10th World Congress on Structural and Multidisciplinary Optimization
May 19 - 24, 2013, Orlando, Florida, USA

Structural optimization of flexible components under dynamic loading within a multibody system approach: a comparative evaluation of optimization methods based on a 2-dof robot application.

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1. Abstract
This paper is dedicated to a comparative evaluation between two methods of optimization to realize the structural optimization of flexible components in mechanical systems modeled as multibody systems. A nonlinear finite element method based formalism is considered for the dynamic simulation of the flexible multibody system. The first method is the Equivalent Static Load method which enables to transform a dynamic response optimization problem into a set of static response optimization problems. The second method treats directly the dynamic optimization problem in an integrated manner where the optimization process is carried out directly based on the time response coming from the multibody system approach. However, the first method proposed by Kang, Park and Arora was developed under the assumption that the multibody system is described using a floating frame of reference. Therefore, in order to carry on the comparison using a unique multibody system approach, a method is first proposed to derive the equivalent static loads when using a nonlinear finite element method based formalism. The comparative evaluation is then carried out on the simple academic example of the mass minimization of a two-arm robot subject to tracking deviation constraints. Conclusions are finally drawn for future work and stringent comparison.
2. Keywords : Structural optimization, dynamic loading, flexible multibody systems, nonlinear finite element method, 2-dof robot.

3. Introduction
Since the early sixties, many works and efforts have been realized in the field of structural optimization. The achieved developments enable to employ sizing and shape optimizations for solving industrial problems while topology optimization is often more employed as a pre-design tool in the industry.
To obtain an optimal design, the most common way is to use a component-based approach and to consider (quasi-)static loading conditions or vibration design criteria. Indeed, even though the majority of loads are dynamic in the real world, a lot of difficulties arise when dealing with system dynamic response optimization.
In Ref. [13], the component-based approach was used to optimize some components of mechanical system. The candidate components were first isolated from the system and then multiple static configurations were selected for the optimization process. This approach can be contested for several reasons. The selection of a few configurations cannot represent the overall system motion of a high-speed system. Moreover, the coupling between rigid and elastic motions are omitted which causes an inaccuracy on the displacements and on the stresses. Another point is that the multiple static postures do not account for the time-dependency of the constraints. Finally, the static postures are chosen in a non-rational and non-automatic way.
In order to better capture the behavior of the whole system, the component-based approach has been recently extended towards a system-based approach which relies on a multibody system (MBS) simulation [3, 6, 8, 9, 10, 11, 14]. This extension is important because Bendsøe and Sigmund [2] pointed out that the optimal design may be very sensitive to the support and loading conditions. The MBS system simulation offers a global approach of the mechanical system dynamics and enables to account precisely for the dynamic loading exerted on the components.
Using this system-based approach, two main optimization methods can be adopted to realize the dynamic response optimization of the mechanical system. The first method is based on the reformulation of the dynamic response optimization problem as a set of static problems in a two-step approach. First, a MBS

simulation precomputes the loads applied to each component and then, each component is optimized independently using a quasi-static approach. A set of equivalent static load cases must thus be defined in order to mimic the precomputed dynamic loads. A possibility is to introduce a set of static loads that gives the same deformation as the one given by the dynamic simulation [10]. The reformulation of the optimization problem allows using the robust and well-established methods of static response structural optimization. Several works have been realized using this two-stage method [8, 9, 11].

The second method considers an integrated approach of the optimization problem where the components are optimized with the response coming directly from the MBS simulation. It has been validated by Brüls *et al.* [3] and they showed that, in order to obtain an integrated approach, it is convenient to work with an optimization loop directly based on the dynamic response of the flexible multibody system. Indeed, the dynamic effects are naturally taken into account with this approach. Other studies on this integrated approach showed that the optimization of MBS is not a simple extension of structural optimization [6]. The coupled problem between vibrations and interactions within the components generally results in complex design problems and convergence difficulties. The design problem is complicated and naive implementations lead to fragile and unstable results. It turns out that the formulation of the optimization problem is essential to obtain good convergence properties.

The paper is dedicated to a comparative evaluation between these two methods based on the mass minimization of a two-arm robot subject to tracking deviation constraints.

Concerning the flexible multibody system simulation, different formalisms can be adopted to analyze the system dynamics. Recently, a strong tendency to merge both finite element analysis and MBS simulation into an unified code has been followed [7]. The integrated simulation tools resulting from this tendency allow analyzing the deformations of mechanism undergoing fast and large joint motions. This formalism based on the nonlinear finite element method is adopted since a development code based on this formalism has been fully implemented in MATLAB© by researchers of our department. The first part of the paper introduces the flexible MBS modeling and the time integration scheme.

In Ref. [10], the Equivalent Static Load method was developed under the assumption that the flexible MBS dynamics is described using a floating frame of reference formulation. This formalism is suitable to derive the equivalent static loads notably as this formulation deals with body reference. In order to realize a comparison between both optimization methods, it is preferable to use a unique MBS approach. However, the method proposed by Kang, Park and Arora can not be directly used with the other formalism. Therefore, the second part of the paper is dedicated to the derivation of equivalent static loads consistent with the nonlinear finite element method based formalism.

The following part introduces the general framework of the dynamic response optimization problem and the different approaches to solve these optimization problems. The comparative evaluation is then performed on the numerical application of a two-dof robot proposed in [10, 11]. Conclusions and perspectives close the paper.

4. Flexible multibody systems approach

4.1. Equations of motion of flexible multibody systems

In this paper, a formulation based on the nonlinear finite element method is employed to model the flexible multibody system dynamics as suggested by Géradin and Cardona [7].

The formulation is based on an inertial frame approach. The vector $\mathbf{q}$ gathers absolute nodal coordinates which correspond to the displacements and the orientations of each node of the finite element mesh.

If the multibody system is not constrained, the motion is governed by the following equation:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{ext} - \mathbf{g}^{int} - \mathbf{g}^{gyr} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ are respectively, the accelerations, the velocities and the displacement, and where $\mathbf{g}$ gathers the external, the internal and the complementary inertia forces. It should be noted that the mass matrix can also depend on the generalized coordinates.

The multibody system is generally constrained and kinematic constraints, denoted by $\mathbf{\Phi}(\mathbf{q}, t)$, are added to Eq. (1), which typically insure the connection of the different bodies. The kinematic constraints introduce a set of nonlinear equations between absolute nodal coordinates.

The resolution of this constrained dynamic problem is based on a Lagrange multiplier method. The product between the derivatives of the constraints $\mathbf{\Phi}_q^T$ and the Lagrangian multipliers $\boldsymbol{\lambda}$ are introduced in the equations of motion in order to impose the constraints. Finally, the equations of motion take the general form of a differential algebraic system (DAE) as follows

$$\begin{array}{rcl}\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{\Phi}_q^T(\mathbf{q},t)\boldsymbol{\lambda} & = & \mathbf{g}(\dot{\mathbf{q}},\mathbf{q},t) \\ \mathbf{\Phi}(\mathbf{q},t) & = & \mathbf{0},\end{array} \tag{2}$$

with the initial conditions

$$\mathbf{q}(0) = \mathbf{q}_0 \text{ and } \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0. \tag{3}$$

4.2. Time integration

Géradin and Cardona suggested using the generalized-α method developed by Chung and Hulbert [5] to solve the set of nonlinear differential algebraic equations (2). Arnold and Brüls [1] demonstrated that, despite the presence of algebraic constraints and the non-constant character of the mass matrix, this integration scheme leads to accurate and reliable results with a small amount of numerical damping.
At the time step $n+1$, the variables $\ddot{\mathbf{q}}_{n+1}$, $\dot{\mathbf{q}}_{n+1}$, $\mathbf{q}_{n+1}$ and $\boldsymbol{\lambda}_{n+1}$ have to satisfy the system of equations (2).
According to the generalized-α method, a vector $\mathbf{a}$ of acceleration-like variables is defined by the following recurrence relation

$$(1-\alpha_m)\,\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\,\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n, \tag{4}$$

with $\mathbf{a}_0 = \ddot{\mathbf{q}}_0$. The vector $\mathbf{a}$ is an auxiliary variable used by the algorithm and has no physical meaning. The integration scheme is obtained by employing $\mathbf{a}$ in the Newmark integration formulae:

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2\left(\frac{1}{2}-\beta\right)\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1} \tag{5}$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\,(1-\gamma)\,\mathbf{a}_n + h\gamma\mathbf{a}_{n+1}, \tag{6}$$

where h denotes the time step. If the parameters α_f, α_m, β and γ are properly chosen according to [5], second-order accuracy and linear unconditional stability are guaranteed.
Going a time step further requires to solve iteratively the dynamic equilibrium at time t_{n+1}. This is performed by using the linearized form (7) of equations (2) and by employing the Newton-Raphson method. The iterations try to bring the residual $\mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda} - \mathbf{g}$ and $\mathbf{\Phi}$ to zero.

$$\begin{array}{rcl}\mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{\Phi}_{\mathbf{q}}^T\Delta\boldsymbol{\lambda} & = & \Delta\mathbf{r} \\ \mathbf{\Phi}_{\mathbf{q}}\Delta\mathbf{q} & = & \Delta\mathbf{\Phi}\end{array} \tag{7}$$

where $\mathbf{C}_t = \partial\mathbf{r}/\partial\dot{\mathbf{q}}$ and $\mathbf{K}_t = \partial\mathbf{r}/\partial\mathbf{q}$ denote the tangent damping matrix and the tangent stiffness matrix respectively.

5. Equivalent Static Load approach

5.1. Introduction and definition.

One of the main problems encountered in structural dynamic response optimization is the problem of dealing with dynamic constraints. The purpose of introducing equivalent static loads is to remove the time component of the problem and to transform the dynamic response optimization problem into a set of static problems [4]. Indeed, all the advantages of static response optimization and all the well-established methods can then be exploited while the problems related to time-dependent constraints are avoided.
A definition of the equivalent static loads can be found in Ref. [10]: *When a dynamic load is applied to a structure, the equivalent static load is defined as the static load that produces the same displacement field as the one created by the dynamic load at an arbitrary time.*
In order to introduce the concept of the equivalent static loads, let us consider the following equilibrium equation of a linear structure* subject to a dynamic load

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t) + \mathbf{K}(\mathbf{x})\mathbf{y}(t) = \mathbf{s}(t), \tag{8}$$

where $\mathbf{s}$ is the applied dynamic load, $\mathbf{x}$ is the design variable vector, $\mathbf{y}$ and $\ddot{\mathbf{y}}$ are respectively the displacement and the acceleration, and where the damping effect is neglected. Eq. (8) can be rearranged as

$$\mathbf{K}(\mathbf{x})\mathbf{y}(t) = \mathbf{s}(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t). \tag{9}$$

*The difference is made between a multibody system and a structure as the latter is composed of only one body. This enables a simplification of the equations for this introductory section.

Eq. (9) has a similar layout as the static equilibrium equation of a structure. By identification and according to the previous definition, the equivalent static load at time t is defined as

$$\mathbf{f}_{eq}(t) = \mathbf{s}(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t). \tag{10}$$

It should be noticed that the equivalent static load $\mathbf{f}_{eq}(t)$ is an implicit function of the design variables and that it involves the external loads and inertia forces. From an analysis point of view, the equivalent static loads seem useless but they are developed in order to deal with a static response optimization problem. Instead of considering a dynamic loading, they offer the possibility of considering a set of static loads that gives at each time step the same displacement field as the one given by the dynamic loading. Therefore, the optimization problem is turned into a multiple static load cases optimization problem, with a load case for each integration time step.

5.2. Derivation of the Equivalent Static Loads using a nonlinear finite element method formalism.
In Ref. [10], the authors derive the equivalent static loads for a multibody system which is described using a floating frame of reference formalism. Unlike the nonlinear finite element method based formalism, the stiffness matrix is constant in the body reference during all the motion. Therefore, independently of the system configuration, only one stiffness matrix per body can be used in the optimization process for all the time step. Furthermore, the deformation of the components are computed in the body reference and can easily be extracted for the computation of the equivalent static loads.
These features are lost when using a nonlinear finite element method based formalism. The equations of motion are developed in an inertial frame and no term is expressed in a body reference with this formalism. However, as our development code for MBS simulation is based on this formalism, our goal is to solve the optimization problem by using it. The method we propose hereafter does not modify the MBS approach but it derives the equivalent static loads in a post-processing step of the MBS simulation.
Let us consider the linearized equations (7) of the equations of motion. At a converged time step t_i, the equilibrium equation is

$$\mathbf{M}(t_i)\Delta\ddot{\mathbf{q}}(t_i) + \mathbf{C}_t(t_i)\Delta\dot{\mathbf{q}}(t_i) + \mathbf{K}_t(t_i)\Delta\mathbf{q}(t_i) + \mathbf{\Phi}_{\mathbf{q}}^T(t_i)\Delta\boldsymbol{\lambda}(t_i) = 0. \tag{11}$$

As realized in section 5.1, by rearranging the terms, we get

$$\mathbf{K}_t(t_i)\Delta\mathbf{q}(t_i) = \mathbf{g}_{eq}(t_i). \tag{12}$$

While this expression has a similar layout as Eq. (9), several problems are encountered.
Firstly, the tangent stiffness matrix is related to the whole system and it evolves with the system configuration. This would lead to a storage of the matrix for each time step and also an update of each matrix during the optimization process iterations. In order to keep a unique matrix for all the time step, only a tangent stiffness matrix of a reference state $\mathbf{K}_t(t_{ref})$ should be kept. When considering another time step, this implies that appropriate transformations have to be applied to the vector $\mathbf{q}$ in order to bring it back to the reference configuration. While the tangent stiffness matrix is related to the whole system, it is possible to extract for each body its tangent stiffness matrix by selecting suitable generalized coordinates.
Secondly, the formalism gives a displacement vector $\mathbf{q}$ where there is no decoupling between rigid body motions and deformations, which is generally an advantage of the FEM approach. However, to carry on the optimization process, the deformations are needed. To obtain a measure of the component deformation, we propose to introduce a corotational frame for each body. This corotational frame also enables to define the transformation in order to switch from the actual configuration to the reference configuration and vice-versa.
Let us introduce the corotational frame definition adopted in this study for each robotic arm. Several definitions of the corotational frame are available and for instance, a definition can be based on the minimization of the strain energy. In the present approach, a tangent frame definition is used instead. In order to apply the boundary conditions related to the multibody system definition in the optimization process, Kang, Park and Arora considered each robotic arm as a fixed free beam [10]. According to this idea, the corotational frame is defined at the fixed beam extremity as follows

$\mathbf{x}_0$ = Position of the fixed beam extremity,
a_0 = Relative rotation angle of the fixed beam extremity compared to reference configuration.

In 2 dimensions (Fig. 1), the relationship between the absolute position ($\mathbf{x}_i$) and orientation (Ψ_i) of node i of the robotic arm and its local displacement $\mathbf{u}_i$ and local rotation ψ_i with respect to the corotational frame is given by

$$\mathbf{x}_i = \mathbf{x}_0 + \mathbf{R}_0 \left(\mathbf{X}_i + \mathbf{u}_i \right), \quad (13)$$

$$\Psi_i = \psi_i + a_0, \quad (14)$$

where $\mathbf{R}_0(a_0)$ is the 2D rotation matrix. Note that this corotational frame is not used within the MBS simulation.

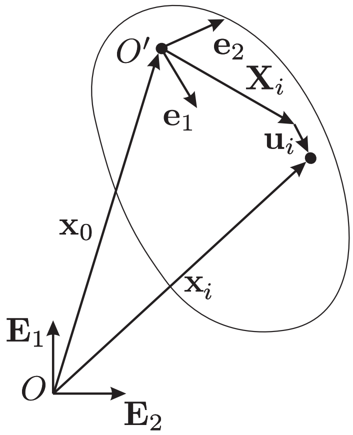

Figure 1: Kinematic description of a corotational frame in 2D.

The Equivalent Static Loads $\mathbf{g}_{eq}^b(t)$ for body b in a multibody system described using a nonlinear finite element method based formalism is therefore defined as

$$\mathbf{g}_{eq}^b(t) = \mathbf{K}_t^b(t_{ref})\mathbf{u}^b(t), \quad (15)$$

in the corotational frame.
The optimization process is then carried out using an equivalent static load at each time step for each body, which leads to a set of multiple static load optimization problems.

6. Integrated optimization method

The integrated optimization method has been proposed by Brüls *et al.* [3] and it follows a natural evolution of virtual prototyping and computational mechanics in which the aim is to define as precisely as possible the loading conditions of the different bodies under service. The authors took advantages of the evolution of numerical simulations and topology optimization codes in order to design optimal truss structures loaded during the MBS motion.
The method introduces a strong coupling between the MBS simulation and the optimization process where the optimization process is carried out directly based on the dynamic response of the flexible multibody system. This method aims at taking properly into account the dynamic coupling between large overall rigid-body motions and deformations. The dynamic effects appears naturally in the optimization process. Furthermore, the objective function and the design constraints can be defined with respect to the actual dynamic problem.
This approach seems to offers more possibilities than an isolated component optimization approach since it is able to capture more complex and coupling behaviors. However, it has been observed that the optimization problem must be carefully formulated to obtain a stable and robust procedure [3, 6]. The optimization of MBS is not a trivial extension of structural optimization. Naive implementations generally lead to inaccurate and unstable results. This may explain why only a few results are available in the literature. Indeed, the coupled vibrations between components generally result in complex design problems and in convergence diffculties. This indicates that specific formulations are required and need to be developed for this extended class of optimization problems.

7. Optimization of flexible multibody systems

7.1. Formulation of the MBS optimization problem

An optimization problem is generally formulated as the minimization of an objective function $\varphi(\mathbf{x})$ subjected to some constraints $c_j(\mathbf{x})$, which typically insure the feasibility of the structural design and some design requirements (Eq. 16). The design variables are gathered in the vector $\mathbf{x}$. Side-constraints limit the values taken by the design parameters and generally reflect technological considerations.

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \varphi(\mathbf{x}) \\ \text{subject to} \quad & c_j(\mathbf{x}) \leqslant \bar{c}_j, \qquad j = 1, \ldots, n_c, \\ & \underline{x}_v \leqslant x_v \leqslant \overline{x}_v, \quad v = 1, \ldots, n_v, \end{aligned} \tag{16}$$

where n_c is the number of constraints and n_v is the number of design variables.
This general formulation allows using different types of optimization algorithms to solve the problem and there is no need to develop specific method. Moreover, this formulation provide a general and robust framework to the solution procedure.
In multibody system optimization, the functions are structural properties and responses like mass, displacements at each time step and stresses for instance. The design variables x_v can be sizing, shape or topological variables. In the paper, only sizing variables are considered.
In this study, the formulation of the functions $c_j(\mathbf{x})$ accounts for all the integration time steps of the MBS simulation. The optimization problem formulation adopted is to minimize the mass of the mechanical system $m(\mathbf{x})$ while the constraints have to be verified at each time step, which is mathematically formulated as follows

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & m(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{\Phi}_q^T(\mathbf{q}, t)\boldsymbol{\lambda} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t), \\ & \mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}, \\ & c_j(\mathbf{x}, t) \leqslant \bar{c}_j, \qquad j = 1, \ldots, n_c, \\ & \underline{x}_v \leqslant x_v \leqslant \overline{x}_v, \qquad v = 1, \ldots, n_v. \end{aligned} \tag{17}$$

for all $t \in [0, t_{end}]$.

7.2. Integrated approach algorithm

When dealing with the integrated approach of the dynamic response optimization problem, the formulation of the optimization problem (17) can be used as it is, except that the variables t has discrete values due to the equation solving. Indeed, the MBS simulation and the optimization process work in an integrated manner without any decoupling.
However, as described in section 6, the design space resulting of this approach is quite complex and the formulation of the optimization functions is essential to obtain good convergence properties.
In this paper, to carry out the comparison, no specific formulation is used and all the time steps are accounted for the optimization process. Indeed, the problem is rather simple and does not require advanced formulations.

7.3. Flowchart of the dynamic response optimization process using the Equivalent Static Load method

When the equivalent static load method is used, the optimization process does not solve directly Eq. (17). As proposed in Ref. [4], the optimization process solves repeatedly the following static response optimization problem where the set of equivalent static loads steps in:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & m(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{K}_t^b(\mathbf{x}, t_{ref})\mathbf{u}^b(\mathbf{x}, t_n) = \mathbf{g}_{eq}^b(t_n), \qquad b = 1, \ldots n_b, \\ & c_j(\mathbf{x}, t_n) \leqslant \bar{c}_j, \qquad j = 1, \ldots, n_c, \\ & \underline{x}_v \leqslant x_v \leqslant \overline{x}_v, \qquad v = 1, \ldots, n_v, \end{aligned} \tag{18}$$

for $n = 1, \ldots, n_{max}$ and where n_b is the number of optimized bodies. It can be observed that for each body, there are as many load cases as the number of time steps.

To solve the dynamic response optimization problem using an equivalent static load method, the algorithm proposed in Ref. [4] is as follows:

1. Initialize the design variables and set $it = 0$.
2. Perform a dynamic MBS simulation.
3. Compute the equivalent static loads.
4. If $it = 0$, go to step 5. If $it > 0$ and if

$$\frac{\sum_{n=1}^{t_{end}} \|\mathbf{g}_{eq,it}(t_n) - \mathbf{g}_{eq,it-1}(t_n)\|}{\|\mathbf{g}_{eq,it-1}(t_n)\|} < \varepsilon, \tag{19}$$

   then, stop. Otherwise go to step 5. Epsilon is set to 0.01 in this paper.

5. Solve the following static response optimization problem

$$\begin{array}{lll} \underset{\mathbf{x}}{\text{minimize}} & m(\mathbf{x}) & \\ \text{subject to} & \mathbf{K}_t^b(\mathbf{x}, t_{ref})\mathbf{z}^b(\mathbf{x}, t_n) = \mathbf{g}_{eq}^b(t_n), & b = 1, \ldots n_b, \\ & c_j(\mathbf{x}, t_n) \leqslant \bar{c}_j, & j = 1, \ldots, n_c, \\ & \underline{x}_v \leqslant x_v \leqslant \overline{x}_v, & v = 1, \ldots, n_v, \end{array} \tag{20}$$

   for $n = 1, \ldots n_{max}$. The iterations to solve this optimization problem are hereafter denoted as inner iterations.

6. Set $it = it + 1$ and go to step 2.

A cycle is composed of the steps from 2 to 6. During step 5, while the equivalent static loads are implicit functions of the dynamic simulation response, they are not updated. Therefore, cycles are needed in order to update the equivalent static loads with respect to the change of the design variables. It has been proved that the solution obtained by this algorithm is an optimum solution of the original dynamic response optimization problem [12].

7.4. Optimization algorithm

Mathematical programming methods which require to compute the derivatives of the design functions are considered here. These methods have been widely employed to solve large scale structural and multidisciplinary optimization problems with conclusive results. Their major advantages are their high speed of convergence and the limited number of iterations and function evaluations required to obtain an optimal solution. The inconvenient of these methods is that they provide local optima due to the local convergence properties of gradient-based algorithms. The robustness of these methods can be a source of difficulties when dealing with highly nonlinear behaviors.
The algorithm adopted in this study is based on the sequential quadratic programming approach.

7.5. Sensitivity analysis

Dealing with gradient-based optimization methods, a sensitivity analysis must be carried out to compute the first order derivatives of the structural responses and to provide them to the optimization algorithm. The sensitivity analysis is an essential step of the optimization process and the computation time can be drastically increased if this part is neglected, especially when the number of variables is large.
While a semi-analytical sensitivity analysis requires less computational efforts in comparison to a finite difference scheme, this second approach is considered in this study. This sensitivity analysis requires one additional simulation per perturbed design variable. This method is easy to use and as the computation time of the numerical applications is quite small, the choice of this method is justified to carry out the investigations.

8. Numerical Applications

8.1. Modeling of the 2-dof planar robot

The numerical application is based on a 2-dof planar robot inspired from Ref. [10, 11]. The material is aluminum with a Young modulus of E=72 [GPa], a Poisson ratio of ν=0.3 and a volumic mass of 2700 [Kg/m^3]. The length of each arm is 600 [mm] and is modeled by beam elements whose cross section is hollow. The beam element model employed is described in Ref. [7].

The hinge A in Figure 2.a has a mass of 2 [Kg] while the end effector has a mass of 1 [Kg]. The gravity field is considered. The functions $\theta_1(t)$ and $\theta_2(t)$ represent the angle variations at the hinges during the robot deployment. The initial position is $\theta_1(t) = 120$ [°] and $\theta_2(t) = 150$ [°]. In Figure 2.b, the ideal tip displacement is illustrated and the trajectory equations are:

$$\Delta x_{tip}(t) = \Delta y_{tip}(t) = \frac{0.5}{T}\left(t - \frac{T}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right), \tag{21}$$

with $t \in [0, 0.66]$ [s] and where the period T is set to 0.5 [s].

The design variables are the outer diameters of the beam elements and the wall thickness of the links is set to 0.1 × the outer diameter. Initial values of the design variables are set to 50 [mm].

A rigid-body kinematic model is used to compute the functions $\theta_1(t)$ and $\theta_2(t)$ resulting from the desired trajectory since rigid-body models are free from deformations and vibrations. These functions are then applied as imposed rotations at the hinges of the flexible robot. Concerning the integration time scheme, a time step of 5E-4 [s] is used with a spectral radius of 0.5.

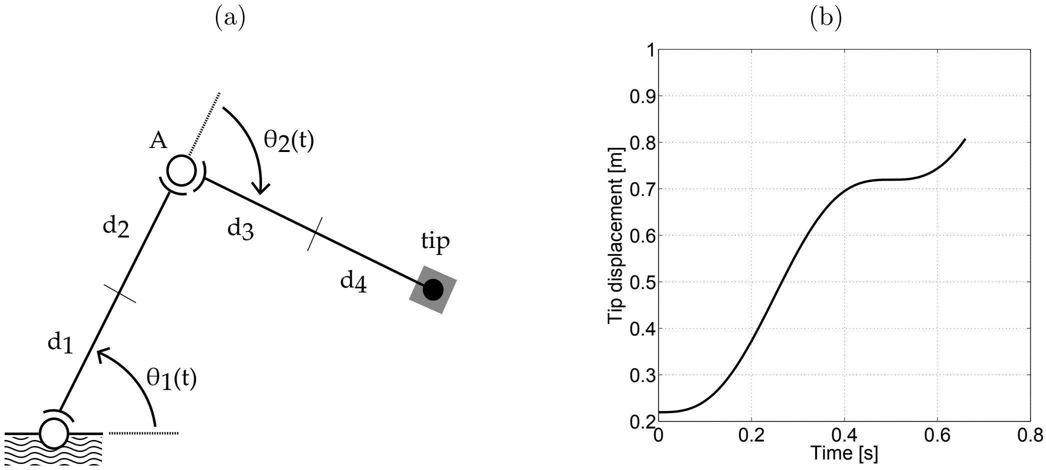

Figure 2: (a) The 2-dof robot model with 2 beam elements per link, (b) The ideal tip displacement with respect to time of a rigid robot.

8.2. Comparison of the optimization methods

The first numerical application considers the following optimization problem where the deviation constraint formulation is suggested by Ref. [10]:

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}} && m(\mathbf{x}) && \\ &\text{subject to} && \sqrt{\delta y_a^2(t_n) + \delta y_{tip}^2(t_n)} \leqslant 0.001 \text{ [m]}, && n = 1, \ldots 67, \\ &&& 0.02 \text{ [m]} \leqslant x_v \leqslant 0.06 \text{ [m]}, && v = 1, \ldots, 4. \end{aligned} \tag{22}$$

where $\delta y_a(t_n)$ and $\delta y_{tip}(t_n)$ are respectively the vertical deflections in the inertial frame of the first link at the hinge A and of the second link at the tip.

The optimization results are shown in Table 1 and are illustrated in Figure 3. For readability reasons of Figure 3.c, the markers have only been printed at each 0.01 [s]. It can be observed that both optimization methods lead rapidly to the same optimal design. The integrated approach has no inner iteration compared to the equivalent static load (EQSL) method. However, the inner iterations are based on static computations and one static analysis is less CPU-time consuming than one dynamic analysis. In order to carry out a fully objective function, we should analyze if it is more interesting to realize a few more dynamic analysis than running several static computations at each cycle.

Table 1: Numerical results - Formulation Eq. (22).

	Mass [Kg]	Iterations	Inner iterations	d_1 [mm]	d_2 [mm]	d_3 [mm]	d_4 [mm]
EQSL Method	1.213	6	61	45.40	32.76	37.99	26.83
Integrated Method	1.214	13	/	45.44	32.69	38.08	26.78

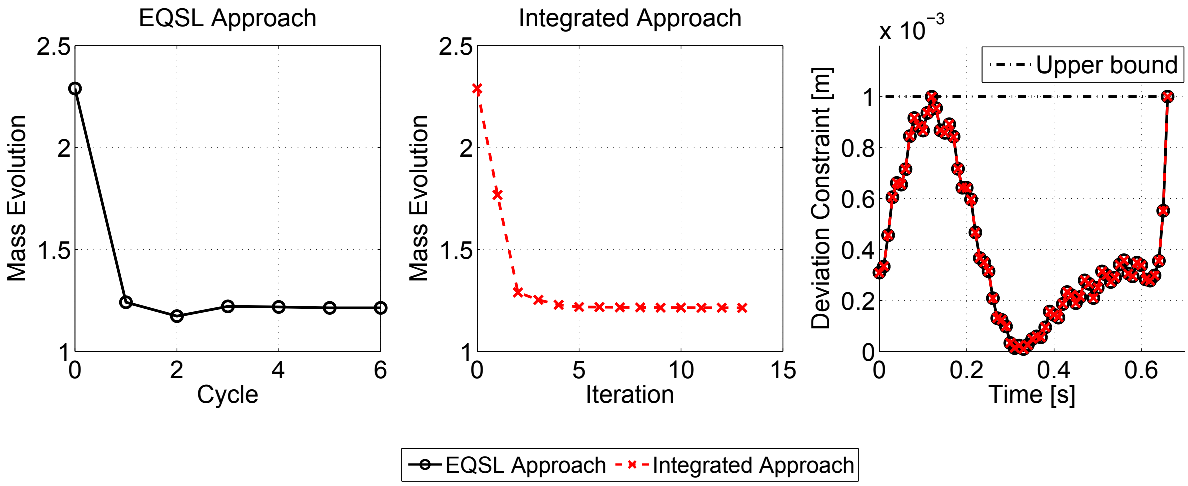

Figure 3: Optimization results - Formulation Eq. (22): Mass evolution of the optimization process and deviation constraint with the optimal design using a MBS simulation.

8.3. Trajectory deviation constraint

Let us now consider the following optimization problem formulation Eq. (23) where the constraint limits the trajectory error of the tip.

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && m(\mathbf{x}) \\ & \text{subject to} && \sqrt{\delta x_{tip}^2(t_n) + \delta y_{tip}^2(t_n)} \leqslant 0.001 \text{ [m]}, \quad n = 1, \dots 51, \\ & && 0.02 \text{ [m]} \leqslant x_v \leqslant 0.06 \text{ [m]}, \qquad i = 1, \dots, 4. \end{aligned} \tag{23}$$

where $\delta x_{tip}(t_n)$ and $\delta y_{tip}(t_n)$ are respectively the horizontal and vertical deflections of the robot tip in the inertial frame.

This application considers a tracking trajectory constraint. Only the extremity of the second robot link is concerned by the optimization constraint.

When using the EQSL method, the components of the system are artificially decoupled during the optimization process. With the tracking deviation constraint, we have to impose a maximal deflection on the tip, which is a constraint on the global system behavior. Using constraints upon the global system response, it is now not clear how to formulate the constraints on the component flexibility within the EQSL method. Indeed, due to the decoupling, considering only the tip deflection would lead to consider that the first link is not subjected to any constraints. While, it obvious that the first member flexibility has a contribution to the tip displacement.

As the deflection of the tip depends on the flexibility of all the components, one can formulate the tracking deviation constraint as a sum of the deflection of all the links. The deflection of the tip is defined as follows:

$$\begin{bmatrix} \delta x_{tip} \\ \delta y_{tip} \end{bmatrix} = \sum_{k=1}^{2} \mathbf{R}(a_{0,k}) \mathbf{u}_{ext,k} \tag{24}$$

where k is the robot link index, $\mathbf{u}_{ext,k}$ is the deformation at the link extremity in the corotational frame and $\mathbf{R}$ is the 2D rotation matrix. However, this does not hold when dealing with complex mechanisms including closed-loops.

Using the integrated optimization method, it is straightforward to consider this type of constraint. Indeed, the generalized coordinates used in the MBS simulation are available in the optimization process and take naturally into account the flexibility of the whole mechanism.
In order to compare the results with Ref. [11], the simulation time is set to 0.5 [s]. The optimization results are shown in Table 2 and are illustrated in Figure 4. For readability reasons of Figure 4.c, the markers have only been printed at each 0.015 [s].
The EQSL method converges after 5 cycles and the integrated method after 15 iterations. While the optimal values of the objective functions are similar for both methods, Figure 4.c illustrates that the time responses of the tracking deviation constraints are a bit different. Although the maximum value of the constraints happens at the same time step, trajectory errors exhibit different oscillations. Considering the same time step, the values of the constraints are different. This can be explained by the small differences between the optimal values of the design variables.
Comparing the results with Ref. [11] is quite difficult as the analysis of the system is different. Nevertheless, the overall trend of the results shows agreement.

Table 2: Numerical results - Formulation Eq. (23).

	Mass [Kg]	Iterations	Inner iterations	d_1 [mm]	d_2 [mm]	d_3 [mm]	d_4 [mm]
EQSL Method	1.411	4	38	47.88	34.51	42.11	30.08
Integrated Method	1.408	15	/	48.59	34.82	41.60	29.02
Ref. [11]	1.602	38	/	54.27	44.15	37.55	26.32

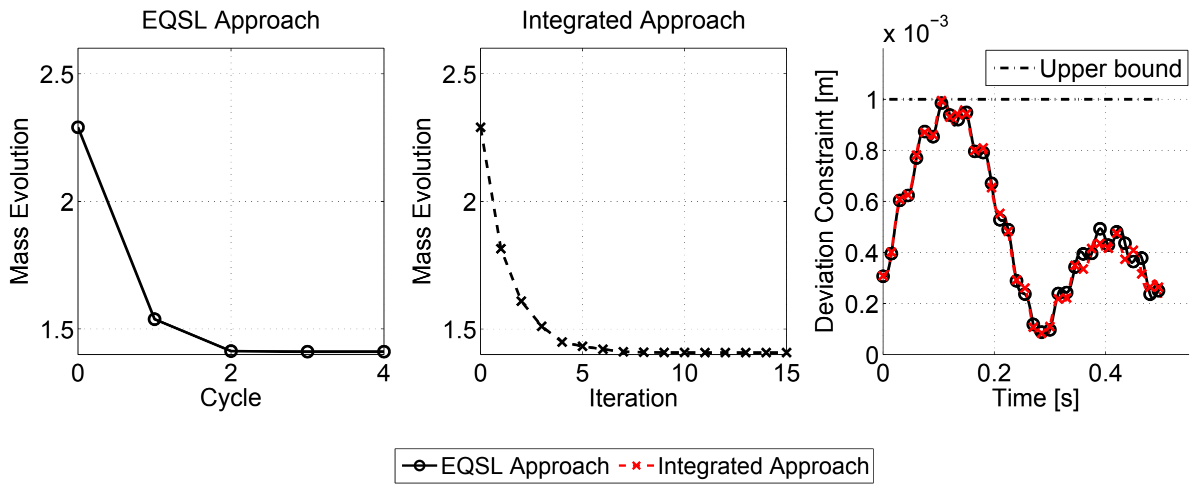

Figure 4: Optimization results - Formulation Eq. (23): Mass evolution of the optimization process and deviation constraint with the optimal design using a MBS simulation.

9. Conclusions and perspectives

A comparative evaluation between two methods for solving the dynamic response optimization of flexible components within a multibody system approach has been carried out.
In this paper, a nonlinear finite element method based formalism is adopted to describe the mechanical system dynamics. Indeed, a development code for MBS simulation based on this formalism is developed by the researchers of our department.
The first optimization method is the Equivalent Static Load method which aims at taking advantage of the well-established techniques of static response optimization by removing the time component from the optimization problem [10]. The second method is an integrated method where the dynamic response optimization is carried out with the system dynamic response coming directly from the MBS simulation [3].
The first contribution is that we propose a definition of the equivalent static loads adapted to the nonlinear finite element method based MBS formalism. Indeed, this method was developed under the assumptions

that the MBS dynamics was described using a floating frame of reference which is a formalism well suited to develop this kind of method.
The numerical application has shown that both methods can converge towards the same optimum for a simple academic problem.
A fundamental difference is that only a single dynamic analysis per iteration is required by the optimizer for the integrated method while a set of static analysis is necessary at each cycle with the equivalent static load method. During the static response optimization stage, the dependence of the equivalent static loads with respect to the design variables is neglected. Therefore, cycles are needed in order to update the equivalent static loads with respect to the effect of the design variables on the dynamic loading. For slowly varying body loads, the equivalent static load method normally requires less dynamic simulations and one dynamic simulation is more CPU-time consuming than one static analysis of the inner iterations. The formulation of global behavior constraints can become rather complex with the equivalent static load method as the components are decoupled during the optimization process.
In order to derive the equivalent static loads, the assumption of small deformation without any non-linearity is made. Therefore, this assumption should be kept in mind.
The comparative evaluation has been carried out on an academic example where both methods converge towards the same optimum. Ongoing work investigates systems with design dependent loading and more advanced cases as we expect different behaviors for the methods.

10. Acknowledgment

The author Emmanuel Tromme would like to acknowledge the Lightcar Project sponsored by the Walloon Region of Belgium for its support.

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