10th World Congress on Structural and Multidisciplinary Optimization
May 19 - 24, 2013, Orlando, Florida, USA

Shape optimization under vibroacoustic criteria in the mid-high frequency range

R. Troian, S. Besset, F. Gillot

D2S/LTDS, Ecole Centrale de Lyon, 36 av., Guy de Collongue, 69134 Ecully Cedex, France
Renata.Troian@ec-lyon.fr Sebastien.Besset@ec-lyon.fr Frederic.Gillot@ec-lyon.fr

1. Abstract
The aim of the conducted research is to develop an original shape optimization method under vibroacoustic criteria within the specific mid high frequency range. The energy density of the acoustic field in the considered system is the quantity to be optimized. We use the Simplified Energy Method (MES), which gives a solution that only depends on the cavity shape, not on the material properties and doesn't need fine meshes. Firstly, the proposed shape optimization method aims at avoiding the remeshing during the optimization process. Secondly, it has to model the acoustic cavity surface exactly. To achieve this goal, we rely on a transformation function which map a 3D cavity surface on a 2D domain. Hence, the optimization is conducted on this function directly. In the present contribution, we first demonstrate the viability of our approach by considering a 3D ellipsoid cavity. The transformation function is the projection function mapping a unit sphere on a 2D area. The considered parameters are the radii of the ellipsoid. The energy density is calculated at several points inside the ellipsoid that are chosen as test points. The input power flow (primary source of acoustic excitation) is situated on the part of the boundary whose position remains unchanged during the optimization process to avoid the respiration effect. As a result, the governing equation to calculate the energy density at the test points is defined on the 2D area and depends on two variables, i.e. the ellipsoid radii. The optimization problem consists in minimizing the objective function (related to the energy density) considering constraints equations linked to the geometry of the system (the volume of the ellipsoid must remain constant). At this stage we are not working on developing new optimization algorithm that is why a standard Matlab optimization function is used for the calculations. The 2D area is discretized once before the first iteration. To study the reliability of the optimization process the constrained condition was reformulated in several ways and the same optimal results were obtained proving the validity of the solution. The method developed within the framework of presented study allows one to optmize the vibroacoustic system of arbitrary shape choosing the appropriate transformation functions. To prove this, the cavity modelization with the help of B-spline surfaces will be the next step in our research.
2. Keywords: energy method; shape optimization; vibroacoustic criteria

3. Introduction
Shape optimization approaches taking into account vibroacoustic criteria are in demand for many engineering applications, for example for the noise reduction in the transporting domains or in the civil engineering. Such approaches usually aim at providing as quick as possible an optimal shape of the part which has a specific and controlled vibroacoustic behavior. The optimization loop involved must then show a balance between the complexity level of the physical models involved, and their computational cost. To deal with the vibroacoustic phenomena the energy methods were developed. They are well adapted to medium and high-frequency situations and yield smaller matrices and fast optimization processes. Moreover, energy methods are often used as alternatives in the high and medium frequency ranges. Among these methods, the most widespread remains the Statistical Energy Analysis (SEA) [12], which provides the mechanical energy of complex built structures. The energy flow variables in the SEA model are well suited for use in optimization algorithms because of their smooth frequency response functions. DeJong in [6] gives the examples of the use of SEA in optimizing the noise reduction and the sound quality of a machinery enclosure and a vehicle body. Chavan and Manik in [5] present the optimization of an automobile model for cabin noise reduction by calculating the design sensitivity vector for damping loss factors of subsystems using the transmission path approach. In this paper, we use a local energy formalism, as first proposed by Nefske and Sung [13] and improved by many subsequent authors [11, 9, 1, 8]. In the following discussion, this energy method will be called the Simplified Energy Method (or MES). The optimization of the vibroacoustic characteristics using MES was presented by Besset and

Ichchou in [2], regarding the optimizing absorption coefficients at the boundaries of an acoustic cavity.

4. Simplified Energy Method. General description.

The Simplified Energy Method method operates two continuous fields to describe the energy transfer inside of the medium. The first energy quantity is total energy density $W(s,t)$ defined as the sum of the potential energy density and the kinetic energy density. The second energy variable $\mathbf{I}(s,t)$ is the energy flow. The energy balance equation from continuum mechanics obeying conservation principle, which govern the energy density in various vibrating system can be described by equation:

$$\Pi_{diss} = \overline{\nabla} \cdot \overline{\mathbf{I}}, \tag{1}$$

where $\overline{\nabla}$ is the gradient operator, Π_{diss} is the power density dissipated. The damping model adopted here is the same as in SEA, *i.e.* dissipated power density is proportional to the energy density and can be neglected. The validity of this simplification has been discussed in the literature about SEA [12]. Equation (1) is the energy balance relation for all elastoacoustic media and it is regarded as a valid method for steady or transient analysis. In order to derive the energy density equations, a wave description of vibrational-acoustical behavior is definitely adopted. In the considered problem symmetrical propagating disturbances in a medium are taken into account. To provide the required vibroacoustic characteristics of

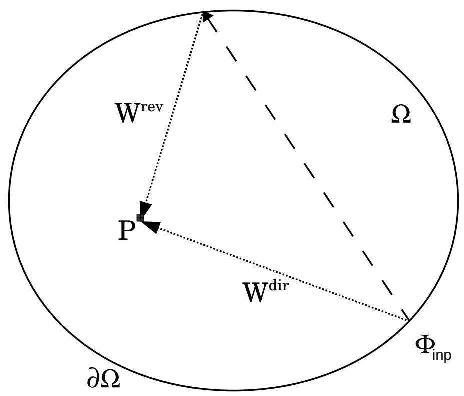

Figure 1: MES formulation: direct and reverberated fields.

the cavity we chose the energy density W to be optimized variable. The energy density W is proportional to the square pressure p^2, which is directly correlated with the sound level inside the cavity. Since W is a quadratic variable made of partial energy quantities corresponding to both direct and reverberated fields (see Figure 1), the superposition principle can be applied:

$$W = W_{dir} + W_{rev}. \tag{2}$$

To define the the wave velocity c the following equation is introduced:

$$\overline{\mathbf{I}} = c \cdot W \cdot \overline{\mathbf{n}}. \tag{3}$$

It was shown to be a valid law for nondamped waves field.

Applying the equations (1) and (3) to the symmetrical waves field we obtain the equation for unloaded lossless media (in this case, the acoustic domain) in the steady-state regime:

$$-c\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 W \overline{\mathbf{n}}\right) = \overline{0}. \tag{4}$$

The solution of equation (4) corresponds to the radiosity method first proposed by Kuttruff [10]. The elementary solution of energy density is denoted G:

$$G(r) = \frac{1}{\gamma_0 c}\frac{1}{r^2}, \tag{5}$$

where γ_0 is a solid angle. The energy density inside the cavity can then be expressed as a function of the primary sources and fictitious sources (reverberated field sources) located on the boundaries:

$$W(P) = \int_{\partial\Omega} \Phi(M)\,\overline{\mathbf{u}}_{PM}\cdot\overline{\mathbf{n}}(M)\,G(M)\,d\partial\Omega + \int_{\partial\Omega} \sigma(M)\,\overline{\mathbf{u}}_{PM}\cdot\overline{\mathbf{n}}(M)\,G(M)\,d\partial\Omega, \tag{6}$$

where P is a point inside of the cavity where W is measured, M is a point of integration on the cavity surface, $\overline{\mathbf{u}}_{PM} = \frac{\overline{\mathbf{PM}}}{\|\mathbf{PM}\|}$, $\overline{\mathbf{n}}(M)$ the unit normal at the point M, $\Phi(M)$ are the acoustic boundary sources and $\sigma(M)$ are the fictitious boundary sources. We will use the term “boundary source“ to denote those sources located on the cavity boundary (which may be due, for example, to external excitations).
For every point M_0 of the boundary $\partial\Omega$, $\sigma(M_0)$ depends on the absorption coefficient α, acoustic boundary sources of the system Φ and fictitious boundary sources in all other points of $\partial\Omega$:

$$\sigma(M_0) = (1-\alpha)\int_{\partial\Omega} \sigma(M)\,\overline{\mathbf{u}}_{M_0M}\cdot\overline{\mathbf{n}}(M)\,G(M)\,d\partial\Omega + +(1-\alpha)\int_{\partial\Omega} \Phi(M)\,\overline{\mathbf{u}}_{M_0M}\cdot\overline{\mathbf{n}}(M)\,G(M)\,d\partial\Omega.$$

5. Optimization method

The present research is oriented on the developing of new shape optimization method under vibroacoustic criteria. The method posseses two important features. It doesn't demand remeshing during the optimization process and it models the acoustic cavity surface exactly. This allows it to be used in the engineering applications.
The geometric parametrization plays an important role in this process and the adequate boundary description is essential. General approach implies the distinction between a design model with its (coarse) geometric description and an analysis model to compute the structural response [7, 14, 3, 4]. Different boundary representations, such as polynomial, Bezier, Bspline and non-uniform rational B-Splines (NURBS) descriptions that proved to be very effective because of their smoothness and boundary regularity appeared for the geometric description. The energy methods chosen for the present study do not need fine meshes and are suitable for geometric parametrization of the acoustic structure.
When studying the vibroacoustical behavior of the 3D cavity by means of MES, only the internal 3D surface has to be modeled to calculate the energy quantity inside the cavity (see Equation 6). Such 3D cavity can be described by parametrized functions of two variables, for example mentioned above Bezier, Bspline and NURBS The shape of the surface thus may be controlled by number of function parameters: when they change, the shape of the surface changes as well. Therefore, we associate the design variables x_i with those function parameters that are allowed to move during the iteration process.
The mathematical formulation of a structural shape optimization problem reads as [14]:

$$\begin{aligned}
\min_S \quad & f(\mathbf{X}, Y(\mathbf{X})); \quad \mathbf{X}\in\mathbb{R}^n \\
\text{such that} \quad & \underline{x_i} \le x_i \le \overline{x_i}, \\
& h_j(\mathbf{X}, Y(\mathbf{X})) = 0, \\
& g_k(\mathbf{X}, Y(\mathbf{X})) \le 0, \\
\text{with} \quad & i = 1,..n, \\
& j = 1..\text{no. of equality constraints}, \\
& j = 1..\text{no. of unequality constraints},
\end{aligned} \tag{7}$$

where f is the objective function and $\mathbf{X}$ are the design variables with n components which control the geometry. The state variable Y describe the structural response. The functions h_j and g_k represent behavioral constraints on quantities describing the characteristics of the structure.
The optimization problem (7) can be solved by a large number of available mathematical programming algorithms. This contribution doesn't aim in developing new optimization algorithms, so classical routines will be used.

6. Optimizational problem for MES formulation.

Let's consider the acoustic cavity that takes the area Ω in $\mathbb{R}^3$, bounded by surface $\partial\Omega$ in $\mathbb{R}^3$ (Figure 1). The surface $\partial\Omega$ is defined by function $\bar{r} = (x, y, z)$ that is parametrized as follows:

$$\bar{r}(\xi_1, \xi_2) = \left[x(\xi_1,\xi_2), y(\xi_1,\xi_2), z(\xi_1,\xi_2)\right], \tag{8}$$

where $(\xi_1, \xi_2) \in S(\xi_1, \xi_2) \in \mathbb{R}^2$ and functions $x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2)$ are piecewise continuous on S. We assume that all the points of surface $\partial\Omega = \overline{r}(S)$ are regular, so that the vectors

$$\overline{\mathbf{a}}_\alpha = \frac{\partial \overline{r}}{\partial \xi_\alpha},\ \alpha = 1, 2, \tag{9}$$

are linearly independent for all points $\xi = (\xi_1, \xi_2) \in S$. These two vectors define the tangent plane to the surface $\partial\Omega$ at the point $\overline{r}(\xi)$.

$$M = \overline{r}(\xi) = (x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2)), \tag{10}$$

$$\overline{\mathbf{n}}(M) = \frac{\overline{\mathbf{a}}_1(M) \times \overline{\mathbf{a}}_2(M)}{\| \overline{\mathbf{a}}_1(M) \times \overline{\mathbf{a}}_2(M) \|}. \tag{11}$$

Here the point P is situated inside the cavity and can't be represented by function $\overline{r}(\xi)$. That is why we take the coordinates (b_1, b_2, b_3) of it as constants and obtain:

$$\overline{\mathbf{PM}} = [x(\xi_1, \xi_2) - b_1;\ y(\xi_1, \xi_2) - b_2;\ z(\xi_1, \xi_2) - b_3], \tag{12}$$

$$\overline{\mathbf{u}}_{PM} = \frac{\overline{\mathbf{PM}}}{\| \overline{\mathbf{PM}} \|} = \frac{[x(\xi_1, \xi_2) - b_1;\ y(\xi_1, \xi_2) - b_2;\ z(\xi_1, \xi_2) - b_3]}{\| [x(\xi_1, \xi_2) - b_1;\ y(\xi_1, \xi_2) - b_2;\ z(\xi_1, \xi_2) - b_3] \|}, \tag{13}$$

$$\int_{\partial\Omega} d\partial\Omega = \int\int_S \| \overline{\mathbf{a}}_1(M) \times \overline{\mathbf{a}}_2(M) \| \, d\xi_1 d\xi_2. \tag{14}$$

After substituting the Equations (10-14) to Equation (6) we have:

$$\begin{aligned} &W(b_1; b_2; b_3) = \\ &\int\int_S \Phi(\xi_1, \xi_2)\, \overline{\mathbf{u}}(\xi_1, \xi_2) \cdot \overline{\mathbf{n}}(\xi_1, \xi_2)\, G(\xi_1, \xi_2)\ \| \overline{\mathbf{a}}_1(\xi_1, \xi_2) \times \overline{\mathbf{a}}_2(\xi_1, \xi_2) \| \, d\xi_1 d\xi_2 + \\ &+ \int\int_S \sigma(\xi_1, \xi_2)\, \overline{\mathbf{u}}(\xi_1, \xi_2) \cdot \overline{\mathbf{n}}(\xi_1, \xi_2)\, G(\xi_1, \xi_2)\ \| \overline{\mathbf{a}}_1(\xi_1, \xi_2) \times \overline{\mathbf{a}}_2(\xi_1, \xi_2) \| \, d\xi_1 d\xi_2. \end{aligned} \tag{15}$$

The similar form can be obtained for equation (7) after the coordinate transformation.
Here the matrix formulation of the problem is used. The surface $\partial\Omega$ is disctetized and integration on the elements is conducted. The meshing criteria applied in FEM or BEM is not useful for energy methods, as they involve only quadratic variables. A coarse mesh size is sufficient and allows to obtain robust results during optimization process quickly. W can then be formulated by discretising Equation (15), i.e.:

$$\overline{W} = \Big([R] + \big([Id] - [T] + [S]\,[\alpha]\,[T] \big)^{-1} \big([Id] - [\alpha] \big)\ [Q] \Big)\, \overline{\Phi} = [M]\, \overline{\Phi}. \tag{16}$$

where $[S]$, $[T]$, $[Q]$ and $[R]$ are matrices corresponding to the discretisation of the integral formulations of Equation (15), $[Id]$ is the identity matrix and $[\alpha]$ the diagonal matrix of the absorption coefficients.
To conclude, the optimization procedure stands as follows:

- First one has to specify the geometry of the cavity Ω with bounding surface $\partial\Omega$ and the function of transformation $r(\xi_1, \xi_2)$ defined on S to describe the cavity shape.
- After, we define the design variables as characteristics of transformation function $\overline{r}(\xi_1, \xi_2) = [x(\xi_1, \xi_2),\ y(\xi_1, \xi_2),\ z(\xi_1, \xi_2)]$, *i.e.* the parameters of the functions $x(\xi)$, $y(\xi)$ and/or $z(\xi)$. These design variables are denoted as x_i.
- The parts of the cavity surface where primary point sources are situated do not change the shape during the optimization process to avoid the effect of respiration.
- To discretize the surface $\partial\Omega$ we have to discretize the area S, where the function $r(\xi_1, \xi_2)$ is defined.
- Objective function has to be specified. The Simplified Energy Method (MES) has been described in Section , and its implementation leads to a matrix formulation (16) of the problem that can be written as follows:

$$\overline{W}(x_i) = [M(x_i)]\, \overline{\Phi}. \tag{17}$$

The energy density vector $\overline{W}$ depends on the geometry of the cavity, and thus on the design variables x_i. Now we can define the quantity F to be minimized:

$$F = \parallel \overline{W}(x_i) \parallel . \tag{18}$$

Function F is computed for a single frequency; a frequency averaging technique may be employed for instance whenever a more realistic definition of the impedance condition is introduced.

7. Numerical example.

The described above method was developed for industrial applications. At the early stage of the research

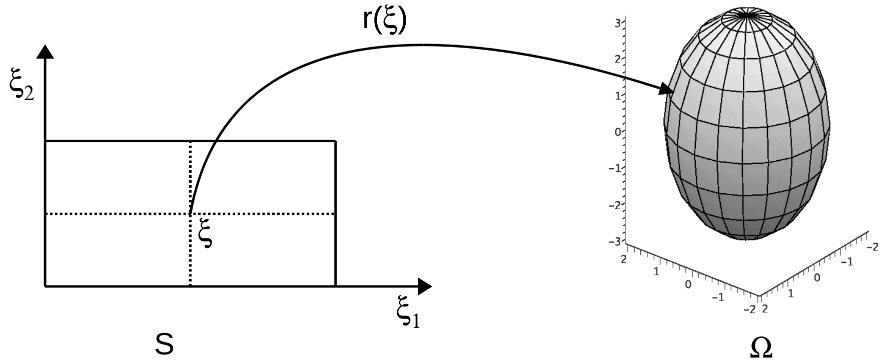

Figure 2: Cavity shape parametrization.

it was tested on the cavity, shape of which can be parametrized with ellipsoid surface.
The function of transformation for the cavity surface is given as:

$$r(\xi_1, \xi_2) = [\, a\cos(\xi_1)\cos(\xi_2);\ b\cos(\xi_1)\sin(\xi_2);\ c\sin(\xi_1)\,], \tag{19}$$

where $\xi_1 \in [0; 2\pi]$ and $\xi_2 \in [0; \pi]$, a, b and c are ellipsoid radii (Figure 2).
The cavity structure was discretized by discretizing the area S on (40×20) elements. Radius a is supposed to be constant ($a = 3$ in the presented case). Radii b and c were chosen to be design parameters, so our optimization problem depends on two design variables $x_1 = b$ and $x_2 = c$.
Acoustic source is applied at the point [3; 0; 0] in Ω which corresponds to the point $[\pi;\ \pi]$ in S; test point [0; 0; 0] in the center of the cavity was chosen to compute energy density vector.
The formulation of the optimization problem takes the form:

$$\begin{aligned} \min(F) = \parallel [M(x_i)]\,\overline{\Phi} \parallel, \\ x_1 \geq 1, \\ x_2 \geq 1, \\ x_1 + x_2 \leq 5. \end{aligned} \tag{20}$$

A constraint condition on the radii is included to put a limit on the amount of material distributed in the design domain $\partial\Omega$ in order to save weight and cost. As we didn't aime to develop new optimization algorithm in this study, we use the standard function $fmincon$ in the Matlab software to compute the optimal values. The optimization algorithm is performed using a local gradient method. The method starts the optimization from the initial value, so it is possible that the results will depend on the initial value.

7.1. Results. Method robustness

Several initial values were used in the optimization process. As can be seen from the Table 1 the result doesn't depend on the initial value and the error for $\parallel W \parallel$ is less than 10^{-8} for the concidered test case. Additionally, the objective function $F(x_1,\ x_2)$ was plotted in order to evaluate the reliability of the results

Table 1: Optimization results with different initial guesses.

Initial guess	Resulting $(x_1; x_2)$	$\parallel W \parallel$, dB
(2.0; 2.0)	(1.0; 5.0)	$3.904923 \cdot 10^{-3}$
(3.0; 2.0)	(1.0; 5.0)	$3.904923 \cdot 10^{-3}$
(5.0; 1.0)	(1.0; 5.0)	$3.904923 \cdot 10^{-3}$

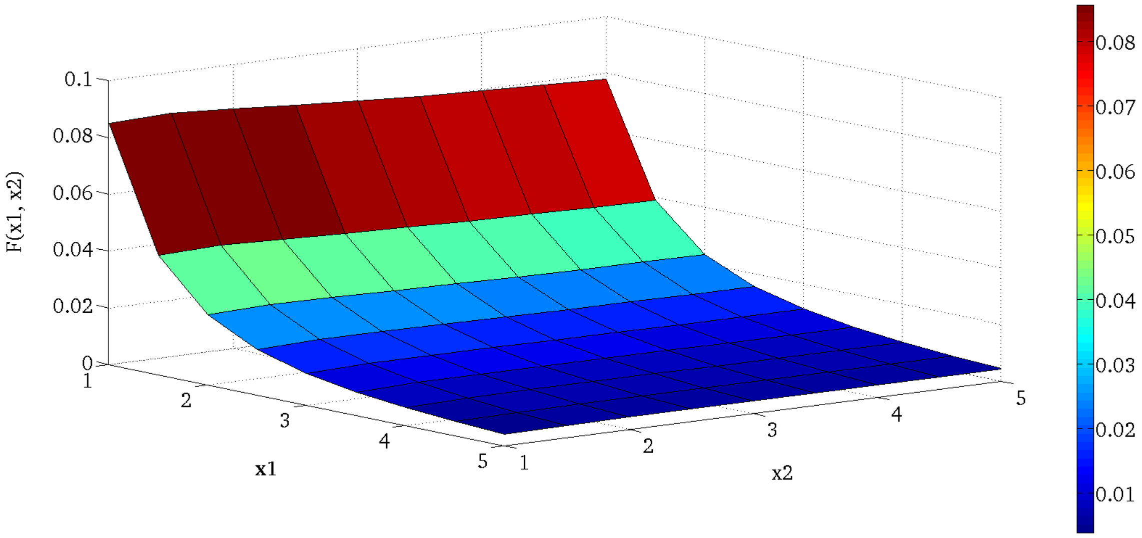

Figure 3: The norm of energy density vector $\parallel W \parallel$ as function of design variables x_1 and x_2.

(see Figure 3). It can be seen that the objective function is quite smooth and has one minimum in the considered region, accordingly to results presented in Table 1. The optimized shape is presented in Figure 4.

8. Conclusion.

The aim of the article was to propose the method for shape optimization under vibroacoustic criteria that will be suitable for engineering applications. Such method has to model easily the shape of the optimized structure, avoid remeshing during the optimization cycle and give realistic shapes as the obtained result. These demands were satisfied by choosing the energetic method to describe the vibroacoustical behavior of the structure and by choosing the isogeometrical-like approach for the optimization part. The developed optimization method was validated for the case of simple cavity, surface of which is modeled with function of parametrization. Two design variables were chosen for optimization problem. The standard optimization procedure of Matlab was used to find the optimal solution.
The realistic applications need to consider the cavity with much more complex surfaces, and bigger number of design variables. To model such surfaces one can use B-spline and NURBS technique, and treating the problems with big number of design variables demands using quick and robust optimization strategies, such as genetic algorithm, for example. These algorithms have the main advantage of being blind algorithms in the sense that they do not require any knowledge about the function to be optimized other than function values at selected points. There is no need for derivatives evaluations, or initial bracketing of optimal points.

9. Aknowledgements.

The authors would like to thank the financial support provided by the French National Research Agency through the framework of its project ANR-12-JS09-0009.

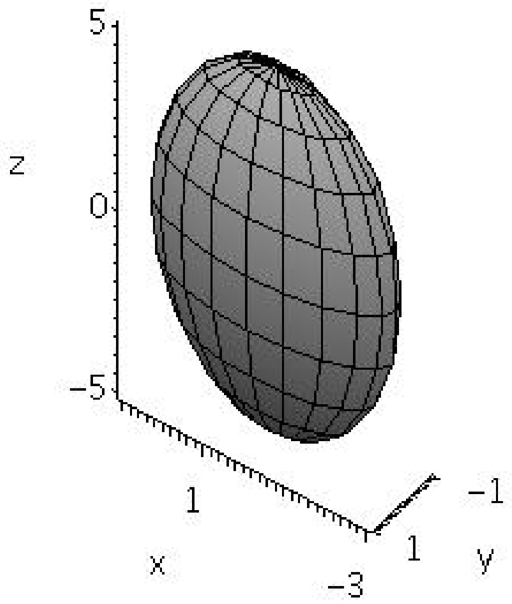

Figure 4: The resulting shape of the cavity after the optimization process. $(x_1;\ x_2) = (1.0;\ 5.0)$.

10. References

[1] S. Besset, M. N. Ichchou, and L. Jezequel. A coupled BEM and energy flow method for mid-high frequency internal acoustic. *Journal of Computational Acoustics*, 18(01):69–85, 2010.

[2] S. Besset and M.N. Ichchou. Acoustic absorption material optimisation in the mid-high frequency range. *Applied Acoustics*, 72(9):632 – 638, 2011.

[3] K.-U. Bletzinger and E. Ramm. Structural optimization and form finding of light weight structures. *Computers & Structures*, 79(2225):2053 – 2062, 2001.

[4] V. Braibant and C. Fleury. Shape optimal design using b-splines. *Computer Methods in Applied Mechanics and Engineering*, 44(3):247 – 267, 1984.

[5] A. T. Chavan and D. N. Manik. Design sensitivity analysis of statistical energy analysis models using transfer path approach. *Electronic Journal Technical Acoustics*, 3, 2005.

[6] R. G. DeJong. Optimization of noise control designs using sea. In *Proceeding of the second International AutoSEA users conference, Detroit-Troy, Marriot-Troy, Michigan, USA*, 2000.

[7] R. T. Haftka and R. V. Grandhi. Structural shape optimizationa survey. *Computer Methods in Applied Mechanics and Engineering*, 57(1):91 – 106, 1986.

[8] M.N. Ichchou and L. Jezequel. Comments on simple models of the energy flow in vibrating membranes and on simple models of the energetics of transversely vibrating plates. *Journal of Sound and Vibration*, 195(4):679 – 685, 1996.

[9] M.N. Ichchou, A. Le Bot, and L. Jezequel. A transient local energy approach as an alternative to transient sea: Wave and telegraph equations. *Journal of Sound and Vibration*, 246(5):829 – 840, 2001.

[10] H. Kuttruff. A simple iteration scheme for the computation of decay constants in enclosures with diffusely reflecting boundaries. *The Journal of the Acoustical Society of America*, 98(1):288, 1995.

[11] Y. Lase, M.N. Ichchou, and L. Jezequel. Energy flow analysis of bars and beams: Theoretical formulations. *Journal of Sound and Vibration*, 192(1):281 – 305, 1996.

[12] R. H. Lyon. *Statistical Energy Analysis of Dynamical Systems: Theory and Application*. Cambridge, Massachusetts, MIT Press, 1975.

[13] D. J. Nefske and S. H. Sung. Power flow finite element analysis of dynamic systems: Basic theory and application to beams. *Journal of Vibration Acoustics Stress and Reliability in Design*, 111(1):94–100, 1989.

[14] W. A. Wall, M. A. Frenzel, and C. Cyron. Isogeometric structural shape optimization. *Computer Methods in Applied Mechanics and Engineering*, 197(3340):2976 – 2988, 2008.