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Reliability based design optimization with experiments on demand

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1. Abstract
In this paper, an algorithm for reliability based design optimization (RBDO) is presented. It incorporates a novel procedure in which experiments are performed one at a time where and when they are needed. The procedure is called experiments on demand (EoD). The experimental procedure utilizes properties specific to RBDO and the problem at hand augmented by the concept of *D*-optimality familiar from traditional design of experiments. Furthermore, an adaptive surrogate model fitting scheme is proposed which balances numerical stability and convergence rate as well as accuracy. Benchmarked against algorithms in the literature, the number of experiments needed for convergence was reduced by up to 80 % for a frequently used problem. The accuracy of the reliability index is in line with the most efficient algorithm against which it was benchmarked but up to 3 % lower than the most accurate algorithm.
2. Keywords: Experiments on demand, Reliability based design optimization, Surrogate model

3. Introduction
In this paper, an RBDO algorithm employing a problem-dependent computer experiment procedure is presented; experiments on demand (EoD). It is a one-experiment-at-a-time approach. The justification for the work is that computer experiments on physics-based models can be so expensive that the algorithms presented to date make RBDO industrially unfeasible, *e.g.* for large scale FE-models. The goal of the proposed algorithm is to reduce the number of computer experiments needed for convergence in RBDO problems. This is achieved by making the most out of the information available before performing additional experiments, and to conduct them one at a time rather than in sets. Moreover, every new experiment is placed (in design space) where it is predicted to add the most useful information, *i.e.* where the demand for new information is the highest. The definition of demand is here made by specifically considering the core of RBDO, which is the most probable failure point (MPFP). Furthermore, if experiments have already been performed in the vicinity of the MPFP, the next experiment is added in a *D*-optimal way. Thus, the *D*-optimality augmented experiments on demand procedure utilized in this paper combines aspects of both RBDO and classical DoE into an advantageous scheme that reduce the expensive computer model evaluations needed for convergence. The particular choice of *D*-optimality is not crucial but to distribute the experiments in a space-filling fashion in the design space is.

4. Reliability based design optimization
In reliability based design optimization (RBDO) the problem can be stated as

$$\begin{aligned} \min_{\boldsymbol{\mu}} \quad & C(\boldsymbol{\mu}) \\ \text{s.t.} \quad & \begin{cases} p_{f,j}(\mathbf{X}) \le \alpha_j, \ j = 1 \dots N_C \\ \mu_i^L < \mu_i < \mu_i^U, \ i = 1 \dots N_X \end{cases} \end{aligned} \tag{1}$$

Throughout this paper, $\mathbf{X} = [X_1 \dots X_{N_X}]^{\mathrm{T}}$ is the design variable vector and its lowercase counterpart $\mathbf{x}$ means realizations thereof, $\boldsymbol{\mu} = [\mu_1 \dots \mu_{N_X}]^{\mathrm{T}}$ is the design variable mean value vector where $\mu_i = E[X_i]$, C is the objective function (cost), $p_{f,j}$ is the j:th failure probability, α_j is the value of the j:th target failure probability, and μ_i^L and μ_i^U are the lower and upper bound of design variable i, respectively. The probability of failure can be formulated using a failure function G and a limit state g separating the safe and the failure domain. Conventionally, $g = 0$ is used, so that

$$\begin{aligned} p_{f,j}(\mathbf{X}) &= \mathrm{P}(G_j(\mathbf{X}) \le 0) \\ &= \int\limits_{G_j(\mathbf{x}) \le 0} f_{X_1 \dots X_{N_X}}(\mathbf{x}) dx_1 \dots dx_{N_X} \ , \end{aligned} \tag{2}$$

where P(•) denotes probability of the event and $f_{\mathbf{X}}$ is the joint probability distribution function of the design variables. In this paper, the integral is solved using the first order reliability method (FORM) for reliability assessment. FORM is based on an isoprobabilistic transformation of design variables to normed normally distributed variables followed by a linearization of the failure function limit state ($g = 0$). For non-linear functions, it was shown in [4] that the point of linearization is of utmost importance. Two points dominate the literature; the *most probable failure point* (MPFP) and the *minimum performance target point* (MPTP). It was shown by [9] that formulations based on the MPTP are less sensitive to the non-linearity of the isoprobabilistic transformation than those based on the MPFP. Additionally, in RBDO formulations, the linearization point is either determined exactly for each iteration in the optimization or gradually converging approximations are used. In both [1] and [8], the so-called single loop-single variable RBDO algorithm, which is based on an approximate MPTP was found to be most efficient and highly stable. Thus, the approximate MPTP approach presented for normally distributed variables by [2] and further developed to apply to general distributions in [7] is used in this work.

4.1 Algorithm description

The RBDO algorithm employed in this work is, apart from the experiment design, that of [3]. The surrogate model used in this work is

$$\hat{r}(\mathbf{x}) = r^0 + a\left(\mathbf{n}^{\mathrm{T}}\mathbf{x} + \overline{p}\right)^{\gamma}. \tag{3}$$

It is a function with a hyperplane as limit state but with non-linearity in the gradient direction. For a justification of the model and a description of the coefficient fitting scheme, see [3].

Based on the assumption that the design variables can be separated into their mean values μ_i and a function of its normed normally distributed counterparts u_i as

$$X_i = \mu_i + H_i(u_i), \tag{4}$$

the equivalent standard deviation matrix components $\hat{\sigma}_{ii}$ are estimated through

$$\hat{\sigma}_{ii,j}^{(k,l)} = \left.\frac{\partial H_i}{\partial u_i}\right|_{\mu_i^{(k,l)}, \mathbf{u}_j^{*(k,l)}}, \tag{5}$$

The condition in Eq. (4) holds for a variety of probability distributions such as the Normal, the Lognormal, the Gumbel, the Uniform and the three-parameter Weibull if the variables are independent. If the design variables are normally distributed, then $H_i(u_i) = \sigma_i u_i$ and the MPTP is independent of l. For other distributions and dependent variables, algorithms have been proposed, see [5]. The important point here is that the approximate MPTP from the previous iteration $\mathbf{u}_j^{*(k,l)}$ is used.

In the RBDO algorithm, $G = r_{max} - r$, where r is the surrogate model presented in Eq. (3), is used to formulate the failure function. Having approximated the equivalent standard deviation matrix $\hat{\boldsymbol{\sigma}}_j$, approximate MPTPs $\mathbf{u}_j^{*(k,l)}$ are computed as

$$\mathbf{u}_j^{*(k,l+1)} = \frac{\hat{\boldsymbol{\sigma}}_j^{(k,l)} \hat{\nabla} r_j^{(k)}}{\left|\hat{\boldsymbol{\sigma}}_j^{(k,l)} \hat{\nabla} r_j^{(k)}\right|} \beta_j = \pm \hat{\boldsymbol{\sigma}}_j^{(k,l)} \hat{\mathbf{n}}_j^{(k)} \beta_j \tag{6}$$

where $\beta_j = \Phi^{-1}(1 - \alpha_j)$ is the reliability index and $\Phi(\bullet)$ is the cumulative probability distribution for a normed normally distributed variable, $\hat{\mathbf{n}}_j^{(k)}$ is the k:th estimate of the j:th limit state normal. It can be noted here that the sign in Eq. (6) depends on the sign of the coefficients $\hat{a}$ and $\hat{\gamma}$, *i.e.* the estimations of a and γ in Eq. (3). Finally, the k,l:th optmization problem can be stated as

$$\begin{aligned} \min_{\boldsymbol{\mu}} \quad & \pm \hat{\mathbf{n}}_c^{\mathrm{T}} \boldsymbol{\mu} \\ \text{s.t.} \quad & \begin{cases} (\Delta r_j / \hat{a}_j)^{1/\hat{\gamma}_j} - \overline{p}_j - \hat{\mathbf{n}}_j^{\mathrm{T}} \hat{\boldsymbol{\sigma}}_j \mathbf{u}_j^* \leq \hat{\mathbf{n}}_j^{\mathrm{T}} \boldsymbol{\mu} \\ \mu_i^L < \mu_i < \mu_i^U \end{cases} \end{aligned} \tag{7}$$

where superscripts k,l have been dropped for readability but are clear from Eqs. (5) and (6), $\hat{\mathbf{n}}_c$ is the normal of the cost function limit state, and $\Delta r_j = r_{max,j} - r_j^0$. The sign in Eq. (7) is determined by the signs of the coefficients $\hat{a}_c$ and $\hat{\gamma}_c$. As can be noted in Eq. (7), the surrogate model employed in this work facilitates the solution of the RBDO problem through a series of linear optimization problems. Thus, linear programming algorithms, such as the simplex method, can be used to solve the optimization problem once the coefficients have been estimated.

5. Experiment scheme

5.1. First iteration

In the first iteration, it is assumed that specific information about the problem is not known. Therefore, a set of experiments is conducted. For the directional surrogate model, the minimum number of experiments needed is $N_X + 2$. Also, the experiments $\mathbf{x}_m$ need to be placed so that the projected design variable experiment vector $[\mathbf{p}] = [p_{m,j} = \mathbf{n}_j^{\mathrm{T}}\mathbf{x}_m]$ has at least $N_X + 1$ unique entries for all constraints $j = 1,\ldots,N_C$. The following experiment design (set of experiments)

$$\mathbf{D}^{(1)} = \begin{bmatrix} \boldsymbol{\mu}^{(1)^{\mathrm{T}}} \\ \left[\boldsymbol{\mu}^{(1)} + \beta\hat{\sigma}_1^{(1)}\mathbf{e}_1\right]^{\mathrm{T}} \\ \vdots \\ \left[\boldsymbol{\mu}^{(1)} + \beta\hat{\sigma}_{N_X}^{(1)}\mathbf{e}_{N_X}\right]^{\mathrm{T}} \\ \left[\boldsymbol{\mu}^{(1)} - \beta\sum_i \hat{\sigma}_i^{(1)}\mathbf{e}_i\right]^{\mathrm{T}} \end{bmatrix}, \tag{8}$$

where $\mathbf{e}_i$, $i = 1,\ldots,N_X$ are the basis vectors for the design variables, fulfills this uniqueness condition. It is a Koshal design augmented with an additional experiment in the negative direction of the sum of the Koshal experiments. For problems where each constraint only depends on one variable, it is the design which maximizes the minimum distance between experiments p_m.

2.2. Determining the demand

A technique for experiment design based on experiments on demand (EoD) requires a way to determine the demand. The approach described here is intended for reliability based design optimization (RBDO). One major difference between traditional deterministic optimization and RBDO is that design variables are considered stochastic instead of deterministic and that the constraints are stated as the probability of an event rather than the complete prohibition of that event. Due to this, information about the constraints at the design variable's mean values is not of primary importance in RBDO, even though it is the design variable means that are altered during the optimization. Instead, information about the constraint at the most probable failure point is of highest importance in an RBDO. This causes a problem since this point may differ in between constraints. The standard way to deal with this problem is to perform a set of experiments, often only enough to estimate the gradients, at each of the constraints or at least the active constraints. However, experiments are expensive and lowered computational cost is along with increased stability and accuracy the driving force for development of new algorithms. Thus, the approach taken here is to extract as much information as possible out of every conducted experiment before making a new one. This means that experiments are not made in sets (as in DoE) but one at a time (EoD). Where to perform the next experiment is determined by where it is most needed, *i.e.* where the demand is the highest. Here, the demand in iteration k has been determined using the absolute change in reliability estimate $\left|\Delta\beta_j\right|$ for all the active constraints between iteration k and $k-1$, as illustrated in Fig. 1. The change in reliability index is stated as

$$\Delta\beta_j^{(k)} = \beta_j(\boldsymbol{\mu}^{(k)}) - \beta_j(\boldsymbol{\mu}^{(k-1)}),\ \forall\, j \in \Omega^{(k)}, \tag{9}$$

where $\Omega^{(k)}$ is the set of active constraints with cardinality $N_\Omega^{(k)}$ and

$$\beta_j(\boldsymbol{\mu}^{(k)}) = \frac{(\Delta r_j^{(k)}/a_j^{(k)})^{1/\gamma_j^{(k)}} - \overline{p}_j^{(k)} - \mathbf{n}_j^{(k)^{\mathrm{T}}}\boldsymbol{\mu}^{(k)}}{\mathbf{n}_j^{(k)^{\mathrm{T}}}\boldsymbol{\sigma}_j^{(k)}\mathbf{n}_j^{(k)^{\mathrm{T}}}\, sign(a_j^{(k)})\, sign(\gamma_j^{(k)})}. \tag{10}$$

with $\Delta r_j^{(k)} = r_{max,j} - r_j^{0(k)}$ and analogously for $\beta_j(\boldsymbol{\mu}^{(k-1)})$. The constraint for which the demand for new experiments is highest is j_d.

5.3. *D*-optimal augmentation

A straightforward way to add experiments to the total experiment design would be to place the next experiment at the MPFP of the constraint for which the demand is the highest. However, there are potential drawbacks with this. There is no guarantee that the experiments are not linearly dependent or even aligned, making estimates of the limit state normal $\mathbf{n}$ poor.

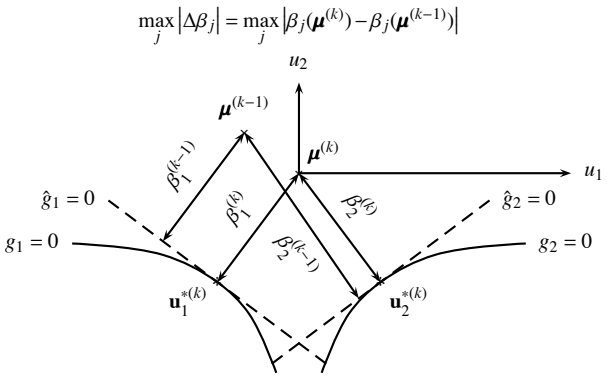

Figure 1: Illustration of the demand as the maximum change in reliability indices β_j with $j_d = 2$.

Hence, the demand condition has in this work been augmented by a *D*-optimality criterion. A *D*-optimal design is the solution to

$$\max \det\left(\mathbf{D}^{\mathrm{T}}\mathbf{D}\right). \tag{11}$$

In Eq. (11), it is not stated in which space the experiment design matrix $\mathbf{D}$ is expressed; the design variable space $\mathbf{x}$, the normed normal space $\mathbf{u}$ or another arbitrarily scaled space $\mathbf{v}$. The space in which the design variables are described will however be important. The normed normal space offers a natural scaling in RBDO applications. Thus, a scaling to normed normal space $\mathbf{u}$ is the starting point for the algorithm used to determine the next experiment in this paper. The origin is shifted to the MPFP and the experiments performed are scaled to unit length as

$$\mathbf{v}_m = \frac{\mathbf{u}_m - \mathbf{u}^*_{j_d}}{\left|\mathbf{u}_m - \mathbf{u}^*_{j_d}\right|}, \tag{12}$$

where $m = 1, ..., N_E$. In RBDO applications, the experiments closest to the MPFP are most important. Thus, the experiments $\mathbf{v}_m$ are moved to $\tilde{\mathbf{v}}_m$ using a radial contraction W_m as

$$\tilde{\mathbf{v}}_m = \mathbf{v}_m W_m, \tag{13}$$

where

$$W_m = w_j e^{-\lambda\left|\mathbf{u}_m - \mathbf{u}^*_{j_d}\right|}. \tag{14}$$

The fictitious experiments make up a experiment design matrix $[\mathbf{D}] = [\tilde{\mathbf{v}}_m]$ composed of all the experiments conducted up to the current point. In Eq. (14), the common factor w_j is chosen so that $\max(W_m) = 1$. The purpose of the weight function is to make those experiments close to the MPFP more influencial in the determinant in Eq. (11). The weight function will be further explained in Section 6, where it has been used for another purpose.

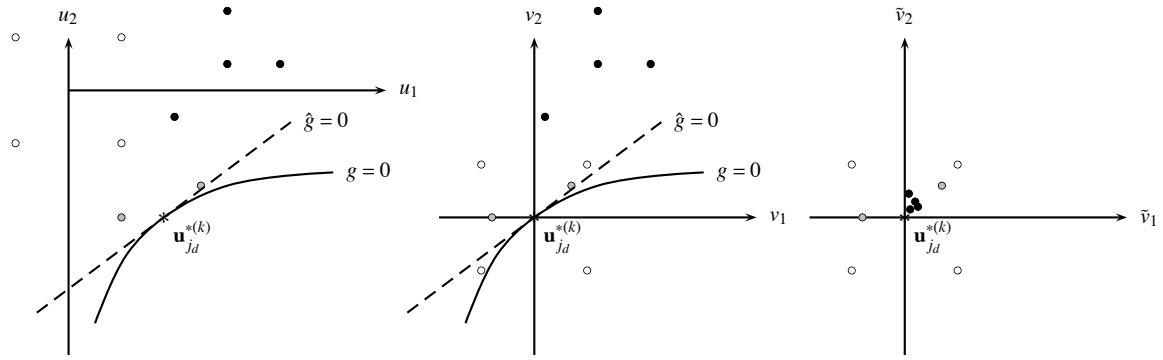

Figure 2: Illustration of the experiments on demand approach and the measures involved. Experiments are represented with dots. The initial experiments are represented in black, the intermediate in gray and the new experiment in white.

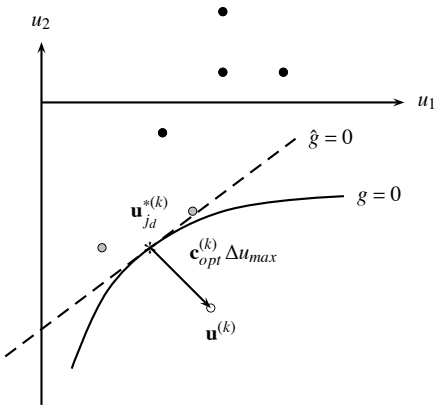

Figure 3: Illustration of the experiments on demand approach and the measures involved. Experiments are represented with dots. The initial experiments are represented in black, the intermediate in gray and the new experiment in white.

In this work, one experiment $\mathbf{d}^{(k)}$ is added in each iteration. The experiment added in iteration k is number $m = N_X + k + 1,\ k > 1$. It is selected as one of the experiments in a full factorial design with absolute component values of one, *i.e.* all the corners of a hypercube with side length 2 centered at the origin. The design matrix $\mathbf{D}$ is thus augmented with each of the hypercube corners $\mathbf{c}$ at a time. The experiment $\mathbf{c}_{opt}$ which renders the largest value for the determinant

$$\det\left(\mathbf{D}(\tilde{\mathbf{v}}_m, \mathbf{c})^{\mathrm{T}} \mathbf{D}(\tilde{\mathbf{v}}_m, \mathbf{c})\right) \tag{15}$$

is chosen as the added experiment. The procedure will place the experiments at a distance $\sqrt{N_X}$ away from the center (the MPFP in RBDO) with components ± 1. However, experiments should preferably be placed closer to the MPFP in RBDO applications. Hence, the solution obtained from Eq. (15) is scaled using a maximum component length Δu_{max} in the procedure employed here. The experiment that is added in iteration k described in u-space; $\mathbf{u}^{(k)}$, is formed using the solution $\mathbf{c}_{opt}^{(k)}$ to Eq. (15) as

$$\mathbf{u}^{(k)} = \mathbf{u}_{j_d}^{*(k)} + \mathbf{c}_{opt}^{(k)} \Delta u_{max}. \tag{16}$$

In design variable space, the added experiment is

$$\mathbf{x}^{(k)} = \boldsymbol{\mu}^{(k)} + \boldsymbol{\sigma}_{j_d}^{(k)} \left(\mathbf{u}_{j_d}^{*(k)} + \mathbf{c}_{opt}^{(k)} \Delta u_{max}\right). \tag{17}$$

6. Surrogate model fit

When experiments have been performed, a surrogate is fitted to the responses. Every surrogate model includes a number of independent coefficients that need to be fitted to the data, and thus a minimum number of responses. If, like the algorithm used in this work, all experiments are used, it is likely that there is more data than needed available. A decision can then be made to attribute higher weight to some data. In RBDO application, the apparent choice is to attribute higher weight to those experiments that are close to the MPFP. Due to these reasons, a two-step weighted least squares method has been used to estimate the surrogate model coefficients in this work. The surrogate model is stated in Eq. (3) and the set of unknown coefficients is $A_j = \left\{\mathbf{n}_j,\ a_j,\ \gamma_j,\ r_j^0\right\},\ j = 1, \ldots, N_C$, where $\left|\mathbf{n}_j\right| = 1$ and the iteration number k has been left out for readability. In the first step, a linear weighted least squares problem is solved analytically. The surrogate model limit state normal estimate $\hat{\mathbf{n}}_j$ is then taken as the (signed) norm of the gradient where the sign is set so that $\hat{\mathbf{n}}_j^{\mathrm{T}} \mathbf{u}_j^* > 0$. The limit state normal estimate $\hat{\mathbf{n}}_j$ is then used as input to the second least squares problem which is solved numerically to obtain the remaining coefficient estimates $\hat{a}_j$, $\hat{\gamma}_j$, and $\hat{r}_j^0$. The weight function matrix $\mathbf{W}_j$, which is used in both steps, is a diagonal matrix with components

$$W_{mm} = w_j e^{-\lambda \left|\mathbf{u}_m - \mathbf{u}_j^*\right|}, \tag{18}$$

where $m = 1, \ldots, N_E$ indicates experiment and N_E is the total number of experiments performed, the common factor w_j is chosen for normalization so that the largest weight is 1 and the remaining weights are smaller. In Eq. (18), $\mathbf{u}_j^*$ is the MPFP approximated using the surrogate model from the prior iteration. Index k has again been

left out for readability. In Eq. (18), a decay factor λ is introduced. In [3], a study was presented in which a larger λ rendered faster convergence and more accurate results. A too large decay factor λ will on the other hand render an ill-conditioned weight matrix. One way to deal with this is to have an adaptive decay factor. It is natural to use the distance from the j:th MPFP to experiment no. m; $\Delta u_{m,j}$. The ratio κ between weights for the furthest experiment that is required to fit the model parameters ($m = N_X + 2$) and the closest one ($m = 1$) is stated as

$$\kappa = \frac{e^{-\lambda_j \left|\Delta \mathbf{u}_{N_X+2,j}\right|}}{e^{-\lambda_j \left|\Delta \mathbf{u}_{1,j}\right|}}. \tag{19}$$

For a particular κ, the decay factor is then found as

$$\lambda_j = ln(\kappa)/(\left|\Delta \mathbf{u}_{1,j}\right| - \left|\Delta \mathbf{u}_{N_X+2,j}\right|). \tag{20}$$

In this work, different values of κ have been used. A reasonable choice is 0.1, *i.e.* the weight is 10 times larger for the closest relative to the furthest of the needed experiments.

7. Examples

The algorithm outlined in this paper has been applied to two frequently used benchmark problems. A target reliability of $\beta = 3 \Rightarrow 1 - \alpha = 0.9987$ has been used for both examples. The RoC, *i.e.* the move limit, reduction scheme of [6] has been applied to the inner loop, whereas the outer loop uses an RoC which is a fixed ratio of the design domain. The first outer loop RoC is 1/10 of the global design domain and the first inner loop RoC is 1/10 of its respective outer loop. Furthermore, the mean values are not changed during the first iteration. It is used to determine the MPFP only. The convergence criteria used are those of [1] and the threshold is 10^{-3}. For problems with multiple limit states, a check of all active constraints is performed when convergence is first achieved. If all the active constraints are not satisfied, the optimization continues.

7.1. Mathematical RBDO benchmark problem

A frequently referenced benchmark problem for RBDO was presented in [9]. The problem is stated as

$$\begin{aligned} \min_{\boldsymbol{\mu}} \quad & C = \mu_1 + \mu_2 \\ \text{s.t.} \quad & \begin{cases} P(G_j(\mathbf{X}) \geq 0) \leq \alpha \\ 0 < \mu_i < 10 \end{cases} \end{aligned} \tag{21}$$

where

$$\begin{aligned} G_1(\mathbf{X}) &= X_1^2 X_2/20 - 1 \\ G_2(\mathbf{X}) &= (X_1 + X_2 - 5)^2/30 + (X_1 - X_2 - 12)^2/120 - 1 \\ G_3(\mathbf{X}) &= 80/(X_1^2 + 8X_2 + 5) - 1 \end{aligned} \tag{22}$$

The optimization results are summarized in Table 1.

Table 1: RBDO results for the mathematical benchmark example.

State (CM)	μ_1	μ_2	C	β_{MC}	No of exps
Start (-)	5	5	10	2.499	-
Converged (Dir SM/EoD)	3.440	3.287	6.726	2.9686	18
Converged SLA	-	-	6.757	2.9998	90
Converged PMA	-	-	6.725	2.9970	540

A study of the influence of the parameters κ and Δu_{max} has been performed. The results were fairly consistent for the ranges $0.1 \leq \kappa \leq 0.2$ and $0 \leq \Delta u_{max} \leq 0.2$. The minimum number of function evaluations needed for convergence was 18 and the maximum was 23. The converged mean values differed only on the third decimal between the solutions, making the reliability estimates highly consistent. The nominal values were chosen to $\kappa = 0.1$ and $\Delta u_{max} = 0.1$ and the results in Table 1 are based on these. The efficiency is improved by 80 % compared to the best algorithm in [1]. However, the accuracy is reduced by approximately 1 %, as measured in reliability index. It is somewhat surprising since the objective function is practically identical. Thus, a Monte Carlo simulation was performed using the solution reported in [9], that is $\boldsymbol{\mu} = [3.441, 3.290] \Rightarrow C = 6.731$ and 10^7 samples. The MC

simulation give the reliability index as $\beta = 2.9707$. It is unclear to the authors how the reliability index in [1], as reported in Table 1, was determined.

The converged solution mean value, limit states and the experiments leading up to it is displayed in Fig. 4.

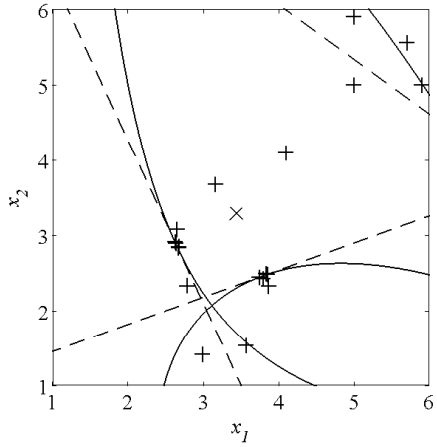

Figure 4: Plot of the limit states $G_j = 0$ (solid lines), the surrogate model limit states $\hat{G}_j = 0$ (dashed lines), the converged solution $\boldsymbol{\mu}$ (x marker), and the experiments (+ markers) obtained with $\kappa = 0.1$ and $\Delta u_{max} = 0.1$. The total number of experiments $N_E = 18$.

8. Discussion

The algorithm for RBDO presented in this work exhibit efficiency and accuracy. It is thus well suited for use in industrial applications with large-scale computations. There are however potential pitfalls with the approach, and they are discussed in this section.

The surrogate model used in this paper is non-linear only in the gradient direction. Thus, all partial second derivatives of the surrogate model with respect to the design variables are completely determined by the gradient and the sign of the exponent γ. This may cause problems in some cases.

A *D*-optimal experiment design gives the most accurate estimate of $\mathbf{n}$ if the response is a linear function with a normally distributed noise. It has been chosen here for just that reason; to obtain an accurate estimate, $\hat{\mathbf{n}}$, of the limit state normal. However, it is not necessarily optimal for obtaining accurate estimates of the other surrogate model coefficients r_j^0, a_j, γ_j. This is because a *D*-optimal design does not ensure that the experiments are well distributed along the $\mathbf{n}$-direction in design space. Also, there is still a risk that the gradient estimate is poor if less than $N_X + 1$ linearly independent experiments have been performed in the vicinity of the MPTP.

Moreover, there is a risk that a large amount of experiments are performed at what is believed to be the MPTP of one of the constraints. Because the experiments are performed at the MPTP for that constraint, the change in surrogate model is likely to be the highest and thus the demand using the definition here. However, when other constraints are eventually checked, it may reveal that the surrogate model estimates are far from accurate at the MPTP and thus large changes in mean values are enforced. This would render the experiments performed at the first high-demand constraint less valuable. This may be viewed as a type of sub-optimization. The fact that all experiments are utilized in the surrogate model fit mitigates this effect to some extent, and no experience with this pitfall has been found during the work on this paper. However, the extent of this risk should be evaluated in future work.

9. Conclusions

In this paper, an RBDO algorithm based on a surrogate model fitted by experiments on demand (EoD) has been presented. The experiment design has further been augmented by a *D*-optimality criterion for improved robustness. Also, the surrogate model is fitted through a weighted least squares procedure in which the weights are adaptive and modified based on the amount of information readily available. The goal is to decrease the number of experiments needed for convergence, which is important in applications where they are computationally expensive. Only then can RBDO become a common practice in simulation-based development of structural components. The results are promising. For a benchmark problem, 80 % fewer experiments were needed compared to the results reported in [1]. The accuracy - measured by deviation from the target reliability index - is on the order of per mille (not counting the FORM error which is about 1 % for the highly non-linear problem used).

10. Acknowledgements
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References

[1] Aoues, Y., Chateauneuf, A., 2010. Benchmark study of numerical methods for reliability-based design optimization. Structural and multidisciplinary optimization 41, 277–294.

[2] Chen, X., Hasselman, T. K., Neill, D. J., 1997. Reliability based structural design optimization for practical applications. In: Proc. 38th AIAA/ASME/ASCE/AHS/ASC Structures, structural dynamics, and materials conference. AIAA-97-1403. pp. 2724–2732.

[3] Dersjö, T., Olsson, M., 2012. A directional surrogate model tailored for efficient reliability based design optimization. Technical report HLF 2012:518, Department of Solid Mechanics, Royal Institute of Technology, SE - 100 44 Stockholm Sweden.

[4] Nikolaidis, E., Burdisso, R., 1988. Reliability based optimization: A safety index approach. Computers and structures 28(6), 781–788.

[5] Noh, Y., Choi, K. K., Du, L., 2009. Reliability-based design optimization of problems with correlated input variables using a gaussian copula. Structural and multidisciplinary optimization 38, 1–16.

[6] Stander, N., Craig, K. J., 2002. On the robustness of a simple domain reduction scheme for simulation-based optimization. Engineering with Computers 19 (4), 431–450.

[7] Wang, L., Kodiyalam, S., 2002. An efficient method for probabilistic and robust design with non-normal distributions. In: Proc. 43rd AIAA/ASME/ASCE/AHS/ASC Structures, structural dynamics, and materials conference. Denver, Colorado, pp. 1–13.

[8] Yang, R., Gu, L., 2004. Experience with approximate reliability-based optimization methods. Structural and multidisciplinary optimization 26(1-2), 152–159.

[9] Youn, B. D., Choi, K. K., 2004. Selecting probabilistic approaches for reliability-based design optimization. AIAA Journal 42 (1).