Reliability-Based Design Optimization Using Adaptive Design Regions

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1. Abstract
In this study, a new approximate reliability-based design optimization (RBDO) algorithm is proposed. Existing RBDO algorithms require a large amount of numerical expenses due to a double-loop structure of the design optimization and the reliability analysis. Because of the numerical expense of RBDO, it is hard to apply existing RBDO algorithms to a practical design problem involving a numerically expensive simulation such as nonlinear crash analysis or computational fluid dynamics analysis. The proposed approximate RBDO algorithm adopts a concept of the sequential approximate optimization which is well-known for its efficiency. The proposed algorithm needs no additional numerical expenses and gradient evaluations except for the generation of the metamodel. The proposed algorithm assigns a design region according to the target reliability index, and sets the size of an approximation region larger than the design region considering the size of sampling region for Monte Carlo sampling or Latin hypercube sampling. The proposed algorithm adaptively updates the location and size of the design and approximation regions considering the convergence history of the approximate RBDO. And then mathematical problem was used to verify the effectiveness and usefulness of the proposed algorithm.
2. Keywords: approximate RBDO; adaptive design regions; sampling methods

3. Introduction
Reliability-based design optimization (RBDO) has been had a lot of numerical burden because of iterative reliability analysis (RA). These numerical burdens became an obstacle to apply RBDO to manufacturing problems needed lots of time to analysis once. To improve numerical efficiency for RBDO and RA, approximate models of performance functions were used in this research. By using approximate model in evaluating the performance function, additional numerical burden is evitable in RA and RBDO procedure.
However, when the number of dimension of performance function is increased, the number of experiment points for generating an approximate model is increased exponentially. Also, in case of generating an approximate model about entire design region, the approximate model might not be sufficiently accurate. Therefore, these problems such as curse of dimensionality and appropriate size of approximate region should be solved for obtaining meaningful RA and RBDO results.
In this research, high dimensional model representation (HDMR) [1-3] was applied to approximate high dimensional function effectively. By using HDMR, high-dimensional performance function can be decomposed to the sum of low-dimensional functions. By approximating low-dimensional functions obtained by HDMR, curse of dimensionality can be solved. The concept of sequential approximate optimization and approximate model is applied to perform RBDO. When sampling method is performed for RA, approximate region and design region are dualized considering the probabilistic characteristic that distribution is generated with design variables as the center. Moreover, technique of managing the size and position of approximate region and design region is proposed in procedure of sequential approximate reliability-based design optimization (SARBDO).

4. Proposed Sequential Approximate Reliability-Based Design Optimization

4.1. High Dimensional Model Representation
High dimensional model representation (HDMR) is general form of the dimension reduction method. Depending on the method to determine the component functions, HDMR can be classified into ANOVA-HDMR, Cut-HDMR, Factorized HDMR and Hybrid HDMR, etc. Generalized form of HDMR is as follows.

$$\begin{aligned} g(\mathbf{x}) &= g_0 + \sum_{i=1}^{n} g_i(x_i) + \sum_{1\le i_1 \le i_2 \le n} g_{i_1 i_2}(x_{i_1}, x_{i_2}) + \cdots \\ &+ \sum_{1\le i_1 < \cdots < i_m \le n} g_{i_1 i_2 \cdots i_m}(x_{i_1}, x_{i_2}, \cdots; x_{i_m}) + \cdots + g_{12\cdots n}(x_1, x_2, \cdots; x_n) \end{aligned} \tag{1}$$

where, g_0 is zero-th order component function or average performance of $g(\mathbf{x})$, $g_i(x_i)$ is first order component function that present independent effect on $g(\mathbf{x})$ with respect to x_i, $g_{i_1 i_2}(x_{i_1}, x_{i_2})$ is second order component function that present an effect on $g(\mathbf{x})$ with respect to x_i and x_j simultaneously, $g_{12\cdots n}(x_1, x_2, \cdots, x_n)$ present an effect with respect to all design variables except an effect of functions from zero-th order to n-1th order.
Among various HDMR methods, cut-HDMR is a representation of the output $g(\mathbf{x})$ in the hyperplane passing through a reference point in the variable space. When reference point is set as $\mathbf{c} = [c_1, \cdots, c_n]$, 0-2 order component functions are presented as equation (2-4).

$$g_0 = g(\mathbf{c}) \tag{2}$$

$$g_i(x_i) = g(c_1, \ldots, c_{i-1}, x_i, c_{i+1}, \ldots, c_n) - g_0 \tag{3}$$

$$\begin{aligned} g_{ij}(x_i, x_j) = g(&c_1, \ldots, c_{i-1}, x_i, c_{i+1}, \ldots, c_{j-1}, x_j, c_{j+1}, \ldots, c_n) \\ &- g_i(x_i) - g_j(x_j) - g_0 \end{aligned} \tag{4}$$

In cut-HDMR [1], performance function cut by hyperplane on reference point is taken as component function. For example, $g_{ij}(x_i, x_j)$ is second order function when the other design variable is equal to reference point except i and j-th design variables. In cut-HDMR, order level of component can be determined by engineers. When order level including component function is getting higher, performance function can be described more accurately, but numerical expenses for composing cut-HDMR more increased. However, first and second order component function which is main effect of probabilistic variables in systems have dominant influence on most engineering system [4]. Therefore, third and higher order components are practically insignificant. First order HDMR and second order HDMR are represented as equation (5) and (6), respectively.

$$g(\mathbf{x}) \approx g_0 + \sum_{i=1}^{n} g_i(x_i) \tag{5}$$

$$g(\mathbf{x}) \approx g_0 + \sum_{i=1}^{n} g_i(x_i) + \sum_{(i,j) \subseteq U, i<j} g_{ij}(x_i, x_j) \tag{6}$$

where, U is universal set has possible all dual combination with respect to n design variables as elements. In second order HDMR, performance function is represented considering all second order elements in universal set. However, cut-HDMR is the way of representing high dimensional function. Therefore, approximation of each component function is needed to evaluate performance function value by using cut-HDMR.

4.2. Dualization of Approximate and Design Regions

4.2.1. Setting the Size of Initial Design Region considering Target Reliability Index

Target reliability index is the distance between the solution of RBDO and most probable point (MPP) in u-space. In x-space, target reliability index means the distance between the solution of RBDO and deterministic optimum. As shown in Figure 1, when initial design region is set as hypercube which edges are determined by standard deviation of each random variables and target reliability index with deterministic optimum as the center, the RBDO solution should be existed in initial design region theoretically.

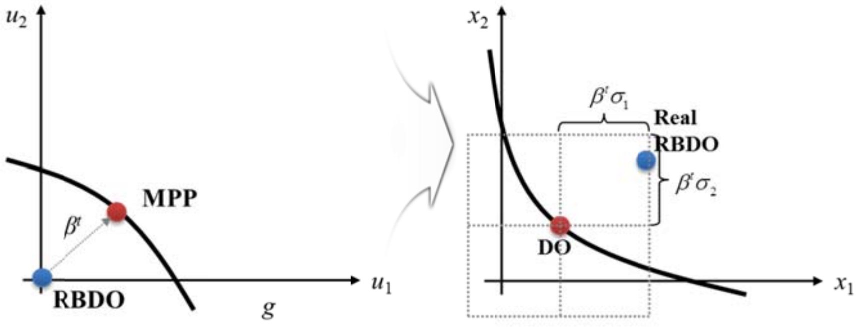


Figure 1: Design region considering target reliability index

4.2.2. Setting the Size of Approximate Region considering Sampling Method

Approximate model is used to calculate probabilistic moments in RA and RBDO. Also, the size of approximate region is set considering probabilistic characteristics. Sampling method on approximate model is used for RA. Therefore, the size of sampling region is very important. Suppose that the size of design region and approximate region are same. Generally, the mean of random variables is used as design variables in RBDO. Joint probabilistic distribution with mean of random variables as center is moved iteratively during approximate RBDO. However, In case the size of design region and approximate region are same, sampling region for RA get out of approximate region when design variables is getting closer to the boundary of design region as Figure 2. In Figure 2, k_a and k_d are approximate region parameter and design region parameter, respectively. If approximate model is generated by interpolation method such as Kriging, predict values in sampling region out of approximate region are possibly very inaccurate.

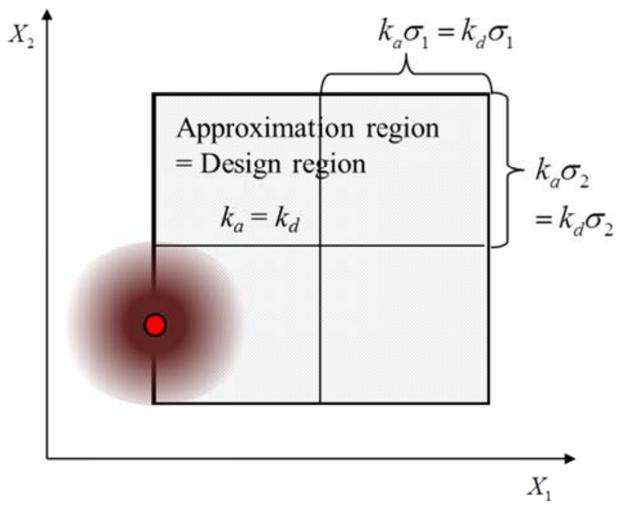


Figure 2 : Occurrence of extrapolation

In order to solve this problem, approximate region in which metamodel is generated, and design region in which design variables are located, are dualized. With dualization of approximate region and design region, sampling region is always located in approximate region in RBDO procedure if design variables are located in boundary of design variables. When approximate region is set as Figure 3, extrapolation of approximate model does not occur. Therefore, accurate result of RA can be expected. In Figure 3, k_{UA} is sampling parameter that means the size of sampling region. The size of approximate region considering the size of design region and sampling region can be expressed as equation 7.

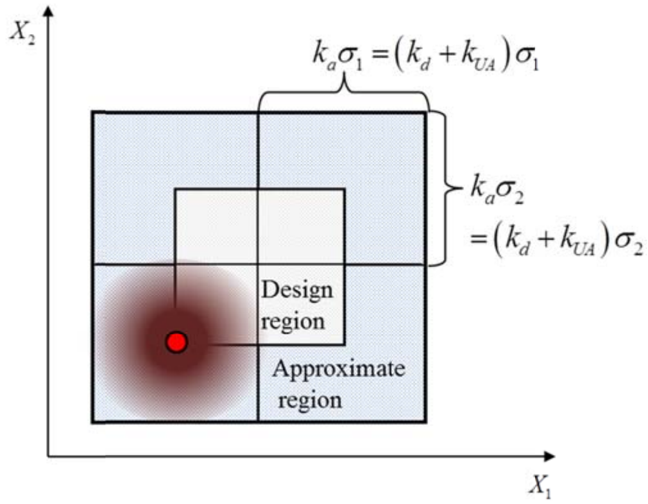


Figure 3 : dualization of approximate region and design region

$$k_a = k_d + k_{UA} \tag{7}$$

5. Numerical Example

In order to verify an effectiveness of proposed RBDO algorithm, highly nonlinear optimization problem is solved. This example consists of two design variables that follow a normal distribution and three limit state functions [5]. The nonlinearity of the limit state function is very severe. The optimization problem is given as

$$\begin{aligned}
&\text{find} && \mathbf{d} = \left[\mu_{X_1}, \mu_{X_2}\right]^T \\
&\text{min} && -\frac{\left(\mu_{X_1} + \mu_{X_2} - 10\right)^2}{30} - \frac{\left(\mu_{X_1} - \mu_{X_2} + 10\right)^2}{120} \\
&\text{subject to} && P\left(g_j(\mathbf{X}) > 0\right) \le P_{f,j}^t,\ j = 1, \ldots, 3 \\
& && \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U
\end{aligned} \tag{8}$$

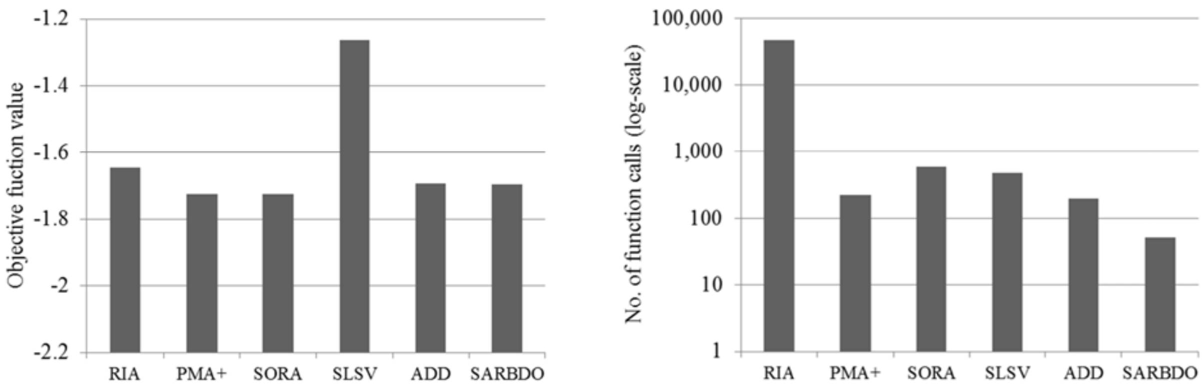
where, the limit state functions are as follows.

$$\begin{aligned}
&g_1(\mathbf{X}) = 1 - \frac{X_1^2 X_2}{20} \\
&g_2(\mathbf{X}) = 1 - (y-6)^2 - (y-6)^3 + 0.6(y-6)^4 + z \\
&g_3(\mathbf{X}) = 1 - \frac{80}{\left(X_1^2 + 8X_2 + 5\right)} \\
&y = 0.9063X_1 + 0.4226X_2 \\
&z = 0.4226X_1 - 0.9063X_2 \\
&\mathbf{d}^0 = [5,5]^T\ \mathbf{d}^L = [0,0]^T,\ \mathbf{d}^U = [10,10]^T \\
&X_i \sim Normal\left(\mu_{X_i}, 0.3^2\right),\ for\ i = 1,2
\end{aligned} \tag{9}$$

Target probability of failure is set to $P_{f,j}^t = 1.350e{-3},\ j = 1, \ldots, 3$. The results of the RBDO are shown in Table 1 and Figure 4.

Table 1: Results of RBDO for numerical example

	RIA	PMA+	SORA	SLSV	SARBDO
OBJ	-1.646	-1.725	-1.725	-1.263	-1.696
X(1)	4.727	4.558	4.558	3.498	4.477
X(2)	2.144	1.965	1.965	3.160	2.034
P_f^{max}	6.410e-4	1.598e-3	1.589e-3	1.489e-3	1.314e-3
Function call	47,311	224	588	477	52
Remarks	infeasible	infeasible	infeasible	infeasible	feasible



(a) Objective function value

(b) Function call

Figure 4 : Comparison of RBDO results for numerical example

Among the applied RBDO algorithms in this numerical example, only proposed RBDO algorithm, SARBDO finds the RBDO solution satisfied with the probabilistic constraints. Total number of function call is the lowest in SARBDO. According to the RBDO results, SARBDO is effectively applicable in case of high nonlinearity of limit state functions. Convergence of SARBDO is shown as Figure 5.

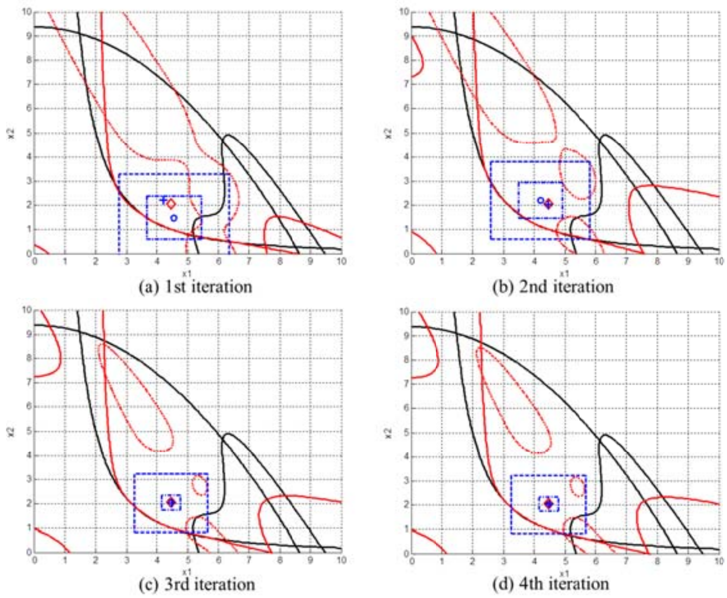


Figure 5 : Convergence of SARBDO for numerical example

6. Conclusions

In this research, new sequential approximate reliability-based design optimization which solve large numerical burden in current reliability-based design optimization algorithms, is proposed. In order to approximate the high-dimension performance function effectively, high dimension model representation is applied. And then, the size of approximate region and design region was determined considering probabilistic characteristics used in reliability analysis and reliability-based design optimization. With highly nonlinear numerical problem, the efficiency of the proposed SARBDO way verified compared to current RBDO algorithms.

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8. References

[1] H. Rabitz and O. F. Alis, General Foundations of High Dimensional Model Representations, *Journal of Mathematical Chemistry*, Vol. 25, No. 2-3, 197-233, 1999.

[2] G. Li, S. W. Wang, and H. Rabitz, High Dimensional Model Representations Generated from Low Dimensional Data Samples. I. mp-Cut-HDMR, *Journal of Mathematical Chemistry*, 30(1), 1-30, 2001.

[3] I. M. Sobol, Theorems and Examples on High Dimensional Model Representation, *Reliability Engineering and System Safety*, 79, 187-193, 2003.

[4] R. Chowdhury and B. N. Rao, High-Dimensional Model Representation for Structural Reliability Analysis, *Communications in Numerical Methods in Engineering*, 25(4), 301-337, 2009.

[5] I. Lee, K. K. Choi, and D. Gorsich, System Reliability-Based Design Optimization Using the MPP-Based Dimension Reduction Method, *Structural Multidisciplinary Optimization*, 41, 823-839, 2010.