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A Stopping Criterion for Surrogate Based Optimization using EGO

Anirban Chaudhuri, Raphael T. Haftka

University of Florida, Gainesville, FL, USA {a.chaudhuri@ufl.edu, haftka@ufl.edu}

Abstract
In Surrogate-based optimization, each optimization cycle consists of fitting a surrogate to a number of simulations at a set of design points, and performing optimization based on the surrogate to obtain one or more new design points. Algorithms like Efficient Global Optimization (EGO) use uncertainty estimates available with the Kriging surrogate to guide the selection of new point(s). The most common EGO variant uses prediction and prediction variance to seek the point of maximum Expected Improvement (EI) as the next point to be sampled in the optimization. A major problem in global optimization has been the lack of an adequate stopping criterion. The traditional goal of stopping criteria has been convergence to the optimum, but this is not practical when each cycle is expensive and convergence is slow. One practical question when considering whether to stop or carry out one more cycle is whether the resources invested in this additional cycle would yield sufficient return to justify it. In this paper we propose a stopping criterion which justifies continuing with one more cycle only if it is expected to yield at least a specified improvement in the objective function. To implement this stopping criterion the most common EGO variant with EI is used along with a specified improvement that makes it worthy to continue with the optimization, based on a criterion suggested by Schonlau. Its efficiency is also compared to a case using a variant of EGO which uses Probability of targeted Improvement (PI) with an adaptive target, EGO-AT. EGO-AT provides two important ingredients for the criterion: (i) a reasonable target for improvement in the next cycle, and (ii) the probability of achieving that target. The effectiveness of the stopping criteria for both algorithms is demonstrated using a few benchmark global optimization problems.

Keywords: Stopping criteria, Surrogate based optimization, Efficient Global Optimization, worth of a cycle, EGO-AT.

1. Introduction

Surrogate-based optimization is increasingly popular in engineering design community due to the savings in computational time [1]-[18]. The goal of surrogate-based optimization is to select new sampling points which would contribute towards global optimization in each cycle. Algorithms like Efficient Global Optimization (EGO) [10], [11] use both the surrogate (also known as meta-models or response surface models) prediction and its uncertainty estimates. The most common EGO variant uses prediction and prediction variance to seek the point of maximum Expected Improvement (EI) as the next point to be sampled in the optimization.

Traditionally stopping criteria for global optimization algorithms have been based on closeness to the global optimum [19], [20]. A discussion of some of the termination criteria based on convergence can be found in *Evolution and Optimum Seeking* by Schwefel [20]. There have been a few stopping criteria proposed for surrogate based optimization algorithms. The number of objective function evaluations is a common stopping criterion. For EGO, Schonlau [21], [22] proposed to stop if the maximum EI is below an absolute tolerance or the ratio of the maximum value of EI to the present best solution is below a relative tolerance. Huang et al. (2006) [23] proposed a similar stopping criterion by putting a relative tolerance on the ratio of the maximum value of so called augmented EI to the active span of the responses. However, when each cycle of surrogate based optimization is very expensive or time consuming, we may not afford to proceed, even if the result is far from the global optimum.

A practical question when considering whether to stop or carry out one more cycle is whether the resources invested in this additional cycle would yield sufficient return to justify it. A good stopping criterion would prevent waste of resources by discontinuing when it is not worthwhile as well as avoid premature stopping. This stopping criterion is implemented here using a specified improvement that makes it worthy to continue with the optimization. This process is applied using the EI criterion with EGO (EGO-EI) following Schonlau's criterion [10] of absolute and relative tolerance. Its stopping efficiency is also compared to another variant of EGO which uses Probability of targeted Improvement (PI) with an Adaptive Target setting (EGO-AT) [18], [24]. EGO-AT has been shown to be quite effective in this decision making [24] as it provides two important ingredients for the criterion: (i) a reasonable target for improvement in the next cycle, and (ii) the probability of achieving that target.

The rest of the paper is arranged as follows. Section II gives the necessary information about selection of points in EGO-AT and explains the optimization method used for the maximization of PI. Section III explains the stopping criterion and ways to judge its success. Section IV provides details about the numerical experiments using benchmark functions. Section V presents results and discussions. Concluding remarks and future work for this research are included in Section VI.

2. Background: Efficient Global Optimization

A brief description of the Efficient Global Optimization (EGO) algorithm developed by Jones et al. [10] is provided followed by a description of the EGO-AT algorithm and the optimization of the probability of improving beyond a given target (PI).

2.1. EGO

Firstly an initial set of data points is fitted with a Kriging [25], [10] model as a realization of a Gaussian Process with a mean of $\hat{y}(x)$ and a standard deviation of $s(x)$. In this work we use Ordinary Kriging (using *DACE* toolbox [1]). Then each cycle consists of selecting additional points based on maximizing the EI (EGO-EI) or PI and refitting the surrogate. Figure 1illustrates the EGO algorithm with EI for a Kriging surrogate for a one dimensional function. Maximizing PI with an adaptive target setting is used in EGO-AT [18] as the selection criterion for new sampling points. After adding the new point to the existing data set, the Kriging model is updated and the process continues until a stopping criterion is met (usually number of cycles). Commonly Kriging is used for EGO, but any surrogate that provides an uncertainty estimate can be used.

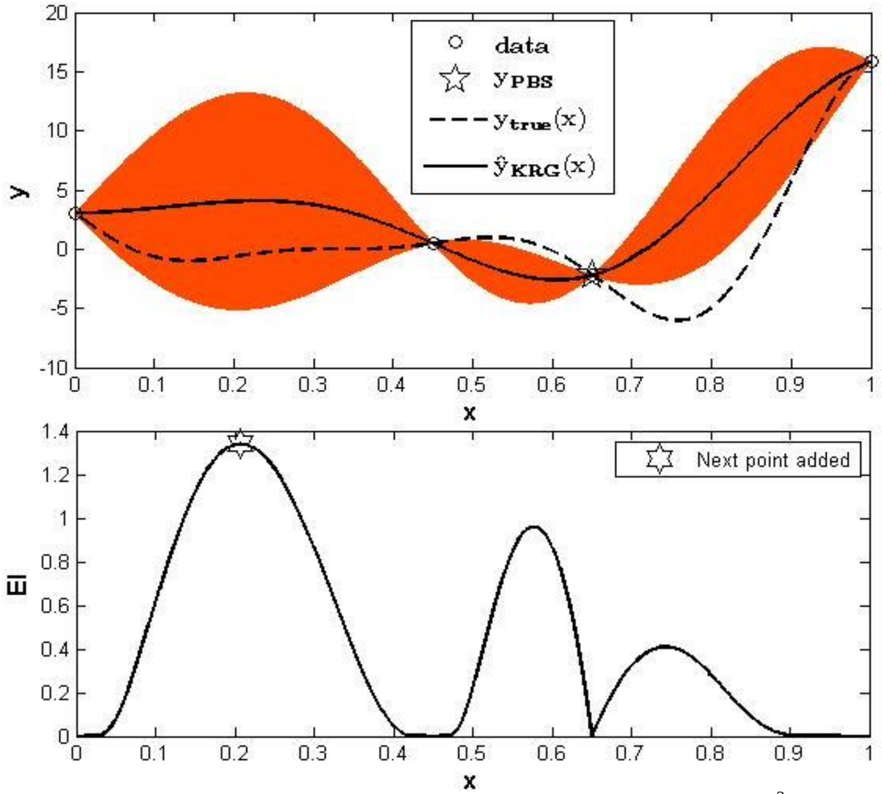

Figure 1. One cycle of EGO using EI for a one dimensional test function [$y(x) = (6x-2)^2\sin(12x-4)$] with initial data set as $x = [0\ 0.45\ 0.65\ 1]^T$. The uncertainty (amplified amplitude of $2*s(x)$) associated with the kriging is plotted in orange.

2.2. Expected Improvement (EI)

The popular way of selecting points is to maximize the expected improvement [10], [11] upon the Present Best Solution (PBS), y_{PBS}. In this case y_{PBS} is given by the lowest mean of the collected data. The Expected Improvement (EI) is given by Equation (1).

$$EI(x) = (y_{PBS} - \hat{y}(x))\Phi\left(\frac{y_{PBS} - \hat{y}(x)}{s(x)}\right) + s(x)\phi\left(\frac{y_{PBS} - \hat{y}(x)}{s(x)}\right) \quad (1)$$

where, $\phi(.)$ is the probability density function of a normal distribution.

2.3. Probability of targeted Improvement (PI)

Another way of selecting points is to maximize the PI. The probability of improving the objective beyond a target y_{Target} at a point x is given by Equation (2) [11], [18], [26].

$$\mathrm{P}\,I(x)=\Phi\left(\frac{y_{Target}-\hat{y}(x)}{s(x)}\right) \tag{2}$$

where, Φ (.) is the cumulative density function of a normal distribution, $\hat{y}(x)$ is the kriging prediction, $s(x)$ is the prediction standard deviation (square root of the kriging prediction variance).

The EI and PI functions have multiple local maxima and this warrants the use of a global optimizer. Here we use Differential Evolution (DE) algorithm [27] which is a stochastic optimizer. To increase its efficiency at locating the global optima we use 4 starts, as multiple starts has been shown to be beneficial for stochastic optimizers [28].

2.4. EGO with Adaptive Target (EGO-AT)

EGO-AT [18], [24] adaptively changes the target of improvement according to the success or failure of the previous EGO cycle. Then it maximizes PI based on the adaptively set target for selecting new points. The target of improvement in cycle $k+1$ is adjusted according to the ratio of actual to targeted improvement η_k in EGO cycle k,

$$\eta_k=\frac{y_{BScycle}-y_{PBS_k}}{y_{Target_k}-y_{PBS_k}} \tag{3}$$

where, $y_{BScycle}$ is the best solution of the points added in the cycle, y_{PBS_k} is the present best solution before adding the points of the present cycle and y_{Target_k} is the target for that cycle which is given by Equation (4).

$$y_{Target_k}=y_{PBS_k}-TI_k \tag{4}$$

The targeted improvement (TI) is assigned an initial value for the first EGO cycle (in this paper, absolute value of 10% of the y_{PBS}). The TI for the next EGO cycle ($k+1$) depends on the value of η_k. If the target is met with a margin TI is increased, while if we fall short TI is decreased. This is given by Equation (5).

$$TI_{k+1}=\begin{cases}1.5TI_k,\ \forall\,\eta_k>2\\ 0.5TI_k(\eta_k+1),\ \forall\,\eta_k\in[0.05,2]\\ 0.525TI_k,\ \forall\,\eta_k<0.05\end{cases} \tag{5}$$

The advantage of EGO-AT is that it automatically provides two bases for deciding whether to undertake another cycle. First, the adaptive target tends to converge towards a realistic estimate of the improvement in the next cycle. Second, PI provides an estimate of the probability of achieving this improvement. It also allows adding multiple points per cycle easily.

3. Stopping Criteria

We assume that the user of EGO has a threshold, I_{worth} that represents the improvement in the objective that should be targeted in order to justify another cycle. The value of I_{worth} is dictated by the economics or time pressures. With EGO-EI the decision whether to stop is based on the Absolute Tolerance ($ATol$) or Relative Tolerance ($RTol$) [21], [22] given in Equations (6) and (7).

$$\max(EI)<ATol \tag{6}$$

$$\frac{\max(EI)}{abs(y_{PBS})}<RTol \tag{7}$$

The two tolerances are set to justify continuing in the following manner:

(a) $ATol$ is equal to a limit defining the worth of a cycle, I_{worth} ($Atol = I_{worth}$).

(b) $RTol$ is equal to a user-defined limit, $RTol_{stop}$ ($RTol = RTol_{stop}$).

With EGO-AT, we proposed that the decision whether the next cycle will meet or exceed this threshold be based on the targeted improvement, TI, set by the adaptive target procedure for the next cycle, and the probability of achieving this target [24]. For EGO-AT, we stop the optimization process:

(a) If the targeted improvement (TI) in the next EGO cycle is below a limit, I_{worth}.

(b) If the probability of targeted improvement (PI) in the next EGO cycle is below a user-defined limit, PI_{stop}. An analysis was done by Chaudhuri et al. [24] to identify the best PI_{stop} and setting it as 20% for all the tested cases was found to be most efficient.

A successful stopping criterion would lead to a high percentage of the right decisions on whether to stop or continue. The decision will not succeed in two ways:

(a) *Waste: A wasted cycle*– EGO is continued for one more cycle and the actual improvement achieved in that cycle is less than I_{worth}. For DOE *i*, success (S_{Waste}) and failure (F_{Waste})in avoiding *Waste* is defined as,

$$S_{Waste_i} = \sum_{j=k_S+1}^{k_{T_i}} I_j[\text{Actual Improvement in cycle } j \geq I_{worth}]$$
$$F_{Waste_i} = (k_{T_i} - k_S) - S_{Waste_i} \tag{8}$$

I is an indicator function which equals 1 when the condition is true and equals 0 when it is false. k_T is the EGO cycle when stopping criterion terminates the algorithm.

(b) *Prem: Premature stopping* – EGO is stopped and the actual improvement achieved in the next cycle (k_T+*1*) is more or equal to I_{worth}. For DOE *i*, success (S_{Prem}) and failure (F_{Prem})in avoiding *Prem* is defined as,

$$S_{Prem_i} = I[\text{Actual Improvement in cycle } (k_{T_i}+1) < I_{worth}]$$
$$F_{Prem_i} = 1 - S_{Prem_i} \tag{9}$$

Note that to determine S_{Prem}, we have to continue the optimization even though the stopping criterion tells us to stop. This will be explained in the next section on numerical experiments.

In order to determine S_{Waste} and S_{Prem} using Equations (8) and (9) for the case of relative tolerance, the equations are altered by replacing I_{worth} with $RTol_{stop}$ to corroborate with the stopping criterion defined for *RTol*. The stopping criterion is implemented only after k_S initial EGO cycles as there is too much uncertainty in the initial stages to trust *TI* or *PI*.

4. Numerical Experiments

We employed the two-dimensional Sasena function, the three-dimensional Hartmann 3 function and the six-dimensional Hartmann 6 function (given in Appendix A) to test the success of the stopping criteria. These functions are often used as a test functions for global optimization. The global optima of Sasena, Hartmann 3 and Hartmann 6 functions are known to be *-1.4565*,*-3.86278* and *-3.32237*, respectively.

To average out the influence of the design of experiment (DOE), 50 different Latin Hypercube DOEs [29] were created using MATLAB function *lhsdesign*. We start sampling Sasena with 8 points, Hartmann 3 with 12 points and Hartmann 6 with 35 points and then let EGO-EI and EGO-AT run for 22, 18 and 45 cycles respectively. The numerical experiments are done by running the optimization for a large number of cycles, and then checking how different combinations of I_{worth} worked with the stopping criterion. For Sasena, Hartmann 3 and Hartmann 6 the stopping criteria are implemented after initially running (k_S) 4, 5 and 9 EGO cycles, respectively. After these initial cycles the stopping criterion is checked in every cycle till terminating EGO-AT cycle, k_T.

5. Results and Discussion

The results for Sasena, Hartmann 3 and Hartmann 6 functions are presented using 50 DOEs to show the effectiveness of the stopping criteria in optimizing the utilization of resources. Different limits for the respective stopping criterion are tried to check the efficiency in each case as shown in Table 1.

The median optimization history of the 3 test functions using EGO-EI and EGO-AT is provided in Figure 2. It can be seen that the median PBS of 50 DOEs converge to the global optimum in all the 3 cases if EGO-EI and EGO-AT are allowed to run long enough with comparable efficiency for both. However, the objective is to stop when it is not worth it to continue even if the solution is not close to the global optimum.

The best solution obtained when EGO is terminated using the stopping criteria mentioned in Table 1for the test functions is presented in Table 2-Table 10. It can be seen that in many cases the solution is not close to the global optimum. This highlights the fact that when we are limited by time or resources we have to be content with an affordable solution rather than an optimal one.

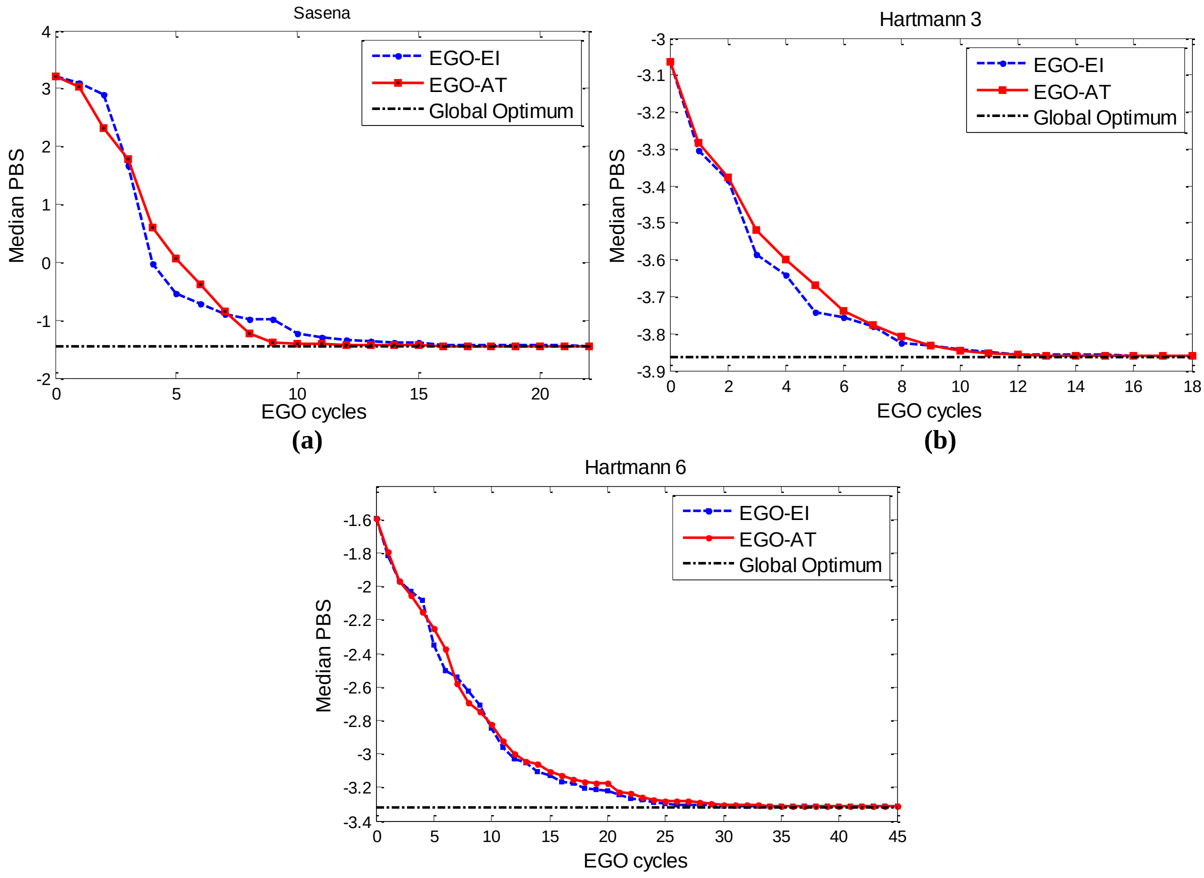

Figure 2. Progression of EGO-EI and EGO-AT optimization history for (a) Sasena, (b) Hartmann 3, and (c) Hartmann 6 functions as a median of 50 DOEs.

Table 1. List of stopping criteria tested for Sasena, Hartmann 3 and Hartmann 6 for EGO-EI and EGO-AT

	Sasena		
	EGO-EI		**EGO-AT**
	ATol = I_{worth} with changing I_{worth}	**Changing limit on RTol**	**Changing I_{worth} and PI_{stop} =20%**
1.	ATol = 0.001	RTol = 0.0001	TI<0.001 or PI <20%
2.	ATol = 0.01	RTol = 0.001	TI<0.01 or PI <20%
3.	ATol = 0.05	RTol = 0.005	TI<0.05 or PI <20%
4.	ATol = 0.1	RTol = 0.01	TI<0.1 or PI <20%
5.		RTol = 0.05	
	Hartmann 3		
1.	ATol = 0.001	RTol = 0.0001	TI<0.001 or PI <20%
2.	ATol = 0.005	RTol = 0.001	TI<0.005 or PI <20%
3.	ATol = 0.01	RTol = 0.005	TI<0.01 or PI <20%
4.	ATol = 0.02	RTol = 0.01	TI<0.02 or PI <20%
5.		RTol = 0.05	
	Hartmann 6		
1.	ATol = 0.001	RTol = 0.0001	TI<0.001 or PI <20%
2.	ATol = 0.01	RTol = 0.001	TI<0.01 or PI <20%
3.	ATol = 0.05	RTol = 0.005	TI<0.05 or PI <20%
4.	ATol = 0.1	RTol = 0.01	TI<0.1 or PI <20%
5.		RTol = 0.05	

The percentage of EGO cycles that successfully avoided *Waste* for all the 50 DOEs, $S_{Waste_{total}}$, is calculated using Equation (10). The EGO cycles starting from when the stopping criterion is first implemented, k_S, till the terminating EGO cycle, k_T, are checked for this case. The instances where the EGO run terminates on cycle k_S are not considered in these calculations. The percentage of cases that successfully avoided premature stopping or

Prem for all the 50 DOEs, $S_{Prem_{total}}$, is calculated using Equation (11). The EGO cycle (k_T+1) is checked for this. The values of S_{Waste}, F_{Waste}, S_{Prem} *and* F_{Prem} for the i^{th} DOE are found using Equations (8) and (9).

$$S_{Waste_{total}}(\%) = \frac{\sum_{i=1}^{50} S_{Waste_i}}{\sum_{i=1}^{50} (S_{Waste_i} + F_{Waste_i})} * 100 \qquad (10)$$

$$S_{Prem_{total}}(\%) = \frac{\sum_{i=1}^{50} S_{Prem_i}}{\sum_{i=1}^{50} (S_{Prem_i} + F_{Prem_i})} * 100 \qquad (11)$$

The results for $S_{Waste_{total}}$ and $S_{Prem_{total}}$ while using the stopping criterion on EGO-EI using absolute and relative tolerances, for Sasena, Hartmann 3 and Hartmann 6 functions are presented in Table 2-Table 7. In these results $S_{Prem_{total}}$ values are mostly above 80% making these stopping criteria very good at not stopping prematurely. But $S_{Waste_{total}}$ values are mostly below 50% for both Sasena and Hartmann 3 showing poor performance. The performance is better for a few cases of Hartmann 6 but the performance is poor overall at avoiding wasted EGO cycles. These tables also provide the mean of the number of EGO cycles performed before stopping and the median of best solution obtained when stopped.

Table 2. Sasena Stopping Criteria cases on absolute tolerance

	Absolute tolerance	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.001	-1.4423	18.1	21.7	91.4
2.	0.01	-1.4326	15.3	24	97.8
3.	0.05	-1.3358	10.9	23.9	84
4.	0.1	-1.1138	7.9	25.9	92

Table 3. Sasena Stopping Criteria cases on relative tolerance

	Relative Tolerance	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.0001	-1.4431	19.6	21.1	95.8
2.	0.001	-1.4423	17.9	22.1	91.9
3.	0.005	-1.4364	15.8	23.4	100
4.	0.01	-1.4308	14.5	22.4	85.1
5.	0.05	-1.2977	9.4	23.7	84

Table 4. Hartmann 3 Stopping Criteria cases on absolute tolerance

	Absolute tolerance	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.001	-3.8586	14.5	36.6	100
2.	0.005	-3.8554	11.6	39.8	91.3
3.	0.01	-3.8450	9.8	39.1	85.1
4.	0.02	-3.8240	8.1	37.3	79.6

Table 5. Hartmann 3 Stopping Criteria cases on relative tolerance

	Relative Tolerance	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.0001	-3.8587	15.6	35.1	100
2.	0.001	-3.8558	11.8	40.4	91.1
3.	0.005	-3.8240	8.2	36.9	79.6
4.	0.01	-3.7769	6.9	34	89.8
5.	0.05	-3.7508	5.2	25	98

Table 6. Hartmann 6 Stopping Criteria cases on absolute tolerance

	Absolute tolerance	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.001	-3.3022	25.2	44.4	85.7
2.	0.01	-3.2190	16.1	49.9	80
3.	0.05	-2.9284	10	**67.4**	**70**
4.	0.1	-2.7842	9.2	**63.6**	**66**

Table 7. Hartmann 6 Stopping Criteria cases on relative tolerance

	Relative Tolerance	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.0001	-3.3050	28	41.6	97.9
2.	0.001	-3.2922	22.4	48	90
3.	0.005	-3.1559	14.1	53.8	80
4.	0.01	-2.9966	11.3	53.1	72
5.	0.05	-2.7624	9.08	**75**	**74**

The results for $S_{Waste_{total}}$ and $S_{Prem_{total}}$ while using the stopping criterion on EGO-AT, for Sasena, Hartmann 3 and Hartmann 6 functions are provided in Table 8, Table 9 and Table 10, respectively. In these results $S_{Waste_{total}}$ and $S_{Prem_{total}}$ values are both mostly above 60% showing much better balance and higher efficiency in making the right decision when it comes to avoiding wasted cycles and premature stopping both as compared to EGO-EI.

Table 8. Sasena Stopping Criteria cases for different I_{worth} with $PI_{stop}=20\%$

	Limit on Targeted Improvement (I_{worth})	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.001	-1.0505	6.3	79	84
2.	0.01	-1.0505	6.3	79	84
3.	0.05	-0.8999	5.8	74.5	80
4.	0.1	-0.8999	5.4	76.1	82

Table 9. Hartmann 3 Stopping Criteria cases for different I_{worth} with $PI_{stop}=20\%$

	Limit on Targeted Improvement (I_{worth})	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.001	-3.8149	7.4	72.7	62
2.	0.005	-3.8149	7.4	71.1	70
3.	0.01	-3.8110	7.3	67.2	68
4.	0.02	-3.8027	6.6	59	68

Table 10. Hartmann 6 Stopping Criteria cases for different I_{worth} with $PI_{stop}=20\%$

	Limit on Targeted Improvement (I_{worth})	**Median of best solution when stopped**	**Mean (k_T)**	$S_{Waste_{total}}$**(%)**	$S_{Prem_{total}}$**(%)**
1.	0.001	-3.0594	14	68.9	70
2.	0.01	-2.9666	11.6	70.2	64
3.	0.05	-2.8162	9.3	50	60
4.	0.1	-2.8132	9.1	66.7	80

6. Conclusions

The traditional goal of a stopping criterion has been to stop not far from the optimum, but when resources are limited and convergence is slow, this may not be practical. We proposed a stopping criterion which justifies doing another cycle only if it yields at least the specified worth in the next cycle.

We used EGO with EI and set limits on absolute tolerance and relative tolerance to check its efficiency at a given worth justifying proceeding with another cycle. We compared the stopping efficiency of EGO with EGO-AT which gives us a value for the targeted improvement in the next cycle that adaptively converges to a realistic value and the probability of achieving that improvement. For three benchmark functions, stopping criterion on EGO-EI, performed exceedingly well when it comes to making the right decision to avoid premature stopping. However, on the percentage of cycles that avoided wasting resources and justified continuing, EGO-EI made the right decisions only around 15% of the time. On the other hand, stopping criterion using EGO-AT gives the correct decisions about 75% of the time as far as avoiding waste of the available resources and avoiding premature stopping both are concerned.

Appendix A: Analytic functions

The Sasena function used in this research is given by Equation (12). The number of design variables for Sasena function is 2.

$$y(x) = 2 + 0.01(x_2 - x_1^2)^2 + (1 - x_1)^2 + 2(2 - x_2)^2 + 7\sin(0.5x_1)\sin(0.7x_1x_2), \quad 0 \le x_1 \le 5,\ 0 \le x_2 \le 5 \tag{12}$$

The equations for the Hartmann 3 and Hartmann 6 functions used in this research are given by Equation (13). For Hartmann 3 and Hartmann 6 the number of design variables, n_{dv} is 3 and 6 respectively. The parameters for Hartmann functions are given in Table 11.

$$\begin{gathered} y(\mathbf{x}) = -\sum_{i=1}^{4} a_i \exp\left(-\sum_{j=1}^{n_{dv}} B_{ij}(x_j - D_{ij})^2\right), \\ \mathbf{a} = \begin{bmatrix} 1.0 & 1.2 & 3.0 & 3.2 \end{bmatrix}, \\ 0 \le x_j \le 1,\ j = 1,\ 2,\ \ldots,\ n_{dv}. \end{gathered} \tag{13}$$

Table 11. Parameters used in Hartman functions

Function	Parameters
Hartman3	$\mathbf{B} = \begin{bmatrix} 3.0 & 10.0 & 30.0 \\ 0.1 & 10.0 & 35.0 \\ 3.0 & 10.0 & 30.0 \\ 0.1 & 10.0 & 35.0 \end{bmatrix}$ $\mathbf{D} = \begin{bmatrix} 0.3689 & 0.1170 & 0.2673 \\ 0.4699 & 0.4387 & 0.7470 \\ 0.1091 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{bmatrix}$
Hartmann 6	$B = \begin{bmatrix} 10.0 & 3.0 & 17.0 & 3.5 & 1.7 & 8.0 \\ 0.05 & 10.0 & 17.0 & 0.1 & 8.0 & 14.0 \\ 3.0 & 3.5 & 1.7 & 10.0 & 17.0 & 8.0 \\ 17.0 & 8.0 & 0.05 & 10.0 & 0.1 & 14.0 \end{bmatrix}$ $D = \begin{bmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1451 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{bmatrix}$

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