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Compliance Based Column Topologies Generated for Maximal Buckling Load

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1. Abstract
This paper presents a new approach to topology optimization of columns exposed to loss of stability. The problem is formulated in a manner allowing to solve it with Cellular Automata (CA) method. CA approach by nature requires local formulation of the design problem, and therefore it is not straightforward how to apply it to maximization of a structure buckling load, which is a global quantity. The challenge is therefore to replace conventional maximization of buckling load by a compliance based topology optimization problem. In order to do this, the standard instability analysis of a compressed column is performed first and buckling mode is determined. Then the compressive loading is replaced by a transverse one which is selected so as to generate bending moment, distribution of which coincides with the one representing considered buckling mode. For the bent structure minimization of compliance is performed, and optimal topology is generated. Finally the critical load for the optimal column is calculated. The preliminary numerical results obtained for selected columns with use of the above technique are presented, showing efficiency of that approach.
2. Keywords: Column buckling, topology optimization, Cellular Automata.

3. Introduction
Topology optimization of structures exposed to loss of stability rarely appears in the literature, hence the number of publications dealing with this subject is rather modest. This also applies to topology optimization of vibrating elements. From among papers discussing this issue one can point out for example Cheng et al. [4] or Du and Olhoff [5]. In the present paper, the problem of topology optimization of columns against instability is formulated in a manner allowing to solve it with Cellular Automata (CA) method.
Cellular Automaton consists of two components. The first is a cellular space: a uniform lattice of cells, each with an identical pattern of local connections to other cells. A particular cell together with the cells which it is connected to forms a neighborhood, and it is assumed that the interaction between the cells takes place only within the neighborhood. The second CA component is an update rule. The local rule is selected in order to control the evolution of each neighborhood state. The rule is identical for all neighborhoods and is applied simultaneously to each of them. The rule operates over a large number of cells which carry only local information. The principle of the CA is therefore that global behavior of the system is governed by cells that only interact with their neighbors. This kind of interaction, where exchange of information is local by nature, can be observed for example in biological tissues. The concept of Cellular Automata was introduced in late forties of 20th century, and since then have attracted researchers from various disciplines. CA can be used also in optimal design as effective and efficient optimization tool. It has been found attractive because of its simplicity and versatility and since CA methodology can be adopted both to optimal sizing and topology optimization. The first application of Cellular Automata to optimization of structures was proposed in mid nineties of 20th century, and the basic idea was described by Inou et al. [6]. Majority of papers dealing with that subject have been published during last decade, and the most common application of Cellular Automata in structural optimal design is topology optimization. This paper presents an implementation of Cellular Automata method to topology optimization paying attention on the novel issue which is generation of compliance based column topologies for maximal buckling load.
It is worth noting that CA approach to structural optimization by nature requires local formulation of the design problem, and therefore it is not straightforward how to apply it to maximization of a structure buckling load, which is a global quantity. Fortunately one can for example observe that for the optimal column, for which critical load has been maximized, the maximal bending stress is uniformly distributed along column axis. Taking that into account it is possible to replace conventional maximization of buckling load by a problem formulated as the fully stressed design. This concept was presented in Bochenek and Tajs-Zielińska [2], where the locally formulated problem of optimal sizing of columns prone to instability was considered. Based on the above observation another locally formulated problem is proposed to be applied for maximization of global buckling load. The detailed description of the proposed concept is given in Sec.5.

4. Generating optimal topologies with local rules of Cellular Automata

In topology optimization (e.g. [1]) one searches for a distribution of material within a design domain that is optimal in some sense. The design process consists in redistribution of a material and parts that are not necessary from objective point of view are selectively removed. Topology optimization usually ends up in finding material/void distribution that is visualized by black and white regions over the design domain. The power law approach defining solid isotropic material with penalization (SIMP) with design variables being relative densities of a material can easily be implemented into Cellular Automata approach. The elastic modulus of each cell element is modeled as a function of relative density d_i using power law, see Eq.(1). This power p penalizes intermediate densities and drives design to a black-and-white structure.

$$E_i = d_i^p E_0, \quad \rho_i = d_i \rho_0, \quad d_{\min} \le d_i \le 1 \tag{1}$$

The local update rule applied to design variables d_i associated with central cells is now constructed based on information gathered from adjacent cells forming the Moore or von Neumann type neighborhood. It is set up as linear combination of design variables corrections with coefficients, values of which are influenced by states of the neighborhood surrounding each cell, as presented in Eq.(2):

$$d_i^{(t+1)} = d_i^{(t)} + \delta d_i^{(t)}, \quad \delta d_i^{(t)} = (\alpha_0 + \sum_{k=1}^{N} \alpha_k)\, m = \tilde{\alpha}\, m \tag{2}$$

$$\alpha_0 = \begin{cases} -C_{\alpha 0} & \text{gdy} \quad U_i^{(t)} < U^* \\ C_{\alpha 0} & \text{gdy} \quad U_i^{(t)} \ge U^* \end{cases} \tag{3}$$

$$\alpha_k = \begin{cases} -C_{\alpha} & \text{gdy} \quad U_{ik}^{(t)} < U^* \\ C_{\alpha} & \text{gdy} \quad U_{ik}^{(t)} \ge U^* \end{cases} \qquad k = 1\ldots N \tag{4}$$

The local compliance values calculated for central cell U_i and neighboring cells U_{ik} are compared to a selected threshold value U^* and depending whether they are larger or smaller than the one selected, positive or negative coefficients are transferred to the design variable update, as given by Eq.(3), Eq.(4). Practical implementation of proposed local rules requires specification of introduced parameters. The values of $C_{\alpha 0}$ and C_{α} are selected so as to provide that their sum equals one. Based on numerical tests performed for von Neumann type neighborhood all coefficients equal to 0.2 seems to be a good choice. The move limit m implemented in the above algorithm controls the allowable changes of the design variables values. If stresses are replaced by local compliances, the above rule can be applied to generation of compliance based topologies. That type of problems have been discussed in former papers e.g. [2], [3], showing good agreement with results presented in literature, obtained with use of various optimization techniques, both evolutionary and based on optimality criteria.

5. A concept of generating compliance based topologies for maximal buckling load

In this paper the problem of topology optimization against instability is formulated in a manner allowing to solve it with Cellular Automata method. The concept is as follows. First the standard instability analysis of a compressed column is performed and buckling mode is determined. Then the compressive loading is replaced by a transverse one which is selected so as to generate bending moment, distribution of which coincides with the one representing particular buckling mode. For the bent structure minimization of compliance is performed, and optimal topology is generated. Finally the critical load for the optimal column topology is calculated.

The following example explains the above idea. A simply supported column has been chosen. The well known solution of the Euler buckling problem leads to instability mode described by a sine function. This refers to both deflection and bending moment. The plane model is now created, with the transverse loading selected so as to reflect bending moment distribution obtained for critical state. The minimal compliance topology optimization is performed, and the final topology is presented in Fig.2. This result is similar to the one shown in Du and Olhoff [5].

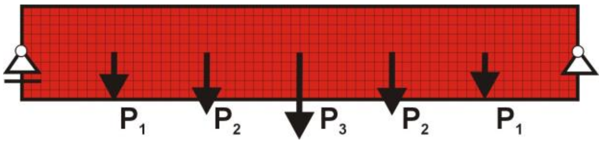

Figure 1: A plane model for topology optimization

Figure 2: The plane topology of minimal compliance, volume fraction 0.6

6. Selected column topologies generated for maximal buckling load

In this section a spatial model of column, which is a direct extension of the plane one presented above, is considered. The first case is a simply supported column shown in Fig.3. The transverse loading is applied in one buckling plane.

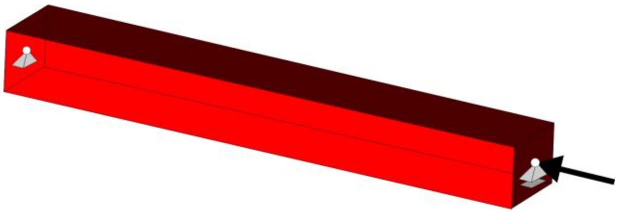

Figure 3: A spatial model of a simply supported column

In Fig.4 the topology found for the volume fraction 0.5 is presented.

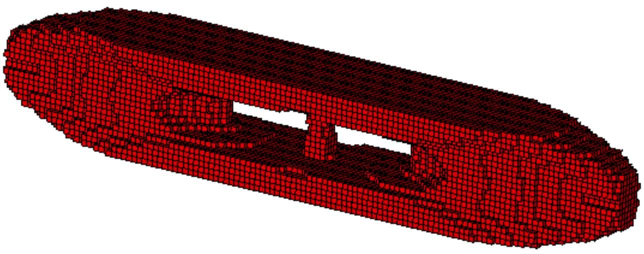

Figure 4: Final topology. The simply supported column 20×20×120 cells, one buckling plane

The calculations are repeated for a more slender column, and the final result is given in Fig.5.

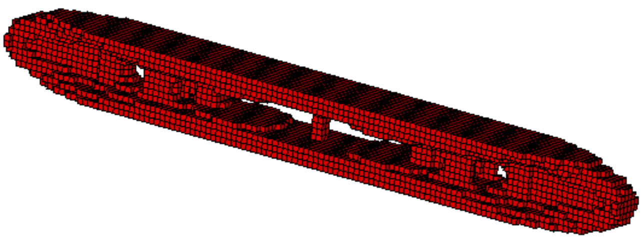

Figure 5: Final topology. The simply supported column 12×12×120 cells, one buckling plane

The next structure is a column simply supported at one end and clamped at the other, see Fig.6.

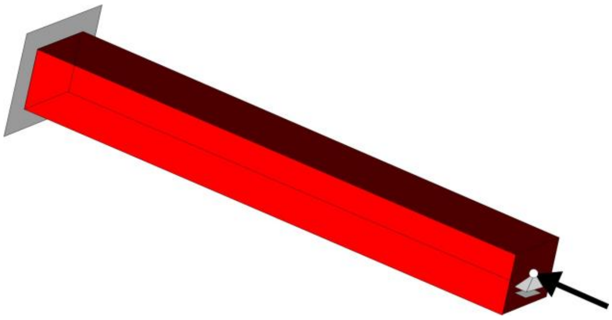

Figure 6: A spatial model of a clamped-simply supported column

As for the previous example, one starts with one buckling plane, what leads to topology shown in Fig.7.

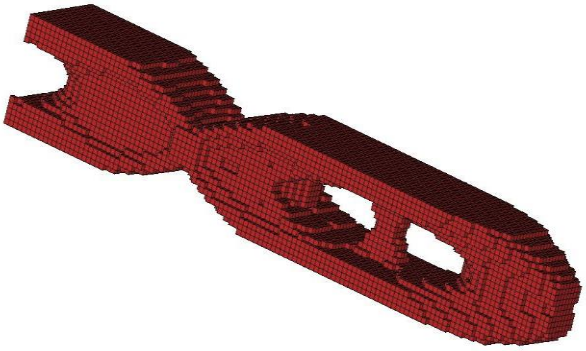

Figure 7: Final topology. The clamped-simply supported column 20×20×120 cells, one buckling plane

It is worth noting that if buckling in both planes is possible the solution found for only one plane considered may not be correct if critical load referring to the second plane has a lower value. In order to eliminate that drawback the formulation of topology optimization problem is extended and loadings applied in both planes are considered. The solution found for the two load case topology is presented in Fig.8.

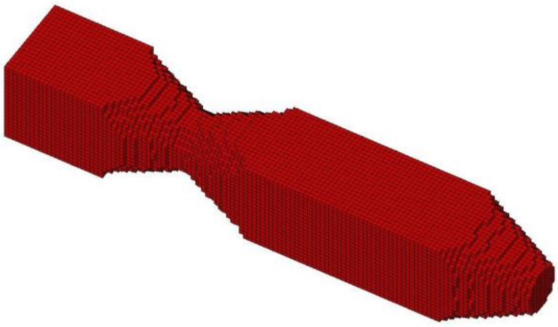

Figure 8: Final topology. The clamped-simply supported column 20×20×120 cells (1cm×1cm×1cm),
two buckling planes, E=200 GPa, critical load 172048 kN

To make details of the final topology visible, a section view is given in Fig.9.

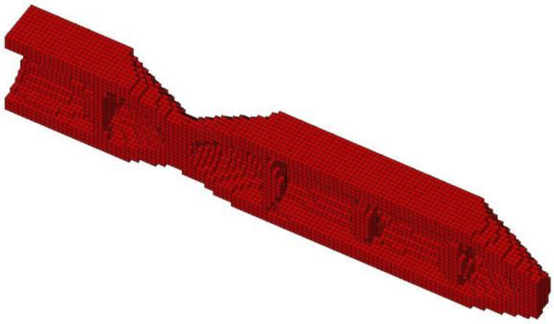

Figure 9: Final topology, section view. The clamped-simply supported column 20×20×120 cells

The critical load for a prismatic column of the same volume is 93470 kN.
Calculations are repeated for a more slender column and results are shown in Figs 10, 11.

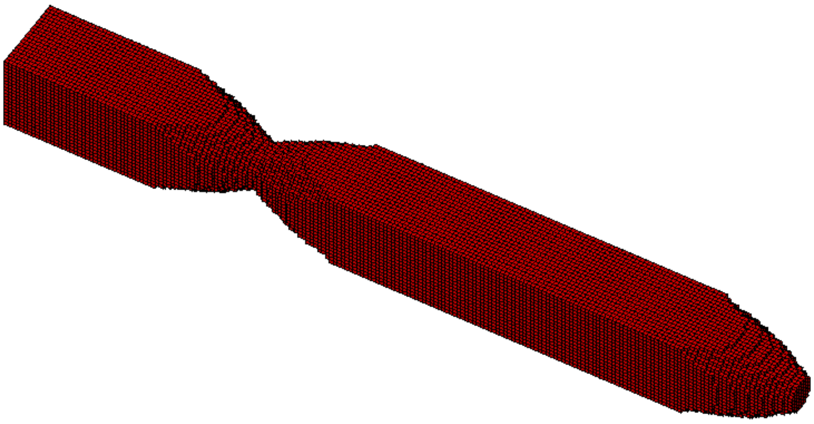

Figure 10: Final topology. The clamped-simply supported column 20×20×200 cells (1cm×1cm×1cm), two buckling planes, E=200 GPa, critical load 86966 kN

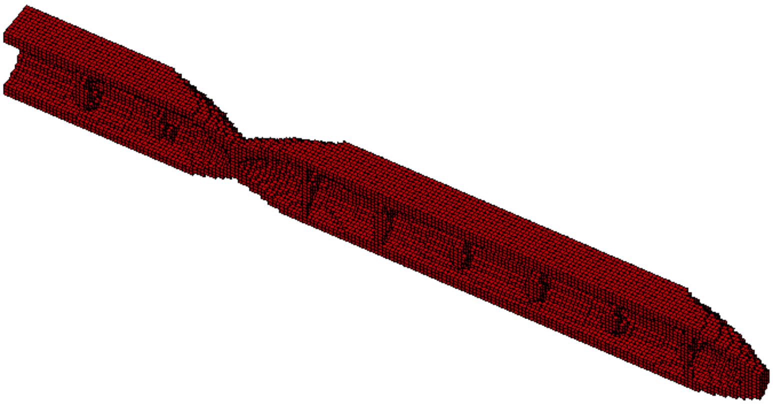

Figure 11: Final topology, section view. The clamped-simply supported column 20×20×200 cells

The critical load for a prismatic column of the same volume is 33650 kN. The critical loads for generated topologies are significantly higher as compared to a prismatic columns of the same volume. It should be also stressed that critical load obtained for the column of optimal topology is higher than the one found for a column for which the standard approach of optimal sizing against buckling has been applied.

7. References

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