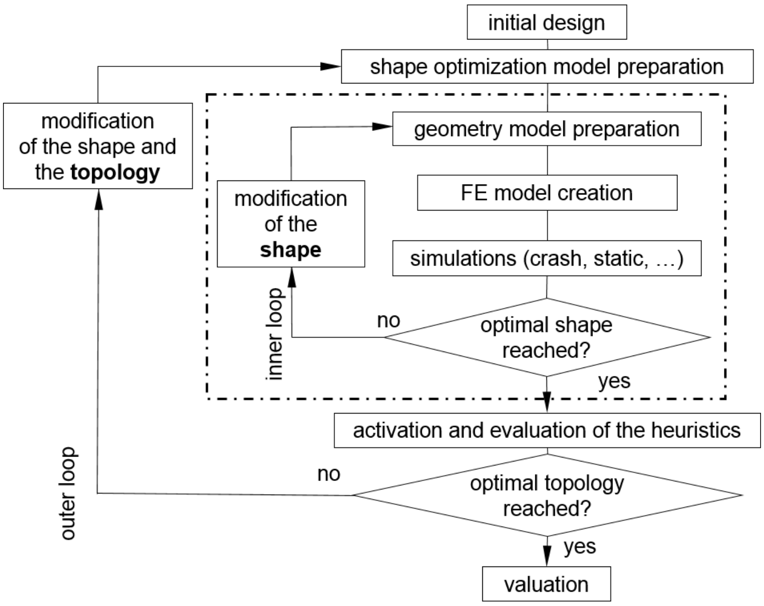


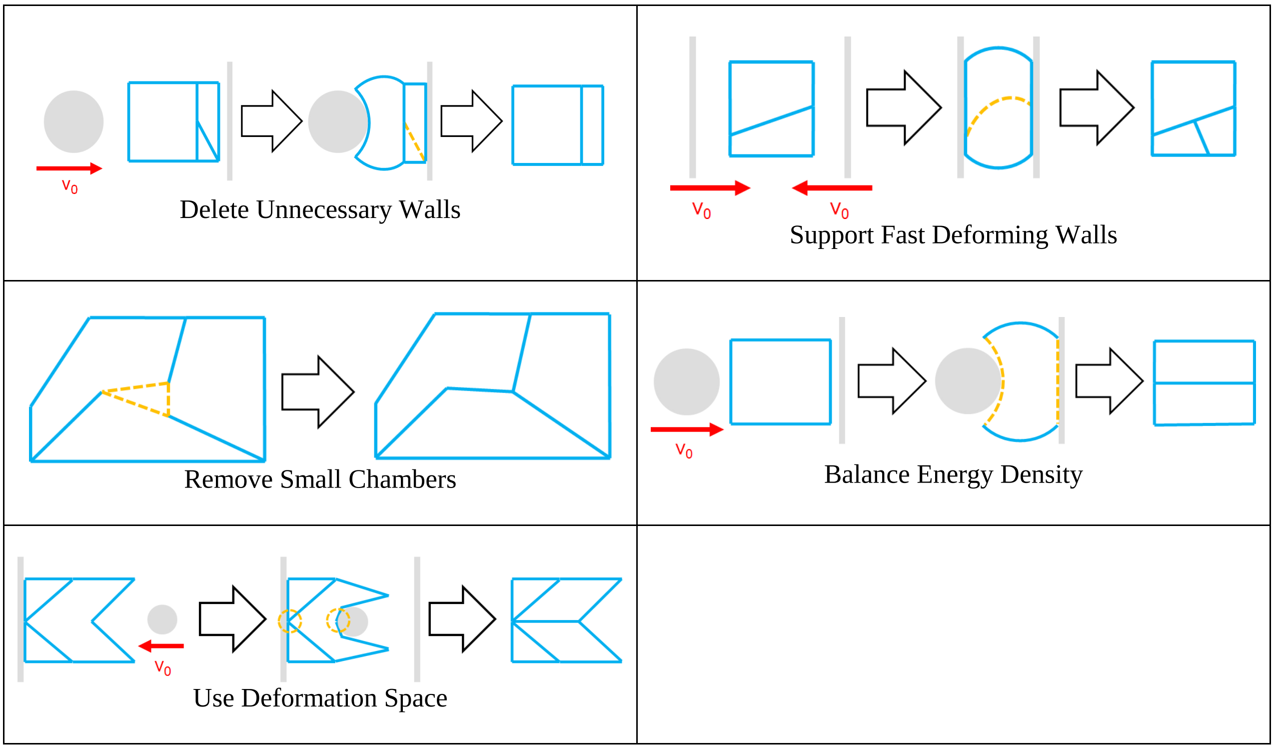
Figure 2: Procedure of a Graph and Heuristic Based Topology Optimization

Because of the characteristics of a crashworthiness optimization problem, especially the huge number of local optima, the bifurcation points and the lack of sensitivity information, the presented method of the Graph and Heuristic Based Topology Optimization will probably find only a good local optimum. In the shape and sizing optimizations of the inner optimization loop genetic algorithms are used. Even in these sub-optimizations the available number of function calls will usually not be high enough to find the global optimum within the given topology class. Therefore the stop criterion of the inner optimization loop is usually a maximum number of function calls. The stop criterion of the outer optimization loop will be fulfilled, if the number of maximum iterations is reached or all heuristics failed in improving the structure further. If this criterion is fulfilled, the optimization procedure will end.

6. Overview of the heuristics

This section briefly introduces the used heuristics. More detailed information including the mathematical structure of the heuristics can be found in [4]. The process of the development and the algorithmization of the heuristics based on expert knowledge is discussed in [6]. Because the heuristics are based on expert knowledge of automotive crash engineers they will only perform useful structural modifications for crashworthiness problems.

In Fig. 3 five concurrent heuristics are shown. The view of the structures (in the figure) is in the extrusion direction. The goal of the heuristic “Delete Unnecessary Walls” is to remove walls from the structure’s profile cross section which contribute little to the structure’s mechanical properties. The heuristic “Support Fast Deforming Walls” detects walls which have a higher deformation speed than the rest of the structure (e.g. walls with a tendency towards buckling) and supports these walls by an additional wall. The simplification of the structure by transforming small chambers into single walls is the goal of the heuristic “Remove Small Chambers”. The intention of the heuristic “Balance Energy Density” is the homogenization of the inner energy density in the strcture. For this purpose walls with a high inner energy density are connected to walls with a low inner energy density. The idea of the heuristic “Use Deformation Space” is the efficient use of the available deformation space by connecting corners of the structure which have a high relative displacement to each other.



Delete Unnecessary Walls

Support Fast Deforming Walls

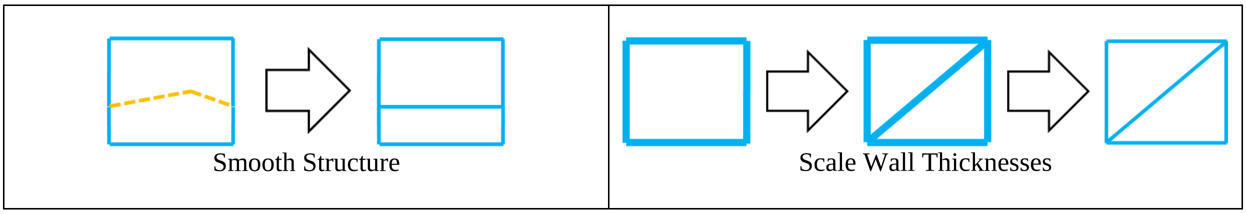
Remove Small Chambers

Balance Energy Density

Use Deformation Space

Figure 3: Concurrent heuristics for topology changes

In Fig. 4 two non-concurrent heuristics are shown. The heuristic “Smooth Structure” has the task to reduce the number of design variables of the current structure by combining two walls which are connected by an obtuse angle like the two dashed walls. Every topology modification of the structure causes a jump-like change of the structure's mechanical behaviour. To reduce these discontinuities in the optimization flow, the heuristic “Scale Wall Thicknesses” scales the wall thickness of the structure such, that the structure's mass does not change despite the topology change.



Smooth Structure

Scale Wall Thicknesses

Figure 4: Non-concurrent heuristics

7. Application examples

The set-up of the application examples consists of a 50x100x5 mm large frame, which is impacted by a rigid sphere with a diameter of 30 mm and clamped on one side (Fig. 5). The frame is meshed with shell elements and the sphere with solid elements. The global edge length of the finite elements is 2.5 mm.

The material law is piecewise linear in the plasticity region and simulates an aluminum extrusion alloy with a yield strength of 240 MPa.

The used manufacturing constraints are: minimum thickness (0.5 mm), maximum thickness (5.0 mm), minimum wall distance (10 mm) and minimum wall connection angle (15°).

The objective function is the intrusion of the sphere into the frame. The intrusion is evaluated as the displacement of the sphere's center in the y-direction. A mass constraint is used, the mass of the frame must not exceed 0.027 kg. Due to the kind of the objective function (stiffness maximization against the intrusion), this restriction must be active for the optimum design. Therefore an option of this optimization method is used which scales the wall thicknesses such, that the mass of all designs is 0.027 kg.

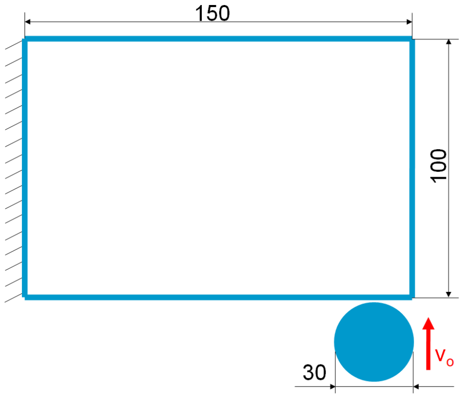


Figure 5: Set-up of the application example

The results of the application examples are summarized in this section and compared to each other. They vary in the maximum number of allowed function calls, the mass and the velocity of the sphere. The initial kinetic energy is the same in every configuration.

Example 1: Sphere mass: 1.757 kg, sphere initial velocity: 6.25 m/s, function calls: about 6500.
Example 2: Sphere mass: 0.1098 kg, sphere initial velocity: 25 m/s, function calls: about 7700.
Example 3: Sphere mass: 1.757 kg, sphere initial velocity: 6.25 m/s, function calls: 20.

The number of function calls in the first two examples is high in order to improve the chance of getting near to the unknown global optimum. In example 3 no shape and sizing optimizations are performed in the inner optimization loop. Only the heuristics in the outer optimization loop are used to change the structure. Therefore the number of function calls is low compared to the other two application examples.

The reference and initial design of the first two application examples is a rectangular frame with a wall thickness of 6 mm and a mass of 0.0405 kg. This violates the mass constraint, but the additional mass is necessary to stop the sphere. The intrusions are 45.68 mm (6.25 m/s) and 44.98 mm (25 m/s and reduced sphere mass). The third application example uses a triangular frame with a mass of 0.027 kg as the initial design. The graphs of the initial designs are shown in Fig. 6.

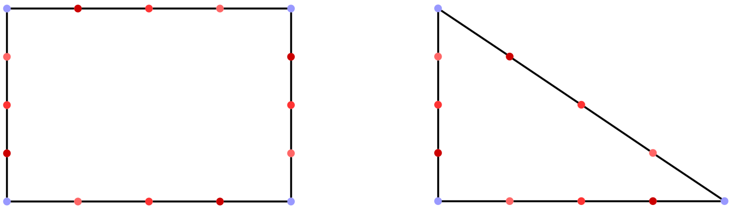


Figure 6: Graphs of the initial designs (left: for application example 1 + 2, right: for application example 3)

The structural changes are shown for each of the three application examples in Fig. 7 - 9. Only the results of accepted iterations, which lead to a reduction of the objective function, are considered. In the first and the second application example the first iteration consists only of a run through the inner optimization loop to find a good design within the given initial topology class. Application example 3 uses only the outer optimization loop and therefore starts directly with a topology modification of the structure.

The optimization histories are summarized in Table 1. Green values indicate, that the result of the specific iteration leads to an improvement of the structure and is accepted whereas red values indicate the opposite. The best result of all iterations is highlighted by blue color.
Compared to their initial designs the intrusion has been reduced by 84.7 % (example 1), 84.3 % (example 2) and 64.1 % (example 3). In the last application example the improvements are significantly lower because of the strongly limited number of function calls compared to the other two examples.

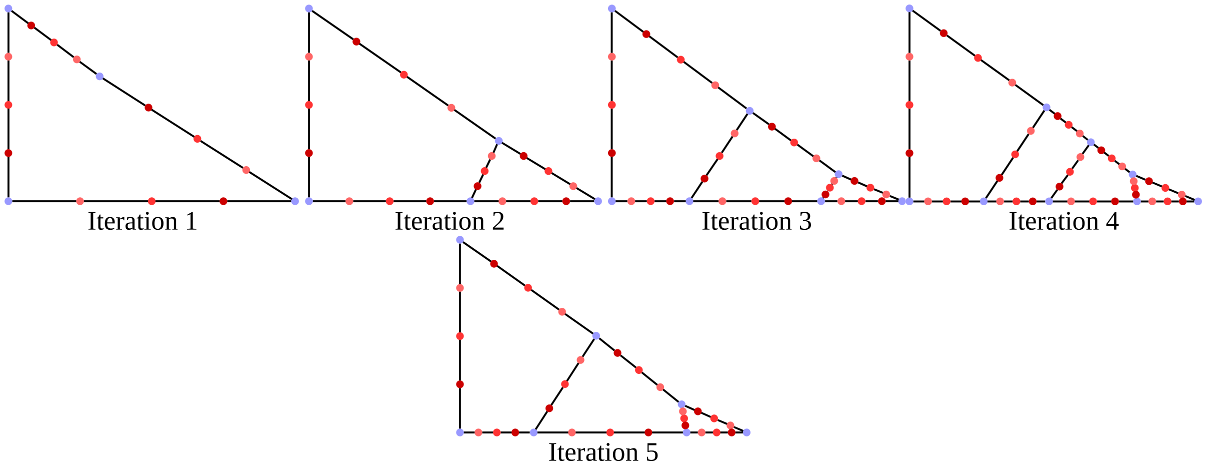

Iteration 1 Iteration 2 Iteration 3 Iteration 4 Iteration 5

Figure 7: Graphs of the results of the iterations of application example 1

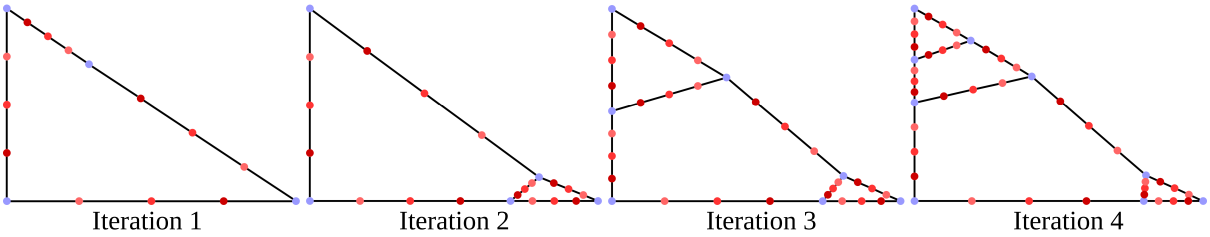

Iteration 1 Iteration 2 Iteration 3 Iteration 4

Figure 8: Graphs of the results of the iterations of application example 2

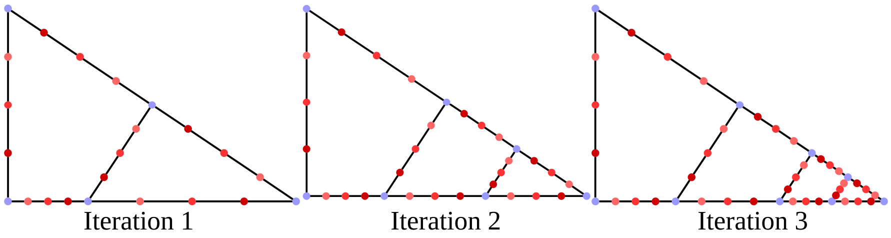

Iteration 1 Iteration 2 Iteration 3

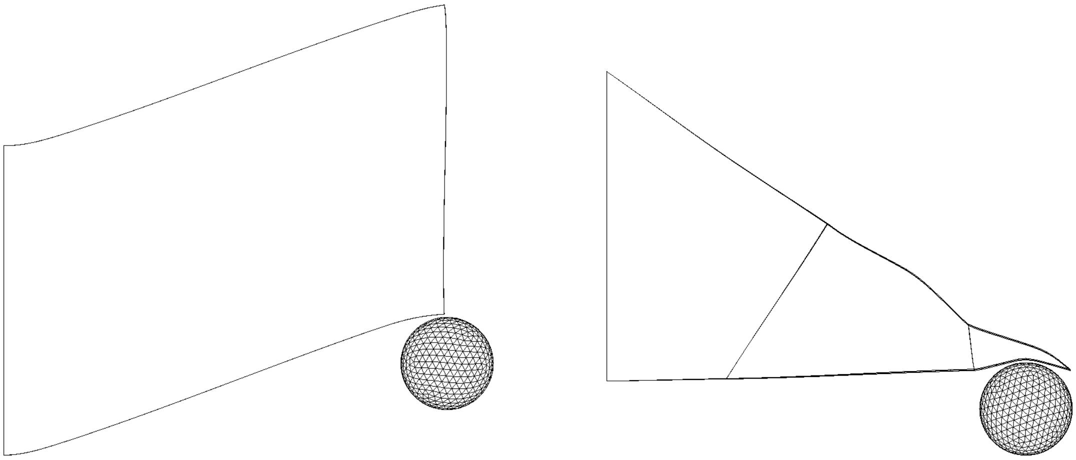
Figure 9: Graphs of the results of the iterations of application example 3

Table 1: Optimization Histories

	Intrusion (y-displacement of the sphere's center)		
	Example 1 [mm]	*Example 2 [mm]*	*Example 3 [mm]*
Initial Design	45.68	44.98	25.24
Iteration 1	21.59	15.37	18.87
Iteration 2	10.15	8.26	10.74
Iteration 3	8.22	7.87	9.05
Iteration 4	7.35	7.08	13.14
Iteration 5	6.98	7.26	15.88
Iteration 6	7.33	8.03	17.02
Iteration 7	7.58	7.10	-
Iteration 8	10.30	8.10	-
Iteration 9	14.11	7.78	-
Iteration 10	8.63	-	-

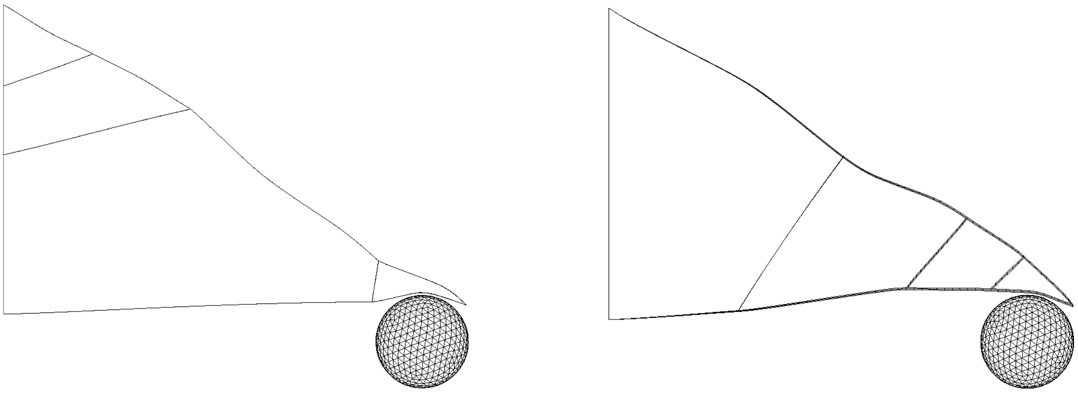
The deformation behaviors of the reference design (for a sphere mass of 1.757 kg and a sphere velocity of 6.25 m/s) and the three optimization results are shown in Fig. 10. The basic principle of the results of the first two application examples is similar although the ratio between sphere mass and sphere initial velocity is different. The majority of the deformation energy is concentrated in the tip of the frame through a controlled deformation mechanism.

Due to the strongly limited number of function calls the result of the third application example is not as stiff as the other results. Here the plasticity is distributed more homogenous over the structure.



Initial design (example 1)

Result of application example 1



Result of optimization example 2

Result of optimization example 3

Figure 10: Deformation behaviors

If the result of application example 1 is impacted by a sphere with the configuration of application example 2 (sphere mass of 0.1098 kg and sphere initial velocity of 25 m/s), the intrusion would be 6.94 mm und thus the result would have the same performance as the result of application example 2, which has been optimized for this specific configuration.

The other way around: if the result of application example 2 is impacted by a sphere with a mass of 1.757 kg and an initial velocity of 6.25 m/s (configuration of example 1), it would have an intrusion of 15.71 mm because the different load would cause one wall to buckle (Fig. 11).

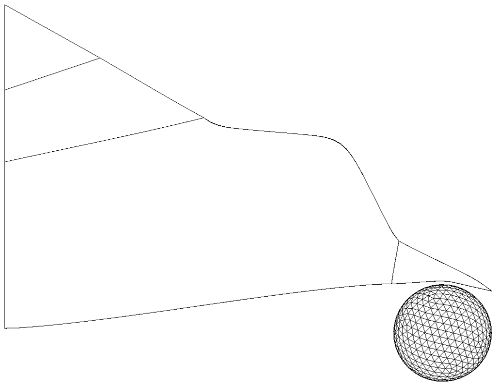


Figure 11: Deformation behavior of the result of application example 2 impacted by a sphere with the configuration of application example 1

Evidently the result of application example 1 is a good local optimum for the example configurations 1 and 2 while the result of the second example is only useful for its own configuration. It can be assumed that both are only local optima.

8. Conclusion

Beside the shown application examples, the method of Graph and Heuristic Based Topology Optimization (GHT) was tested for several crash applications coming from the crash development of car bodies. The results are promising and interesting and we could not find better results by using other methods. But we have to note, that the presented method will find most probably only local optima.

It has been shown that the mechanical performance of the optimization procedure result can be scaled by the number of maximum function calls. For the simple optimization problem of the application example presented here (minimization of the intrusion with a mass constraint), a good result can be reached with 20 function calls by using purely the heuristics and no shape and sizing optimization in the inner optimization loop.

Further research activities will focus on the improvement of the computational efficiency of the method and the finding of the global optimum in crashworthiness topology optimization problems.

9. Acknowledgements

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