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Level Set Based Topology Optimization for Optical Cloaks Containing a Large Scattering Object

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1. Abstract

This paper presents level-set based topology optimization for optical cloaks containing a large scattering object. A large object whose typical scale is larger than the wavelength of incident light is assumed as a scattering object. A topology optimization method based on the level set expression of dielectric structures is used to optimize the configuration of optical cloaks. A finite element method is used for light scattering analyses, the computation of adjoint fields, and updating level set functions. Level set functions are defined on grid points of grids to express dielectric structures. Dielectric boundaries are interpreted as lines on the iso-surface of the level set functions.

2. Keywords: Optical Cloak, Topology Optimization, Level Set Method.

3. Introduction

Optical cloaks [1, 2, 3] can render an object invisible by controlling and minimizing the scattering of light around an object. The cloaking devices have been received much attention as one of the most interesting and remarkable topics in the field of optical engineering. Several optical cloaking devices are proposed as microwave cloaking shell [3] composed of split ring resonators [4], fishnet metamaterials [5], and dielectric carpet cloaks [6, 7, 8, 9].

In order to realize high performance optical cloaks, it is important to design cloaking structures appropriately. Optimization methods by using numerical computaions are effective approach for the design of optical cloaks. Among of the methods, topology optimizations are one of the most powerful ones. Andkjær et al. applied density and gradient based topology optimization to the design of optical cloaks [10, 11]. Objective functionals are defined as the integrated intensities of light scattering around a scattering object, and the scattering of light is minimized. They also succeeded to remove grayscales [11] by using Heaviside filtering scheme, and polarization-free optical cloaks are presented. On the other hand, we developed level set based topology optimization for optical cloaks [12] and succeeded to obtain grayscale-free optimal design of optical cloaks. The optimized optical cloaks in some cases can minimize the amount of light scattering less than one-tenth of those presented in previous studies. The optimal designs of optical cloaks presented in the above studies render a wavelength-scale object invisible. However, cloaking structures for a scattering object which is larger than wavelength scale are not presented. As the size of a scattering object becomes larger, it becomes more difficult to minimize the scattering of light. The enlargement of scattering objects make it difficult to render the objects invisible.

In this work, we present the design of optical cloaks for a large scattering object whose typical scale is larger than the wavelength of incident light by using level set based topology optimization [13]. The integrated intensity of light scattering around a large scattering object is considered as an objective functional in order to minimize light scattering. We employ a finite element method (FEM) and a perfectly matched layer absorbing boundary condition (PMLABC) to simulate the scattering of light in an open region. Dielectric boundaries between dielectric materials and air are interpreted as the iso-surface of level set functions given on grid points, and finite elements are created based on the dielectric boundaries.

4. Optimization

4.1. Cloak Scheme and Dielectric Boundary

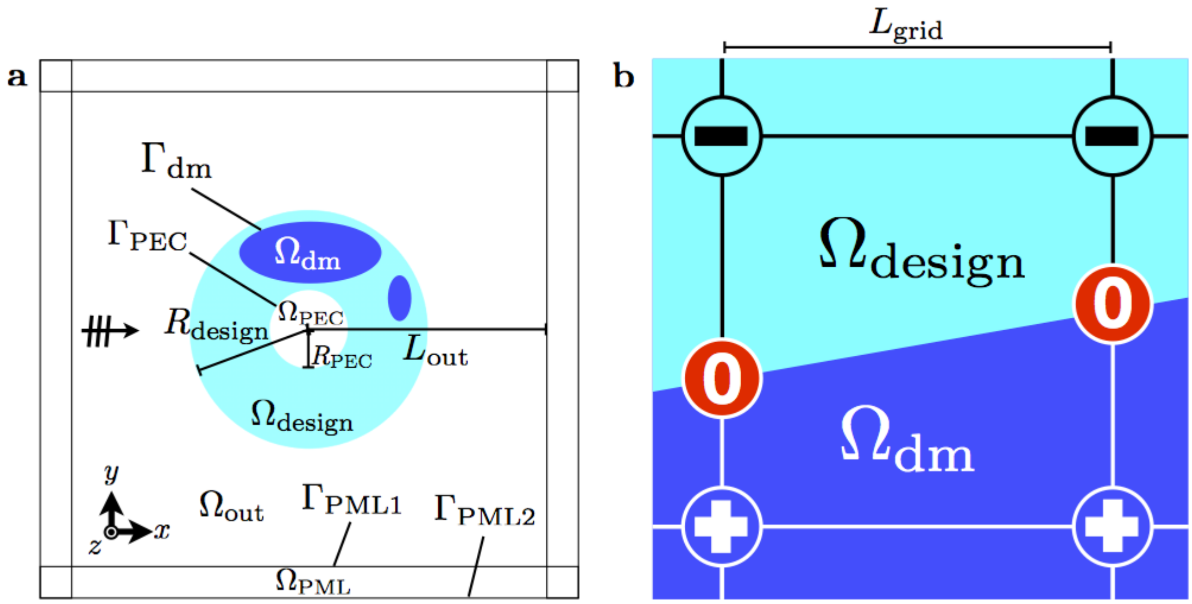

Figure 1: (a) Cloak scheme. The sizes of domains are $R_{\mathrm{design}} = 3R_{\mathrm{PEC}} = L_{\mathrm{out}}/2$. (b) Dielectric boundary defined by level set functions. The functions are linearly interpolated in a grid. The edge length of the grid is $L_{\mathrm{grid}} = R_{\mathrm{design}}/200$.

In this optimization problem, we assume light scattering in an open region. Figure 1(a) shows a schematic illustration of optical cloaks. Light waves are assumed to propagate in xy plane as transverse magnetic (TM) waves. We simulate light scattering in an open region by means of a FEM and a PMLABC. A perfect electric conductor (PEC) are assumed as a scattering object and Ω_{PEC} represents the domain in PEC. The PEC is located at the center of design domain Ω_{design} in which cloaking structures are designed and transform to minimize the scattering of light. The domains Ω_{dm} and Ω_{out} represent dielectric materials of optical cloaks and an outer domain of Ω_{design}. The objective functional, the amount of light scattering, is computed in Ω_{out}. R_{design} and R_{PEC} are the radii of the circular domains Ω_{design} and Ω_{PEC}, respectively.

Figure 1(b) shows dielectric boundary obtained from level set functions. The zero point of level set functions is determined in terms of linear interpolation of neighboring level set functions whose signs are different each other. The lines connecting two zero points of level set function are understood as dielectric boundaries. Finite elements are produced based on the dielectric boundaries.

4.2. Formulation for the Design of Optical Cloaks

We optimize the dielectric structure of two dimensional optical cloaks. Light waves are assumed to be TM mode and propagate in the xy plane. We solve a Helmholtz equation as follows:

$$\nabla^2 E_{\mathrm{s}} + \frac{\omega^2}{c^2}\epsilon(\mathbf{x})E_{\mathrm{s}} = -\frac{\omega^2}{c^2}[\epsilon(\mathbf{x}) - \epsilon_{\mathrm{air}}]E_{\mathrm{i}},$$

where E_{s} and E_{i} are the scattering and incident waves of electric field, c is the light velocity in a vacuum, ϵ_{air} is the relative permittivity of air. Position-dependent relative permittivity $\epsilon(\mathbf{x})$ is given as follows:

$$\epsilon(\mathbf{x}) = \begin{cases} \epsilon_{\mathrm{air}} + \chi(\epsilon_{\mathrm{dm}} - \epsilon_{\mathrm{air}}) & \mathbf{x} \in \Omega_{\mathrm{design}}, \\ \epsilon_{\mathrm{air}} & \mathbf{x} \in \Omega_{\mathrm{out}}, \end{cases}$$

where ϵ_{dm} is the relative permittivity of a dielectric material and χ is a characteristic function defined in the design domain as

$$\chi(\phi(\mathbf{x})) = \begin{cases} 1 & \text{if} \quad \mathbf{x} \in \Omega_{\mathrm{dm}}, \\ 0 & \text{if} \quad \mathbf{x} \in \Omega_{\mathrm{design}} \backslash \Omega_{\mathrm{dm}}, \end{cases}$$

where $\phi(\mathbf{x})$ is the level set function defined as piecewise constant values to dielectric material boundaries such that

$$\begin{cases} -1 \leq \phi(\mathbf{x}) < 0 & \text{for} \quad \forall \mathbf{x} \in \Omega_{\mathrm{design}} \backslash \Omega_{\mathrm{dm}}, \\ \phi(\mathbf{x}) = 0 & \text{for} \quad \forall \mathbf{x} \in \Gamma_{\mathrm{dm}}, \\ 0 < \phi(\mathbf{x}) \leq 1 & \text{for} \quad \forall \mathbf{x} \in \Omega_{\mathrm{dm}} \backslash \Gamma_{\mathrm{dm}}. \end{cases}$$

In order to design optical cloaks, we consider the problem of minimizing light scattering as follows:

$$\begin{aligned} \underset{\phi}{\text{minimize}} \quad & F = \tfrac{1}{F_0} \textstyle\int_{\Omega_{\mathrm{out}}} E_{\mathrm{s}} E_{\mathrm{s}}^{*} d\Omega \\ \text{subject to} \quad & a_{\mathrm{H}}(\phi, E, \tilde{E}) - b_{\mathrm{H}}(\phi, E, \tilde{E}) = 0 && \text{in } \Omega_{\mathrm{design}},\ \Omega_{\mathrm{out}} \\ & G = V_{\mathrm{dm}} - V_{\mathrm{max}} \leq 0 \\ & E_{\mathrm{s}} = -E_{\mathrm{i}} && \text{on } \Gamma_{\mathrm{PEC}} \\ & a_{\mathrm{PML}} = 0 && \text{in } \Omega_{\mathrm{PML}} \\ & E_{\mathrm{s}} = E_{\mathrm{PML}} && \text{on } \Gamma_{\mathrm{PML1}} \\ & E_{\mathrm{PML}} = 0 && \text{on } \Gamma_{\mathrm{PML2}}, \end{aligned}$$

where E_{s}^{*} is the complex conjugate of E_{s}, V_{max} is the upper limit of volume constraint, and the other notations are defined as follows:

$$\begin{aligned} & a_{\mathrm{H}}(\phi, E_{\mathrm{s}}, \tilde{E}) = \int \nabla^2 E_{\mathrm{s}} \tilde{E} d\Omega + \int \frac{\omega^2}{c^2} \epsilon(\mathbf{x}) E_{\mathrm{s}} \tilde{E} d\Omega \\ & b_{\mathrm{H}}(\phi, E_{\mathrm{s}}, \tilde{E}) = \int -\frac{\omega^2}{c^2} [\epsilon(\mathbf{x}) - \epsilon_{\mathrm{air}}] E_{\mathrm{i}} \tilde{E} d\Omega \\ & V_{\mathrm{dm}}(\chi) = \int_{\Omega_{\mathrm{design}}} \chi(\mathbf{x}) d\Omega \\ & a_{\mathrm{PML}}(E_{\mathrm{PML}}, \tilde{E}, \gamma_x, \gamma_y) \\ & = \int_{\Omega_{\mathrm{PML}}} \frac{1}{\gamma_x} \frac{1}{\partial x} \left(\frac{1}{\gamma_x} \frac{\partial E_{\mathrm{PML}}}{\partial x} \right) \tilde{E} d\Omega + \int_{\Omega_{\mathrm{PML}}} \frac{1}{\gamma_y} \frac{1}{\partial y} \left(\frac{1}{\gamma_y} \frac{\partial E_{\mathrm{PML}}}{\partial y} \right) \tilde{E} d\Omega + \int_{\Omega_{\mathrm{PML}}} \frac{\omega^2}{c^2} E_{\mathrm{PML}} \tilde{E} d\Omega \\ & F_0 = \int_{\Omega_{\mathrm{out}}} E_{\mathrm{s}} E_{\mathrm{s}}^{*} d\Omega \bigg|_{V_{\mathrm{dm}}=0}, \end{aligned}$$

where V_{dm} is the volume of a dielectric material, F_0 is the integrated intensity of the scattering field in Ω_{out} in the case of no cloak ($V_{\mathrm{dm}} = 0$), γ_x and γ_y are absorbing functions proposed in Ref. [14].

Topology optimization method must be regularized, because it is an ill-posed problem. Tikhonov regularization method is employed to regularize a topology optimization method. Based on the formulation of level set based topology optimization [13], a regularization term is introduced to the objective functional as follows:

$$\underset{\phi}{\text{minimize}} \qquad F_{\mathrm{r}} = \frac{1}{F_0} \int_{\Omega_{\mathrm{out}}} E_{\mathrm{s}} E_{\mathrm{s}}^{*} d\Omega + \int_{\Omega_{\mathrm{design}}} \frac{1}{2} \tau |\nabla \phi|^2 d\Omega,$$

where F_{r} is the regularized objective functional, τ is a positive regularization parameter which represents the ratio of the objective functional to the fictitious interface energy term.

The optimization problem for optical cloaks is replaced with an optimization problem using Lagrange' s method of undetermined multipliers as follows:

$$\underset{\phi}{\text{minimize}} \qquad \bar{F} = F + 2\mathrm{Re} \left\{ a_{\mathrm{H}}(\phi, E, \tilde{E}) - b_{\mathrm{H}}(\phi, E, \tilde{E}) + a_{\mathrm{PML}}(E_{\mathrm{PML}}, \tilde{E}, \gamma_x, \gamma_y) \right\} + \lambda G(\phi).$$

The Karush-Kuhn-Tucker (KKT) conditions required for the above optimization problem of optical cloaks are written as follows:

$$\bar{F}' = 0, \quad a_{\mathrm{H}}(\phi, E, \tilde{E}) - b_{\mathrm{H}}(\phi, E, \tilde{E}) + a_{\mathrm{PML}}(E_{\mathrm{PML}}, \tilde{E}, \gamma_x, \gamma_y) = 0, \quad \lambda G(\phi) = 0, \quad \lambda \geq 0, \quad G(\phi) \leq 0.$$

4.3. Updating level set functions and configurations

The level set functions representing the configurations of optical cloaks are updated by solving a following time-evolutional equation as

$$\frac{\partial \phi}{\partial t} = -K(\phi)(\bar{F}' - \tau \nabla^2 \phi),$$

where t is a fictitious time, and the level set functions ϕ becomes t-dependent functions. The fictitious time t is discretized by a time step Δt as follows:

$$\frac{\phi(t + \Delta t)}{\Delta t} = \frac{\phi(t)}{\Delta t} - K(\phi) \left[\bar{F}' - \tau \nabla^2 \phi(t) \right].$$

The above time differential equation is solved by using FEM. The level set functions are updated from $\phi(t)$ to $\phi(t + \Delta t)$.

4.4. Optimization algorism

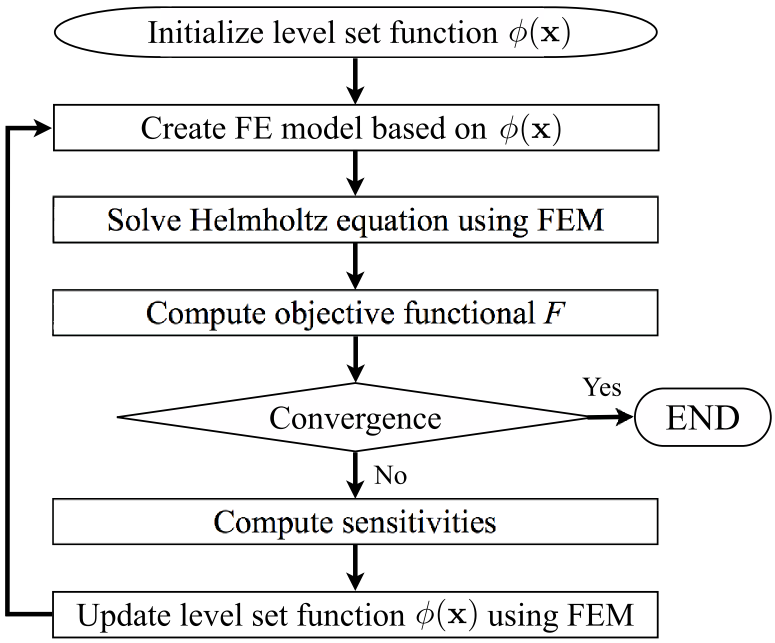

Figure 2: Flow chart of level set based topology optimization process.

Figure 2 shows the flow chart of a level set based topology optimization process. A initial configuration is determined by setting initial level set functions. Dielectric boundaries represented by the zero point of level set functions are derived from the initial level set functions as shown in Fig. 1(b). A FE model is created based on the dielectric boundaries. The scattering of light is simulated by means of FEM, and the solution of the Helmholtz equation is obtained. The value of objective functional representing the amount of light scattering is computed. The process of optimization is finished when the objective functional value converges, otherwise, the sensitivity is computed by means of adjoint variable method. Level set functions are updated by solving time-evolutional equation based on the computed sensitivity by using FEM.

5. Results

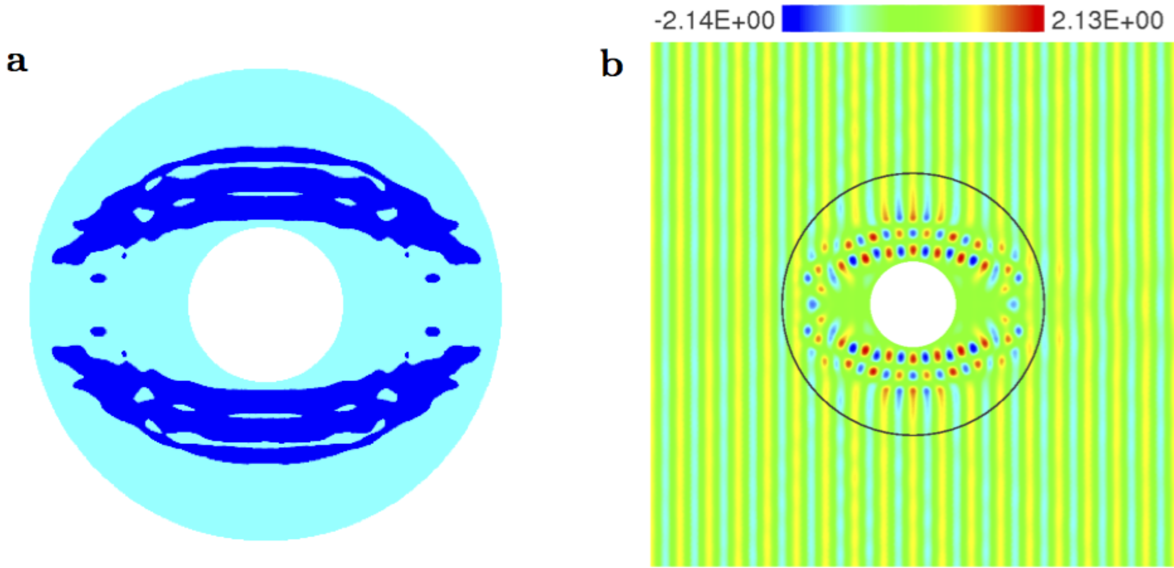

Figure 3: (a) Obtained optimal configuration of $\tau = 3.0 \times 10^{-6}$. (b) Normalized total electric field $\mathrm{Re}(E_z)/|E_\mathrm{i}|$ examples. Objective functional value is $F = 2.0656 \times 10^{-2}$. The normalized frequency is $\omega R_\mathrm{design}/2\pi c = 4.5$. The relative permittivity values are $\epsilon_\mathrm{dm} = 2.0$ and $\epsilon_\mathrm{air} = 1.0$. The black circle represents the design domain.

Figure 3 shows a obtained optimal configuration and the simulated result of light scattering minimized by the optimal configuration. Incident plane waves are propagate in the positive x direction. The normalized frequency and wavelength are $\omega R_\mathrm{design}/2\pi c = 4.5$ and $\lambda = R_\mathrm{design}/4.5 = R_\mathrm{PEC}/1.5$. The diameter of a scattering object is equal to 3λ. The total electric field in the outer domain of the design domain in Fig. 3(b) seems to correspond to incident plane waves, that is, the amount of light scattering is minimized.

6. Conclusion

This paper presented level set based topology optimization for optical cloaks containing a large scattering object. We developed level set based topology optimization for optical cloaks and designed an optical cloak which can render a scattering object whose radius is larger than wavelength of incident light. We succeed to obtain an optimal configuration for optical cloaks containing a large scattering object.

7. References

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