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Application of X-FEM in Isoline/Isosurface Based Topology Optimization

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Abstract
In this paper, application of the eXtended Finite Element Method (X-FEM) is considered for the structural optimization of continuum structures using an isoline/isosurface topology optimization algorithm. First, the methodology is described for 2D design domains in which the design boundary is represented by isolines of strain energy density (SED) in a fixed-grid design space. The optimization algorithm operates by gradually removing inefficient material from low SED regions of the design domain which is separated from the solid sub-domain using isolines of a minimum SED level in each iteration. The X-FEM is used to improve the accuracy of FE solutions on the boundary of the design. In order to represent the topology optimization of real-life structures, the proposed method is then extended to 3D using isosurface boundary representation and a 3D X-FEM scheme. The proposed method is effectively used for topology optimization of several 2D and 3D benchmark problems. Efficiency and accuracy of this method is shown by comparing the converged solutions with standard BESO solutions. The results suggest that the proposed method can be of benefit when using a coarse mesh to generate topologies with smooth and clearly defined boundaries whilst avoiding time consuming remeshing approaches. This is an important advantage when the method is used for the optimization of 3D structures in which the computational time can be significant.
Keywords: topology optimization, X-FEM, isoline, isosurface.

1. Introduction

In recent years, structural optimization has become a rapidly growing field of research with application in many areas such as mechanical, civil and automotive engineering. Topology optimization is one of the most challenging aspects of structural optimization, in which one needs to find the best topology as well as shape of a design domain. Up to now, several approaches have been proposed for the topology optimization of continuous structures such as homogenization [1], Solid Isotropic Material with Penalization (SIMP) [2,3], level set method [4,5] and evolutionary structural optimization methods (e.g. ESO/BESO) [6:8].

ESO is based on the assumption that the optimal layout of the design domain can be obtained by gradually removing inefficient material from the design domain [9]. In the original ESO method, the elements of the design space are ranked in terms of their sensitivity, and those with lower sensitivity are removed from the design domain until a desired optimum is obtained. Bi-directional evolutionary structural optimization (BESO) is an extension of ESO in which the elements are allowed to be added and removed simultaneously. These heuristic methods are easy to program and provide a clear topology (no grey regions of intermediate densities as in SIMP) in the resulting optimal designs. Conventional ESO/BESO algorithms have been successful since they can be easily combined with the finite element model of a structure. However they suffer from a weak capability of boundary representation. In these methods the geometrical information of the boundaries is not clear during the optimization process and the boundaries of the optimal solution are represented by the jagged edges of the finite elements. This limitation causes difficulties in combining these methods with CAD and the obtained solutions require post processing to manufacture a smooth design.

The fixed grid finite element method (FG-FEM) allows the boundaries of the design to cross over finite elements. This capability has been used in boundary based optimization methods such as the level set method, and element based optimization methods such as the fixed grid evolutionary structural optimization (FG-ESO) method. FG-ESO or Isoline/Isosurface approach [10, 11] is an alternative to ESO in which the inefficient material is allowed to be removed/added within the elements of the design domain during an evolutionary process. The boundaries are defined by the intersection of isoline plane with the criteria distribution of the design domain. Since in this approach the boundary of the design is no longer consistent with the fixed finite elements as in ESO, a classical finite element analysis may result in poor FE approximation on the boundary. Conventionally in the fixed grid finite element approach, the element stiffness is assumed proportional to the area fraction of the solid material within the element (also called the density scheme). Although this approach is widely accepted and implemented in many works [5, 10], studies have shown that it cannot provide accurate results for the boundary elements [12, 13].The Extended finite element method (X-FEM) is another fixed grid approach which can be used to model void/solid interfaces. X-FEM extends the classical finite element approach by adding special shape functions which can represent a discontinuity inside finite elements. In this approach, the geometry of the discontinuity is often described by a level set method. Combination of the level set description of the geometry and the fixed mesh framework of X-FEM has been used in recent level set based topology optimization work [13,14].

This study presents a simple and effective evolutionary optimization approach in a fixed grid domain, representing the topology optimization of 2D and 3D continuum structures. The novelty of this work is to apply X-FEM in the evolutionary optimization algorithm. The proposed method doesn't require a level set framework for geometry description in the X-FEM and the boundaries of the design can be simply represented by isolines of a desired structural performance. The comparison study of the final solutions obtained using the proposed scheme and standard BESO shows the efficiency of the proposed algorithm.

2. Structural Optimization Problem

The topology optimization problem where the objective is to minimize the strain energy can be written as:

$$\text{Minimize: } c = \frac{1}{2} U^T K U \tag{1}$$

$$\text{Subject to: } \frac{\sum_{e=1}^{N} v_S^{(e)}}{V_0} = V^* \tag{2}$$

where c is the total strain energy, and U and K are the global displacement and global stiffness matrices, respectively. N denotes the number of finite elements in the design domain, $v_S^{(e)}$ the volume of the solid part of the element, V_0 the design domain volume and V^* the prescribed volume fraction. While in ESO/BESO methods, the presence/absence of each element in the design domain is considered as a design variable, in our proposed method the distribution of material inside each element is considered as a design variable. In our study, we have used strain energy density (SED) as the criterion for finding the efficient material distribution within the design domain. Therefore, the solid material will be gradually removed from the low SED regions and added to the high SED regions during the evolutionary procedure. The effective removal of material can be achieved by assigning a weak material property to low SED regions (soft-kill method). The strain energy density of the elements can be calculated from

$$SED_e = \frac{1}{2} u_e^T k_e u_e / v_e \tag{3}$$

with u_e the element displacement vector and k_e the element stiffness matrix which is calculated using an XFEM scheme.

3. Isoline Topology Design

The basic idea of isoline design is to represent the shape and topology of the structure using the contours of desired structural behavior. This idea has been suggested in several studies [15, 16]. The isoline optimization algorithm that we use in this paper is originated from the isoline topology design (ITD) algorithm [10]. The ITD approach can be summarized into the following steps:

1- An extended finite element analysis is performed to find the distribution of strain energy density within the design domain.
2- A minimum SED level (MSL) is determined and the new structural boundary is obtained from intersection of SED distribution and MSL (figure 1).
3- The regions of the domain having the criteria level less than MSL are not included in the design domain. Therefore their material property is set to the weak material. The regions where the criteria level is more than MSL are inside the design domain and their material property is set to the solid material.
4- Steps 1-3 are repeated by gradually increasing the MSL until a desired optimum is obtained.

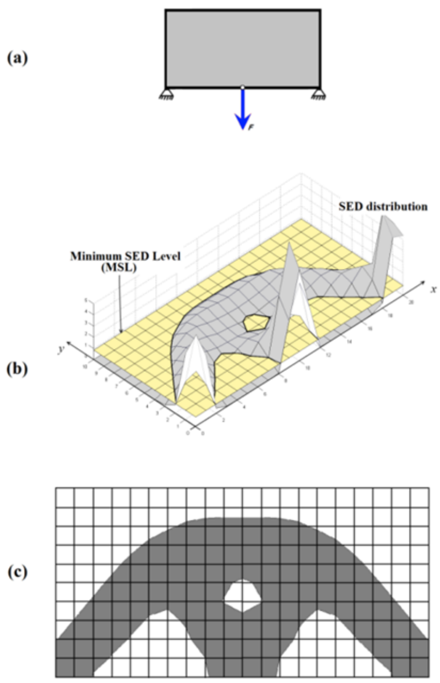


Figure 1: a- initial design domain with boundary conditions. b- Structural boundary represented by intersection of criteria (SED) distribution and MSL. c- Final solution shown in a fixed grid domain.

4. X-FEM Integration Scheme

The extended finite element method (X-FEM) is a fixed grid approach proposed by Moës et al [17]. It was originally developed to represent crack growth in a fixed grid domain without meshing the internal boundaries. X-FEM has also been implemented for other kinds of discontinuities such as fluid structure interaction [18] and modeling holes and inclusions [19]. In our case, the X-FEM scheme for modeling holes and inclusions can be implemented for modeling the boundary of the design (weak/solid material interfaces) during the optimization process. In this approach, the displacement field is approximated by the following equation:

$$u(x) = \sum_i N_i(x)\, H(x) u_i \tag{4}$$

where N_i are the classical shape functions associated to degree of freedom u_i, and the Heaviside function *H(x)* has the following properties:

$$H(x) = \begin{cases} 1 & if\ x\ \in \Omega_S \\ 0 & if\ x\ \notin \Omega_S \end{cases} \tag{5}$$

where Ω_S is the solid sub-domain. Since there is no enrichment in the displacement approximation equation of X-FEM in modeling holes and inclusions, there will be no augmented degrees of freedom during optimization. Eq.(5) defines a zero displacement field for the void part of the element, which means that only the solid part of the element contributes to the element stiffness matrix. Thus we can use the same displacement function as FEM and simply remove the integral in the void sub-domain of the element.

$$K_e = \int_{\Omega_S} B^T D_S B t d\Omega \tag{6}$$

with B the displacement differentiation matrix, D_S the elasticity matrix for the solid material and *t* the thickness of the element. When an element is cut by the boundary, the remaining solid sub-domain is no longer the reference rectangular element. So we partition the solid part of the boundary element into several sub-triangles (figure 2) and use Gauss quadrature to calculate the integral given by Eq.(6).

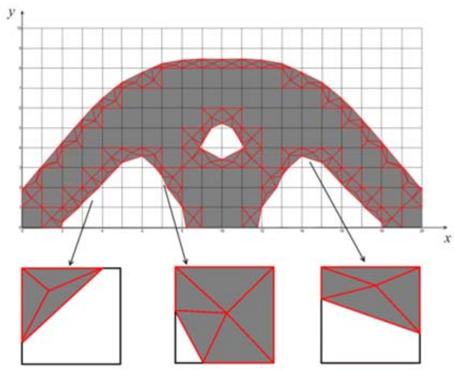


Figure 2: The solid sub-domain of the boundary elements are partitioned into several sub-triangles.

In our study, the second order Gauss rule with 3 midline Gauss points was implemented (figure 3).

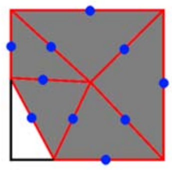


Figure 3: X-FEM integration scheme for 2D quadrilateral elements

5. Combining X-FEM and Optimization Algorithm

Figure 4 illustrates the topology optimization procedure used which in general consists of initialization, X-FEM structural analysis, and isoline update scheme. In initialization, the initial material distribution within the design domain and the discretization of the design domain, as well as the necessary parameters for the isoline topology design are defined.

In the X-FEM structural analysis, by using nodal criteria numbers, the elements are categorized into three groups: solid, void and boundary elements. Solid and void elements are treated using classical finite element approximation. The stiffness matrix of the boundary elements are calculated by partitioning the solid sub-domain into several sub-triangles and applying the Gauss quadrature integration scheme described in the previous section. The minimum SED level (MSL) is calculated by increasing the value from the last iteration. The new structure is obtained from the intersection of the MSL and current criteria distribution. The process is continued until the target volume is achieved.

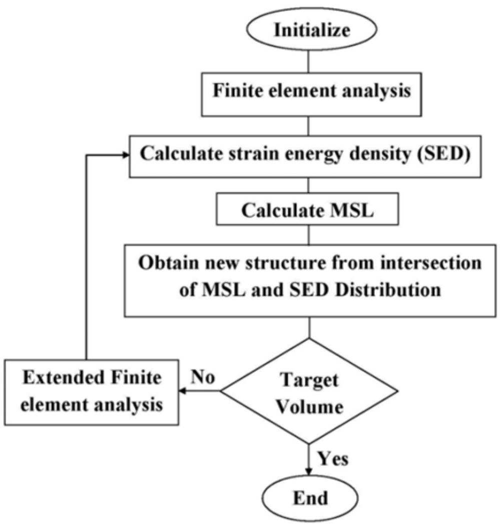


Figure 4: Flowchart of optimization algorithm

6. The 2D Test Case

The proposed method of combining X-FEM and evolutionary optimization algorithm was implemented in a MATLAB code to present the topology optimization of 2D rectangular domains. A set of dimensionless parameters are used for all test cases of this study. The 2D test case used in this study is a Michell type structure [20] with the design domain an boundary conditions shown in figure 5(a). A 120x60 mesh is used for the finite element model of the design. The optimized final design for a volume fraction of 20% of the initial volume, obtained using the proposed method is shown in figure 5(b). Figure 5(c) shows the BESO solution for the same problem and same mesh size. It can be seen that the proposed X-FEM based approach has resulted in a final solution having smooth and clearly defined boundaries. The evolution histories of the objective function and volume fraction are shown in figures 7 and 8, respectively. It can be seen that as material is slowly removed from the design domain, the strain energy increases, then reaches a constant value at convergence. It can also be seen that there are a few jumps in the objective function, which can be attributed to removing a link from the body of the structure.

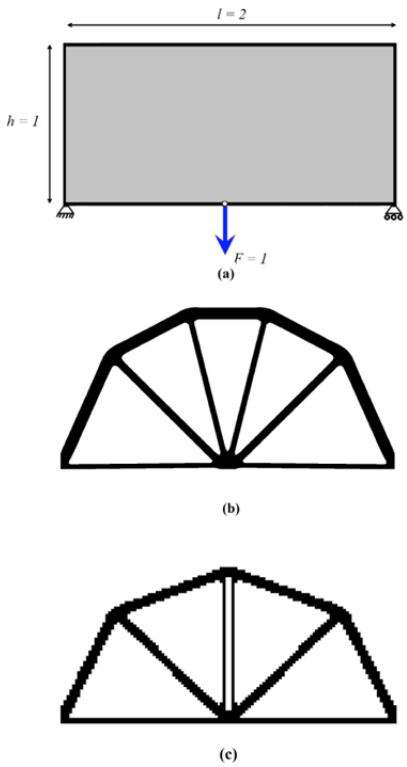


Figure 5: (a) Initial design domain, load and boundary conditions. (b) Topology optimized solution for final volume of 20% of initial volume, obtained using the proposed method. (c) The optimized solution obtained using BESO.

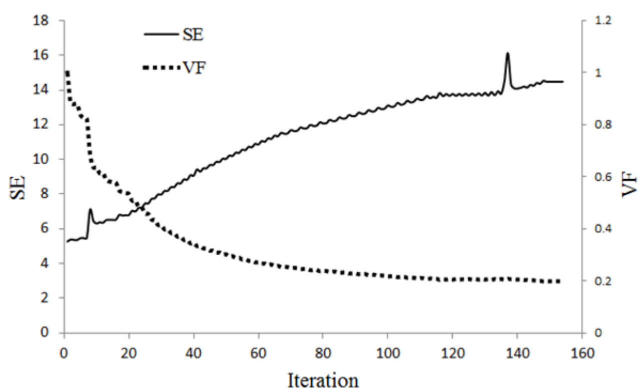


Figure 6: Evolution history of objective function (SE) and volume fraction (VF) of the 2D test case.

7. Extension to 3D

The proposed optimization method can be extended to 3D considering the following changes:

1. The boundary of the design is represented with isosurfaces of strain energy density. It will be a surface that represents the points of a constant value ($SED = MSL$) in a 3D design space.
2. An 8-node hexahedral element is used for finite element modeling of the structure. The X-FEM scheme is realized by decomposition of the solid part of a boundary element into several sub-tetrahedra and performing numerical integration on tetrahedra using 4-point Gauss quadrature, as shown in figure 7. The stiffness matrix of the element is obtained by summing the integrations of all sub-tetrahedra in the elements.

7.1. 3D test case

The 3D test case is a Michell type structure having length l=40, height h=20 and thickness t=5 where a unique distributed force is applied to the middle of the bottom surface as shown in figure 8. The design domain is discretized with 40x20x5 hexahedral finite elements and the final volume fraction is set to 20% of the initial volume. Figure 9(a) shows the final optimized design, obtained after 231 evolutionary iterations. Figure 9(b) is the BESO solution for the 3D test case. Evolution history of the objective function and volume fraction are illustrated in figure 10. It shows that the objective function converges when the desired volume fraction is obtained.

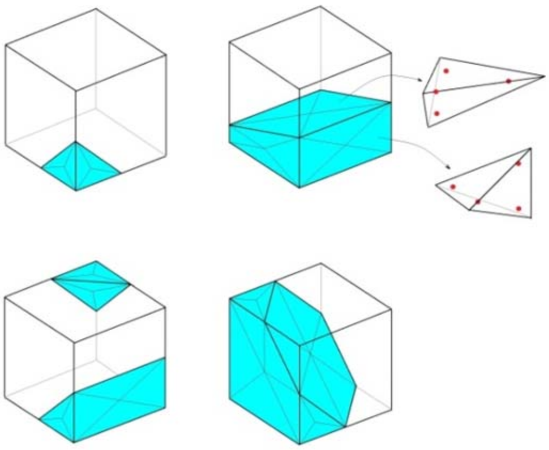


Figure 7: 3D X-FEM integration scheme: solid sub-domain of a boundary element decomposed into sub-tetrahedra.

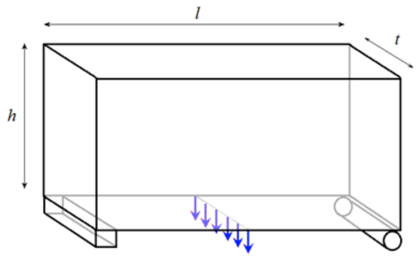
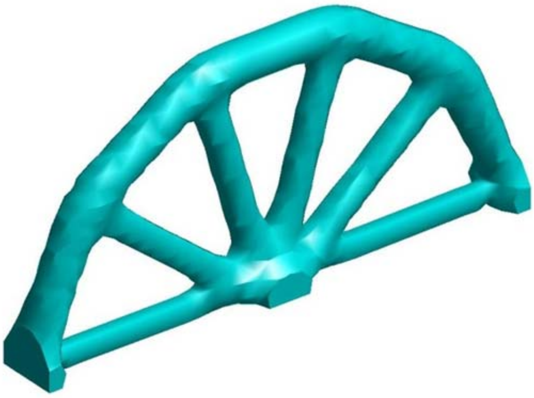
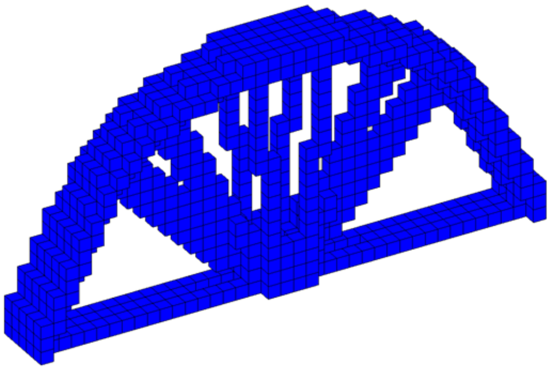


Figure 8: The Michell type structure used as a 3D test case.



(a)



(b)

Figure 9: (a) The optimization result of the 3D Michell type structure obtained using the proposed method. (b) The BESO solution for the same problem

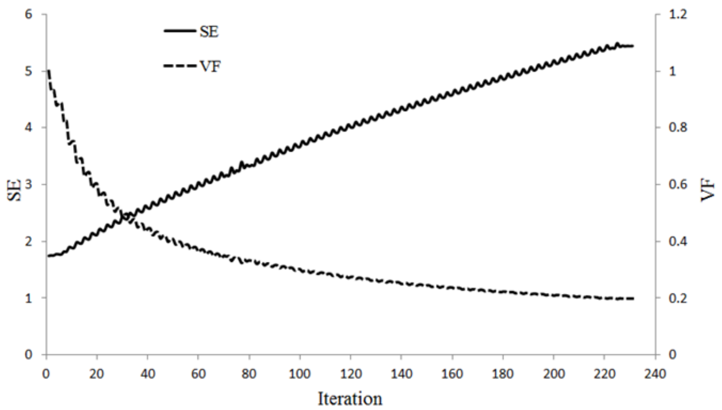


Figure 10: Evolution history of the objective function (SE) and volume fraction (VF) for the 3D test case.

8. Conclusion

In this study, X-FEM and Isoline design are implemented for the topology optimization of 2D and 3D continuum structures. By applying the proposed X-FEM scheme there is no need to use time consuming remeshing or moving mesh approaches to improve the FE solution. The generated structures have smooth boundaries which need no further interpretation and post-processing. The comparison of the optimal solutions with those obtained using standard BESO shows the efficiency of the proposed method.

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