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Interior Value Extrapolation - A new method for stress evaluation during topology optimization

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1. Abstract
Much research in the field of topology optimization has in the later years been focused on efficient solution techniques for problems involving stress constraints. Although not yet fully developed, several methods exist to take stresses into account when optimizing a structure, both using local stress constraints and aggregated stress measures. However, one fundamental problem not exhaustively studied is how to correctly calculate stresses in the non-smooth structures that arise during topology optimization. The jagged structural boundary leads to artificially high stresses in the finite element solution, which may lead the optimizer to solutions that, when smoothed, are not optimal.

This article presents a new method for evaluating stresses during topology optimization. The new method, Interior Value Extrapolation, IVE, exploits the fact that in the interior of the structure, the artificial stresses caused by the jagged structural boundary vanish, and as a result, the stresses calculated for the interior are more accurate than those calculated for the boundary. In order to eliminate the influence of the artificial stresses present at the boundary, the proposed method is based on extrapolating stress results for the boundary from results in the interior of the structure, resulting in a more stable and accurate stress measure.

The extrapolation of stresses from the interior to the boundary of the structure may be conducted in a number of ways. In this work, evaluating the stresses at points along a line pointing inwards from the element is proposed. A functional is fitted to the stresses along this line, and a stress value for the boundary element is estimated by extrapolating this functional. A fast method for calculating an approximate normal at each boundary element during a topology optimization process is also proposed.

To compare the accuracy of the extrapolation technique to a conventional stress calculation, a number of geometries are simulated, for which the stresses are known. It is shown that the new stress evaluation technique is more accurate than a conventional stress calculation, and that the impact of the jagged edges of the mesh is drastically reduced.

Finally, an optimization example is solved using the new stress evaluation process. The results are compared to an optimization using a conventional stress evaluation, and it is shown that nontrivial changes in optimization result can occur when the stress evaluation is improved.

In conclusion, the proposed method for stress evaluation during topology optimization offers a significant improvement existing methods in terms of accuracy, and is also shown to give rise to different structures.
2. Keywords: Topology optimization, Stress evaluation, Finite Element Methods

3. Introduction
The field of topology optimization has in the last decades gone from academic research to standard practice in many industrial applications. In a topology optimization problem, the goal is to distribute material in a given region, so that a given objective function is minimized while given constraints are respected. An overview of the field is given in [1]. Much due to simplicity in implementation, the special case of minimizing the compliance of a structure subject to a volume constraint has been widely used to design load-carrying structures. However, in many cases, there is a desire to optimize a structure, taking stresses into account. Regardless of whether the problem is to minimize stresses subject to a volume constraint, or to minimize the volume subject to stress constraints, some fundamental hurdles need to be overcome. One of these is the issue of formulating the problem in such a way that good solutions can be reached by the optimizer. It has been shown that stress constraints can lead to so called "singular topologies" [2, 3], which require special considerations in formulating the problem. One approach that

has proven successful is the so called ϵ-relaxation [4, 5]. Another relaxation used to overcome this problem is the so called *qp*-approach [6].

Another issue is the local nature of a stress constraint. Solving topology optimization problems with local stress constraints for every element is a daunting task, because of the huge number of constraints involved. Local constraints have been used in smaller examples, see for example [5, 7]. However, for larger problems, an attractive approach from a computational point of view is to aggregate the stress constraints. A common way to achieve this is to use some aggregate measure of the stresses in the model, like the p-norm, which is shown in [8] to be able to approximate the maximum stress in the model while being sufficiently smooth for application in optimization.

In this work, the *qp*-approach, combined with the p-norm, is used to formulate stress-constrained optimization problems which are solvable using standard solution techniques, such as the Method of Moving Asymptotes (MMA) [9]. Besides the difficulties mentioned above, which are mainly related to the optimization problem, another problem arises when evaluating stresses during topology optimization, which is related to the model used for the structure. When using the finite element method for topology optimization, problems arise in evaluating stresses, because the structure is in general not aligned with the element mesh. The approximation of an arbitrary structure using a regular mesh will be jagged, which in turn causes unphysical stress concentrations in the finite element solution. This work attempts to address this problem by proposing a new method for evaluating stresses in topology optimization, using the stress results from the interior of the structure to extrapolate results to the boundary, thereby reducing the impact of the jagged structural edges. This new method is used for solving an optimization problem involving stress, comparing the result to a conventional stress evaluation. The structure of the paper is as follows: In Section 4, some background in the field of topology optimization with stresses is given, and the methods chosen for the numerical examples in the article are presented. In Section 5, the problem of stress evaluation in topology optimization is described, and a new method for evaluating stress is developed. In Section 6, a number of example geometries are evaluated using the new method, and results are compared to conventional stress evaluation. A numerical example is defined in Section 7 and the results are found in Section 8. Conclusions on the performance of the new method are finally drawn in Section 9.

4. Stresses in topology optimization

In this work, topology optimization problems involving stresses are solved, using a finite element based relaxation/penalization approach, as described in [1]. A design domain is discretized using finite elements, and each element is assigned a variable, $\tilde{\rho}$, interpreted as a density, with $\tilde{\rho}_j = 1$ being interpreted as: "element j consists of material", and $\tilde{\rho}_j = 0$ as "element j consists of void". The optimization problem is reduced to finding the binary vector $\tilde{\boldsymbol{\rho}}$, containing densities for each element in the mesh, which minimizes some objective function subject to a number of constraints. The problems are solved by relaxing the integrality constraints, and instead penalizing intermediate densities. In this work, the penalization of stiffness is made according to a modified simplified isotropic material with penalization (SIMP) scheme [10]: $E_j = E_{min} + (E_0 - E_{min})\tilde{\rho}_j^p$, where E_j is the modulus of elasticity of element j, E_{min} is a small stiffness, in this work chosen as 10^{-9}, E_0 is the modulus of elasticity of the design material, here chosen equal to 1, $\tilde{\rho}_j$ is the density variable of element j, and p is a parameter controlling the amount of penalization.

An important factor in the formulation of a topology optimization problem involving stresses is the calculation of stresses in elements with intermediate densities. To obtain stresses consistent with the applied load, the same elasticity tensor should be used for the displacement solution as for the stress evaluation. However, in topology optimization, there are sometimes reasons to consider other choices of stress measures. In [5], it is shown that as the density tends to zero, the stresses in a porous micro structure tend to a finite, non-zero value, and to be physically consistent, so should the stresses in the optimization formulation. The form $\sigma_{ij} = \frac{\rho^p}{\rho^q} E_{ijkl}\epsilon_{kl}$ is proposed for the "local" stresses, where $q = p$. However, this formulation creates a need for other measures such as the ϵ-relaxation to solve the so called singularity problem, which manifests itself in problems for the optimizer to remove low density regions of the structure. In this work, the so called *qp*-approach [6] proposed by Bruggi has been used. Using this method, one chooses $q < p$ in the expression above, leading to a vanishing stress at zero density. Bruggi shows that this approach is similar to the ϵ-relaxation, in that it generates a larger feasible design space, and avoids the discontinuity of stresses at zero density.

It is well known that to avoid mesh-dependent solutions and checkerboards, special measures have to be taken to regularize topology optimization problems. Many methods have been proposed, among the most used are sensitivity filtering [11] and density filtering [12, 13, 14, 15]. When using a density

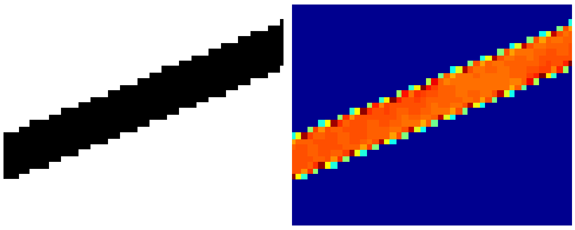

Figure 1: Left: Density distribution for a discretized straight bar. Right: First principal stress evaluated at element centroids for the bar subjected to uniaxial tension

filter, the solution is restricted to the range of a filter function, by introducing non-physical variables ρ_j, controlling the physical densities $\tilde{\rho}_j$ via $\tilde{\boldsymbol{\rho}} = F(\boldsymbol{\rho})$. The variables in the optimization problem are then ρ_j. The function F is chosen to give desired properties to the structure, for example, minimum structural member sizes and minimum hole sizes. Many density filters for topology optimization may be assigned to two main categories, dilate filters and erode filters. The dilate filters add material, thickening the structure and providing a minimum member size, and the erode filters remove material, providing a minimum inner radius to the structure. Combining a dilate and an erode filter of equal filter radius does not give both minimum hole size and minimum member size, however, combining a dilate and an erode filter of different radii accomplishes this task. For the examples in this work, an erode filter with radius 3.5 element lengths followed by an dilate filter with radius 2.5 element lengths is used, giving a minimum member size of 5 element lengths and a minimum inner radius of 1 element length. The filters used are the harmonic erode (1) and dilate (2) filters proposed by Svanberg and Svärd[16]. The physical densities $\tilde{\boldsymbol{\rho}}$ are thus obtained by first using the harmonic erode filter

$$\frac{1}{\bar{\rho}_i + \alpha} = \sum_j \frac{h_{ij}}{\rho_j + \alpha}, \tag{1}$$

obtaining the intermediate variables $\bar{\boldsymbol{\rho}}$, that are in turn filtered using the harmonic dilate filter

$$\frac{1}{1 - \tilde{\rho}_i + \alpha} = \sum_j \frac{h_{ij}}{1 - \bar{\rho}_j + \alpha}, \tag{2}$$

obtaining the physical densities $\tilde{\boldsymbol{\rho}}$.

The weighting factors h_{ij} were chosen according to:

$$h_{ij} = \begin{cases} \frac{1}{|N_i|} & d(i,j) \in N_i \\ 0 & j \notin N_i \end{cases}, \tag{3}$$

where $N_i = \{j | d(i,j) \leq r\}$, r being the filter radius. The parameter α, controlling the discreteness of the filters, is during the optimization process slowly decreased from a large value, which helps convergence but gives solutions with large areas of intermediate densities, to a small value, which enables solutions that are almost completely black and white.

5. The problem of stress evaluation

Using the ground structure approach to topology optimization, the resulting structure consists of a subset of the elements in the mesh, and is generally not smooth, but has jagged edges. These jagged edges give rise to artificially high stress results on the boundary of the optimized structure, as can be seen in the example in Figure 1. These artificial stress contributions vanish in the interior of the structure, where the calculated stresses are correct. Applying a constraint on the element stresses without addressing this issue will generate heavier structures than necessary, since extra material has to be added to compensate for the artificial stress concentrations. The calculated stresses in the interior of the structure are influenced by the overall topology, but not by the jagged nature of the edges. The quality of the stress measure can therefore be improved by disregarding the stress calculated by the finite element method in the elements on the boundary of the structure, and instead extrapolating these results from the interior of the structure. In this section, a method for extrapolating stresses from the interior is developed, using stresses at points along a normal of the surface to generate a stress measure for the element at the boundary. Since

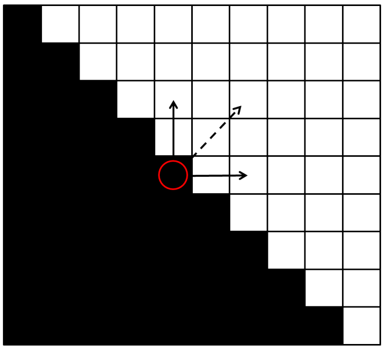

Figure 2: Normals at element level. For the encircled element, the normals at element level are shown with solid arrows, and the normal of the structure is indicated with a dashed arrow.

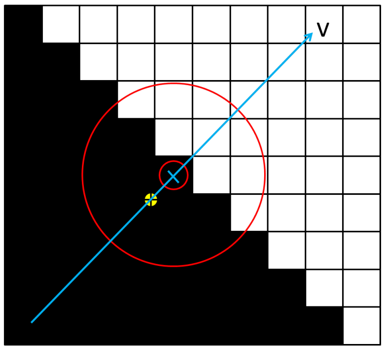

Figure 3: Calculating an approximate normal by using center of gravity of the neighbourhood. A neighbourhood of the element is shown with a large red circle. The center of gravity of the neighbourhood is shown with a yellow dot, and the resulting approximate structural normal is shown with a blue line.

the main idea of the method is extrapolating values from the interior of the structure to the boundary, the name Interior Value Extrapolation, or IVE, is proposed, and used in the remainder of this work.

5.1. Estimating the normal of the surface

Consider a structure represented as a 0/1 density distribution for a given mesh. The mesh is created beforehand, without knowledge of the optimal topology, and thus, the element edges are not generally aligned with the boundaries of the optimal structure. The optimization result will therefore be a jagged representation of the real optimal structure, which is assumed to be smooth. At the element scale, the edges of an element at the boundary will be facing in arbitrary directions, and will not provide information about the normal of the surface of the optimal structure, see Figure 2.

However, using the density distribution in a neighbourhood of the element, an approximate normal of the underlying optimal structure can be calculated. This can be done by computing the position of the center of gravity of the elements in the neighbourhood. In the limit, as the mesh is refined, if the boundary of the underlying structure is continuous and smooth, a small enough circular neighbourhood centered at a point on the boundary will consist of a half circle of void and a half circle of material, and the center of gravity of the neighbourhood will lie in the negative normal direction from the point. The difference between the position of the point and the position of the center of gravity of its neighbourhood gives a vector pointing into the structure, approximately orthogonal to the surface, see Figure 3.

5.2. Estimating the stress in the boundary element

Once the normal direction of the boundary at an element is known, the stresses in elements along a line pointing inwards from the element can be extracted from the finite element solution. Let v be a coordinate along the line normal to the structure boundary at the considered element. Let the center of the element be situated at $v = 0$, and $v < 0$ for points in the interior of the structure. The objective is then to make an estimate of the real stress at $v = 0$, denoted $\tilde{\sigma}$, using the information of the numerically calculated stress, $\sigma(v)$.

A natural way of doing this is to fit some function to the data points from the interior of the structure, where the FE-solution is evaluated, and extrapolate the value of this function to the element at the boundary.

One suggestion is to use a polynomial for the extrapolation. The polynomial can be formulated as $\tilde{\sigma}(v) = [1, v, v^2, ..., v^n] \cdot [c_0, c_1, ..., c_n]^T$.

When fitting the polynomial to the data, the minimum residual to the system $\mathbf{V} \cdot \mathbf{c} = \boldsymbol{\sigma}_\perp$ is sought. Here, $\mathbf{v}$ is a vector of v-coordinates at which stresses are evaluated. Using a notation where $\mathbf{v}^k$ denotes a vector whose components are v_j^k, the matrix may be written $\mathbf{V} = [\mathbf{1}, \mathbf{v}, \mathbf{v}^2, ..., \mathbf{v}^n]$, $\mathbf{c} = [c_0, c_1, ..., c_n]^T$, and $\boldsymbol{\sigma}_\perp$ is the vector of stress values at the coordinates in the vector $\mathbf{v}$.

The optimal solution, in the least squares sense, is given by the solution to the system $\mathbf{V}^T\mathbf{V} \cdot \mathbf{c} = \mathbf{V}^T\boldsymbol{\sigma}_\perp$, hence the optimal polynomial coefficients are $\mathbf{c}^* = (\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T\boldsymbol{\sigma}_\perp$. The extrapolated stress at the element centroid, $v = 0$, is then given by $[1, 0, 0^2, ..., 0^n] \cdot (\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T\boldsymbol{\sigma}_\perp$, that is, the extrapolated stress is just a weighed sum of the stresses along the normal. The choices to make are what degree of polynomial to use, and which positions should be used to fit the polynomial, i.e. the vector $\mathbf{v}$.

5.3. Description of method

In summary, the proposed method for evaluating stresses during topology optimization, IVE, consists of the following steps.

1. Given a density distribution, assemble the stiffness matrix $\mathbf{K}(\boldsymbol{\rho})$.
2. Calculate the displacement vector by solving $\mathbf{Ku} = \mathbf{f}$.
3. Calculate the stress in the centroids of all elements. This gives the vector $\boldsymbol{\sigma}$, containing the centroid stresses of all elements in the model.
4. Calculate the center of gravity for circular regions surrounding every element. Let $\mathbf{x}_{CG,j}$ be the position of the center of gravity of the region centered at element j.
5. Divide the elements in two groups: boundary elements, where interpolated stresses are to be used, and regular elements, where stresses are left as is. Let $\mathbf{x}_j$ be the position of the centroid of element j, and let elements j for which $||\mathbf{x}_j - \mathbf{x}_{CG,j}|| > d$ and $\rho_j > \rho_{lim}$ belong to the boundary, and let all other elements be regular.
6. For boundary elements, use the position of the center of gravity to approximate the normal of the structure. Along this normal, construct a line with coordinate $v = 0$ at the element and $v < 0$ for elements in the interior of the structure. For each v-coordinate in the vector $\mathbf{v}$, note which element element coincides with this point. For any boundary element i: $\tilde{\sigma}_i = [1, 0, 0^2, ..., 0^n] \cdot (\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T\boldsymbol{\sigma}_\perp$. Thus, the new stress measure is a weighted sum of the ordinary stresses $\boldsymbol{\sigma}$, and one may write: $\tilde{\sigma}_i = \mathbf{w}_i^T\boldsymbol{\sigma}$.
7. For regular elements j, let $w_{jj} = 1, w_{jk} = 0, \forall k \neq j$
8. Calculate the new stress measure: $\tilde{\boldsymbol{\sigma}} = \mathbf{W}\boldsymbol{\sigma}$

6. Verification of stress extrapolation

In order to verify the accuracy of the new stress evaluation procedure, a series of example geometries, for which the maximum stresses are known, were evaluated. The example geometries, shown in Figure 4, were chosen from a solid mechanics handbook [17]. The upper eight geometries were subjected to a uniaxial tensile load, and the lower six were subjected to a pure bending moment. The test geometries were modelled in a topology optimization setting, by discretizing them on a square grid of four node finite elements. The density of elements with centroid in the interior of the geometry was set equal to one, and the density of all other elements was set to zero. Since the structure in a topology optimization can take on arbitrary angles towards the mesh, it is important that the stress evaluation is not affected by a rotation of the structure within a given mesh. The example geometries were therefore generated at different angles with respect to the element mesh, to test the abilities of the different stress evaluation procedures to give correct results. Two stress evaluations were compared:

1. The centroid stress in the critical element, as evaluated by standard FEM.
2. The extrapolated stress in the critical element, using a first degree polynomial fitted to the stresses in $v \in [-3.5, -1.75]$, where v is measured in element lengths.

Representing the geometry with a regular mesh, the calculated stress varies significantly along the boundary, see Figure 5, where the centroid stresses are shown for the example geometry rotated 17° relative to the horizon. In Figure 6, the stresses along lines normal to the surface are shown for an example geometry at three rotation angles; 17°, 27° and 37°, and polynomials are fitted to each of the stress curves. As can be seen, although the stresses in the outermost element differ by large amounts, the extrapolated stresses remain more stable. The stress in the 14 examples was calculated at 30 different angles between 0 and 45 degrees, summing up to a total of 420 stress calculations for each of the element

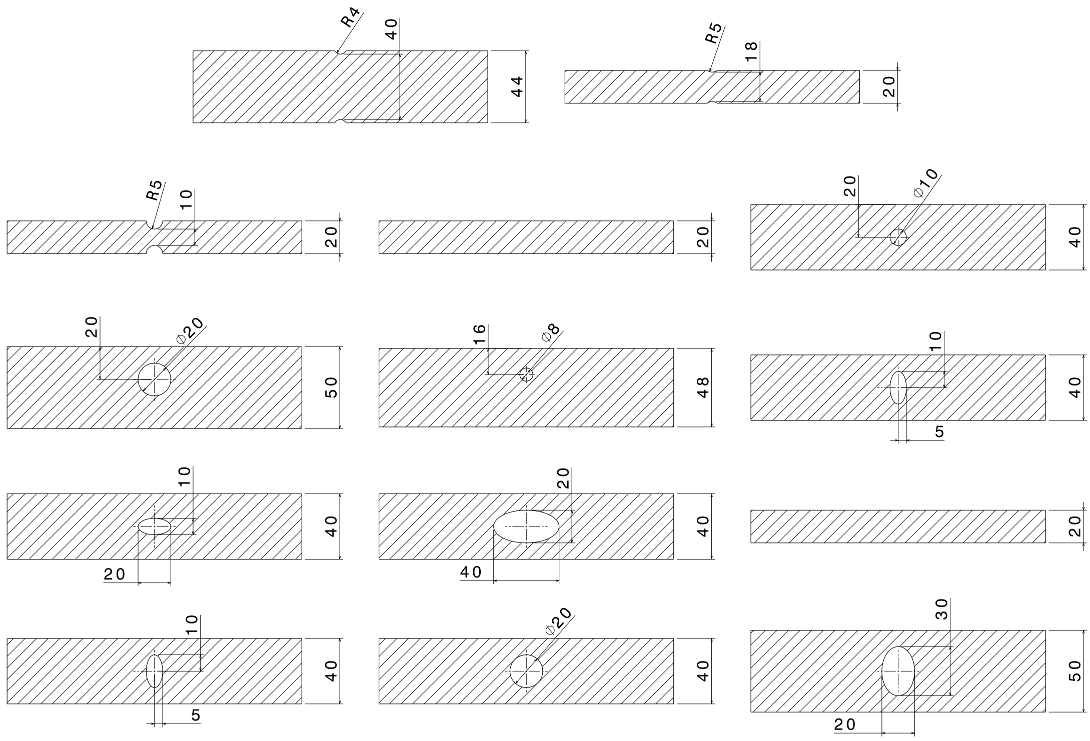

Figure 4: Test geometries used for the verification of IVE.

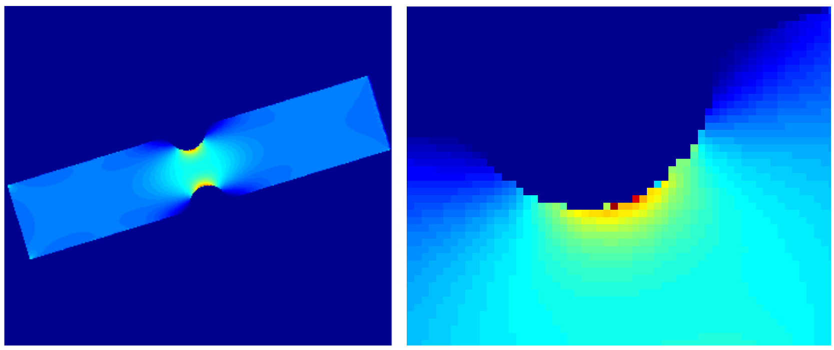

Figure 5: Stresses for example geometry at angle 17°. Left: Whole geometry, right: Zoom in on notch.

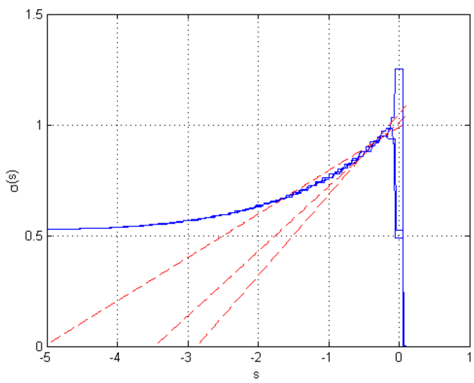

Figure 6: Stresses along the normal for the angles 17, 27 and 37 °. Blue lines show element stresses from the FE solution, red dashed lines show linear extrapolations. The exact maximum stress according to the handbook solution is equal to 1.

sizes $l_e = \{1, 1/2, 1/3, 1/6\}$. The mean error and standard deviation of the error was calculated for each element size.

6.1. Results

The results of the study are reported in Table 1, where the mean and standard deviation of the relative error of the different stress evaluation methods are shown. The table shows that for these examples, the extrapolated stress is both closest to the true stress in average, and, perhaps more importantly, the scatter is lower. For example, for the smallest element size tested, both the mean error and the standard deviation of the error in evaluated stress are reduced by more than 60 %. Moreover, for the conventional evaluation method, the scatter is virtually unaffected by the mesh refinement, pointing out that the problem of stress evaluation in jagged meshes does not disappear with smaller elements. Using the extrapolation technique, however, the scatter is reduced with smaller elements. Since the quality of the optimization result is dependent on accurate objective and constraint function evaluations, this improvement should lead to better optimization results, with a smaller discrepancy between optimization model and verification computations. It should be pointed out that the choice of extrapolation, *i.e.* the order of polynomial (or other choice of approximating function) and which data points to use for fitting, has not been optimized in any way, but rather chosen as simple as possible, to show that there is a potential in the general idea. There are probably gains to be made, both in terms of scatter and mean value correctness, if a more thorough study is made regarding these factors.

Table 1: Mean(μ) and standard deviation(σ) of the errors in stress value calculated with the conventional method and IVE.

	Conventional		IVE	
	μ	σ	μ	σ
$l_e = 1$	-22.9 %	21.9 %	-26.1 %	19.7 %
$l_e = 0.5$	-15.5 %	21.4 %	-14.6 %	14.0 %
$l_e = 0.33$	-11.2 %	21.0 %	-9.5 %	10.9 %
$l_e = 0.167$	-8.4 %	20.2 %	-3.3 %	7.3 %

7. Optimization example

To evaluate the newly developed stress extrapolation method, an example optimization problem has been solved, aimed at illustrating differences compared to standard evaluation of stresses.

The example consists of minimization of the p-norm of the first principal stress on an L-shaped domain subjected to a vertical force, see Figure 7. The optimization is conducted under two constraints: a volume

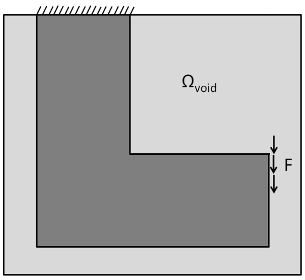

Figure 7: The L bracket load case. The whole rectangular region is discretized, but a constraint on the total volume of elements inside Ω_{void} forces the structure to be inside the darker L-shaped region.

constraint on the total volume of the structure, and a constraint on the volume of the structure inside the Ω_{void} domain. By enforcing the L-shaped design domain in this fashion, all edge effects that tend to appear close to the boundary of the mesh are avoided in the optimization. Two questions need to be answered: first of all, will the two methods generate different topologies, or significantly different shapes? Also, perhaps more importantly, when designs are based on the optimization results, is there a difference in performance of the resulting structures? To answer the second question, a post processing has been chosen, mimicking the design process typically used when designing mechanical components.

1. The optimization is conducted, resulting in a density distribution.
2. An image of the optimization result is created, and the edges of the structure are found using an image processing software.
3. Using the same image processing software, the edge is smoothed, to avoid the jaggedness from the optimization result.
4. The smooth outline is imported in a FE-preprocessor, and a very fine finite element mesh of the resulting structure is created.
5. The commercial finite element code Abaqus is used to calculate the stress in the post processed structure.

7.1. Optimization problem
The optimization problem to be solved is

$$\min_{\boldsymbol{\rho}} \quad (\sum_{i=1}^{n} \tilde{\sigma}_j(\boldsymbol{\rho})^P)^{1/P}, \tag{4}$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} \tilde{\rho}_i(\boldsymbol{\rho}) \leq V^* \\ \sum_{i\in\Omega_{void}} \tilde{\rho}_i(\boldsymbol{\rho}) \leq V^*_{void} \\ \boldsymbol{\rho} \in [0,1]^n \end{cases}, \tag{5}$$

where the physical densities $\tilde{\rho}_j$ and stresses $\tilde{\sigma}_j$ are obtained as in the first example.

7.2. Solution parameters
The design domain was discretized using 110×120 square four node elements, with the size of the actual L-domain being 100 ×100 elements. For this example, P in the p-norm was chosen to 6, and the penalization parameters were chosen as $p = 3$ and $q = 2.5$. The allowable amount of material in the Ω_{void} region was chosen as $V^*_{void} = 0.5$ elements, and the allowable total volume fraction was chosen as 20 % of the total domain, which is equal to 41.25 % of the L-shaped domain. The filter parameter α was initiated at $\alpha = 100$ and reduced with a factor of 0.5 every 50 iterations until $\alpha = 0.0005$, and the optimization was aborted when the maximum difference between iterates was smaller than 0.01.

8.Results
In Figure 8 the results of the optimization of the L-bracket using the two stress measures are shown. Also shown in the same figures are the results of the simple post processing used for the verification computations. As can be seen, changing the stress evaluation leads to large differences in optimization result.

The results of the verifying FE-calculations on the smoothed structures are shown in Figure 9. To enable comparisons the stress contour scale is the same in both pictures. The maximum stresses calculated for the conventional stress optimization is 2.47 MPa, and for the IVE optimization it is 2.15 MPa.

In Figure 10, three stress contours for the optimization result obtained using IVE are shown; conventional stress on the optimization mesh, IVE stress on the optimization mesh, and stress on the verification

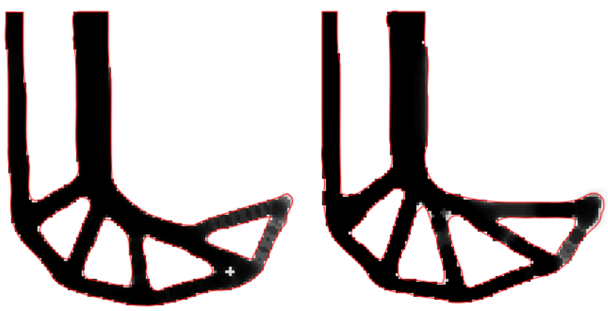

Figure 8: Results of the optimization, using conventional stress calculation (left), and IVE (right).

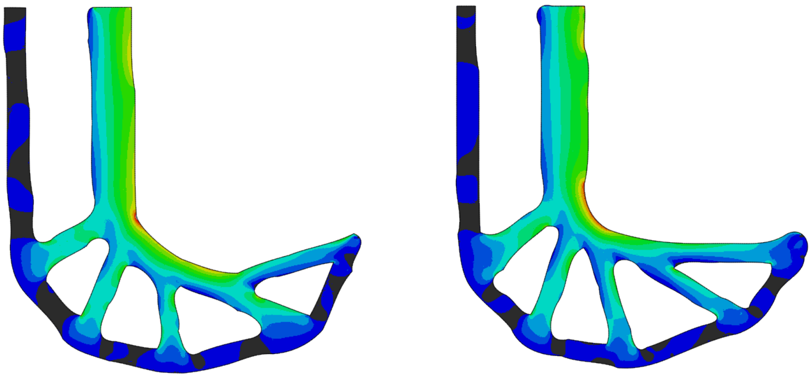

Figure 9: Results of the verification calculation, conventional stress result (left), and IVE result (right).

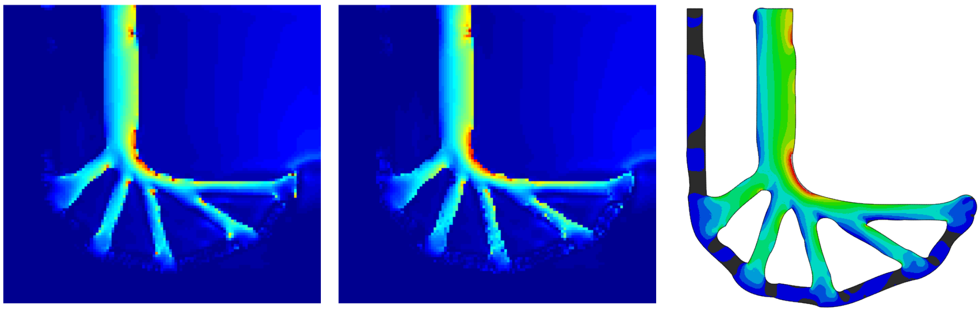

Figure 10: Stress results. left: conventional stress on the optimization mesh, middle: IVE stess on the optimization mesh, right: stress from the verification calculation.

mesh. The contour scale is chosen equal in the three pictures, to allow for comparisons between figures. As can be seen, some of the stress concentrations in the leftmost picture are not present in the verification results, indicating that they are artificial, resulting from the coarse discretization. The leftmost picture also shows low-stressed elements in the middle of the corner, which is obviously not a correct result. In the middle picture, it is visible that the stresses calculated using IVE better approximate the true stresses. There are not any artificial stress concentrations, and the stresses oscillate less between elements than in the leftmost picture.

9. Conclusion

The paper describes a new way to evaluate stresses during topology optimization, which is aimed at reducing the impact of artificial stresses emanating from the jagged representation of the structure when using the ground structure approach to topology optimization. The verifying stress calculations reported in Section 6 indicate that the new method improves the accuracy of the stress evaluation, both in terms of mean value and scatter in results. The results also show that while the problem of scatter in stress results does not seem to vanish with mesh refinement using the conventional method, the new method exhibits a positive trend with regard to accuracy as the mesh is refined.

The results of the optimization example show that significantly different solutions can be the outcome of changing the stress evaluation. Although one must be careful when drawing any conclusions from an isolated example, it is promising that the structure resulting from using the new stress evaluation technique is better with regards to maximum stress when smoothed out structures are compared. As an added benefit, it has also been observed that the smoothing effect that is achieved by the stress interpolation has some beneficial effects for the optimization. Tendencies to diverge seem to be reduced, which is a positive side effect for the very non-convex problem of stress minimization.

Although some positive effects have been shown in this report, many aspects of the proposed method are candidates for further work. For example, other extrapolation functions or filtering techniques could be used, to give even better stress estimates.

It may be noted that although stress evaluation is the motivation for this work, the proposed method of extrapolating results should be applicable to any result of a finite element calculation, for example temperature or flow velocity. Also, while not yet implemented by the author, the proposed method may be generalized to 3-dimensional problems with no major modifications.

10. Acknowledgements

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