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Two-level design optimization of aircraft structures under stress, buckling and aeroelasticity constraints

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1. Abstract
Two-level approach to structural optimization with stress/aeroelasticity and panel buckling constraints has been developed. The problem is solved by using different level models in multidisciplinary design optimization environment. The design optimization of supersonic airplane wing is presented to demonstrate the application of the proposed two-level method. Optimal panel sizes, number of stringers in panels and ribs steps have been determined.
2. Keywords: structural optimization, two-level approach, multidisciplinary design optimization, buckling, aeroelasticity

3. Introduction

The complexity of aircraft design is due to increased interaction between the different individual disciplines determining the performance of the aircraft. Traditionally aircraft design is performed by teams in which experts deal with a specific discipline. Each expert in the team uses his/her own experience and arguments of other team members to develop practicable design. This process is usually sequential and it can lead to non-optimal design. In contrary, the modern multidisciplinary approach to aircraft design requires the integration of multiple technical disciplines such as aerodynamics, structures, propulsions, etc. The general problem of multidisciplinary design optimization (MDO) is extraordinarily complex. It is due to highly large number of design variables and constraints proceeding from the different disciplines. MDO requires repetitive generation of models and analysis as design variables vary. The process can be very time-consuming. Therefore, in practice it is necessary to simplify the design problem and to form the MDO framework that automates the process.
In this paper we are restricted by one of the typical problems concerning the determination of structural parameters for weight minimization with taken into account the variety of strength, buckling and aeroelasticity requirements. Practical optimization problems can have from hundreds to thousands design variables and thousands of highly nonlinear and implicit constraints from various disciplines. We have a multidisciplinary aero-structural optimization problem where stress, aeroelasticity and buckling constraints should be taken into consideration together. The best way to solve such complex problem is to consider all these constraints and design variables simultaneously in one design optimization process. However, there is no possibility at present time to do this due to very large dimension of the optimization task. In such cases the optimization problem can be divided into several smaller subproblems by using different strategies [1-4].
Here, to solve this problem it is proposed to use different level models in multidisciplinary design optimization procedures. The paper describes a developed approach of multilevel optimization which include both calculation of design constraints in different level models and two-level optimization scheme. In this approach the optimization problem can be reformulated as a series of smaller subproblems for separated subsystems of structure. To coordinate coupling between the subsystems a coordination problem is added. Each subproblem having its own goal function and constraints is solved independently at the first level. The coordination problem is solved at the second level.
In the paper the two-level approach to structural optimization with stress/aeroelasticity and panel buckling constraints is demonstrated in details. The design constraints are written for a stiffened panel which can be modeled by equivalent rectangular stiffened plate. In this case the Ritz-Timoshenko method can be used to calculate critical stresses for global buckling of panel, local buckling of skin between stringers and local buckling of stringer elements. The approximate equations of interaction of different load components are used to compute critical load parameter of buckling. Main relationships for the constraints are given.
The design optimization of supersonic airplane wing is presented to demonstrate the application of the proposed two-level method. Such important structural wing parameters as optimal panel sizes, number of stringers in panels and ribs steps are determined.

4. Two-level structural optimization

The main idea of the method is previously used in the problems of linear and dynamic programming. For instance, linear programming includes a set of independent linear subproblems coupled by the constraints which include the common variables. To solve the general optimization problem it is necessary to solve individual problems for subsystems and the system problem combining common variables for subsystems.

The decomposition of initial optimization problem can be multilevel because any subproblem in some level can be decomposed. Consider only two-level approach when system problem is divided into a set smaller subproblems with their own goal functions and constraints. The individual optimization of subsystem is performed independently on the first level and coordinate problem is solved on the second (system) level.

In general the nonlinear programming problem is formulated by the following way:

$$\text{Minimize} \quad f(\mathbf{X}) \tag{1}$$

subject to constraints

$$\begin{aligned} &\mathrm{g}_j(\mathbf{X}) \le 0, \quad j = 1, 2, \ldots, m \\ &\mathrm{h}_k(\mathbf{X}) = 0, \quad k = 1, 2, \ldots, p \\ &x_i^l \le x_i \le x_i^u, \; i = 1, 2, \ldots, n \end{aligned} \tag{2}$$

where x_i^l и x_i^u denotes upper and lower limits of x_i, which are components of the vector of design variables $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}^T$. In many technical systems the vector $\mathbf{X}$ can be subdivided into two subvectors $\mathbf{Y}$ and $\mathbf{Z}$:

$$\mathbf{X} = \begin{Bmatrix} \mathbf{Y} \\ \mathbf{Z} \end{Bmatrix}, \tag{3}$$

where the vector $\mathbf{Y}$ includes the interaction variables between subsystems and the vector $\mathbf{Z}$ has the variables belonging only to subsystems. The vector $\mathbf{Z}$ can be partitioned by following way:

$$\mathbf{Z} = \{\mathbf{Z_1}, \mathbf{Z_2}, \ldots \mathbf{Z_k}, \ldots \mathbf{Z_K}\}^T \quad , \tag{4}$$

where $\mathbf{Z}_k$ are the vectors with variables related with only k-th subsystem and K is number of subsystems. Such partitioning of variables permits us to regroup the constraints as

$$\begin{Bmatrix} g_1(\mathbf{X}) \\ g_2(\mathbf{X}) \\ \vdots \\ g_m(\mathbf{X}) \end{Bmatrix} = \begin{Bmatrix} \mathbf{g^{(1)}}(\mathbf{Y}, \mathbf{Z}_1) \\ \mathbf{g^{(2)}}(\mathbf{Y}, \mathbf{Z}_2) \\ \vdots \\ \mathbf{g^{(K)}}(\mathbf{Y}, \mathbf{Z}_K) \end{Bmatrix} \le 0 \,, \quad \begin{Bmatrix} h_1(\mathbf{X}) \\ h_2(\mathbf{X}) \\ \vdots \\ h_m(\mathbf{X}) \end{Bmatrix} = \begin{Bmatrix} \mathbf{h^{(1)}}(\mathbf{Y}, \mathbf{Z}_2) \\ \mathbf{h^{(2)}}(\mathbf{Y}, \mathbf{Z}_2) \\ \vdots \\ \mathbf{h^{(K)}}(\mathbf{Y}, \mathbf{Z}_K) \end{Bmatrix} = 0 \tag{5}$$

In Eq.(5) the vector of variables $\mathbf{Y}$ may appear in all constraint functions while the vectors $\mathbf{Z}_k$ appear only in the constraint sets $\mathbf{g^{(k)}} \le 0, \quad \mathbf{h^{(k)}} = 0$. The bound constraints in Eq.(2) can be written as:

$$\begin{aligned} &\mathbf{Y}^{(l)} \le \mathbf{Y} \le \mathbf{Y}^{(u)} \\ &\mathbf{Z}_k^{(l)} \le \mathbf{Z}_k \le \mathbf{Z}_k^{(u)}, \quad k = 1,2, \ldots K \end{aligned}$$

The goal function can be expressed as

$$f(\mathbf{X}) = \sum_{k=1}^{K} f^{(k)}(\mathbf{Y}, \mathbf{Z}_k) \,, \tag{6}$$

where $f^{(k)}(\mathbf{Y}, \mathbf{Z}_k)$ is the contribution of the k-th subsystem to the general goal function.

By using Eq.(3)-Eq.(6) the two-level method can be formulated as follows.

First level problem. Tentatively fix the values of the vector $\mathbf{Y}$ at values of vector $\mathbf{Y}^*$. The problem Eq.(1)-Eq.(2) can be reformulated as K independent optimization problems as follows:

Find the vector $\mathbf{Z}_k$ which minimizes function $f^{(k)}(\mathbf{Y}, \mathbf{Z}_k)$ at satisfying the constraints:

$$\mathbf{g}^{(k)}(\mathbf{Y}, \mathbf{Z}_k) \le 0, \quad \mathbf{h}^{(k)}(\mathbf{Y}, \mathbf{Z}_k) = 0, \quad \mathbf{Z}_k^{(l)} \le \mathbf{Z}_k \le \mathbf{Z}_k^{(u)}, \quad k = 1,2, \ldots K \tag{7}$$

Second level problem. The following problem is solved in the second stage:

Find new vector $\mathbf{Y}^*$ which minimizes the function

$$f(\mathbf{Y}) = \sum_{k=1}^{K} f^{(k)}(\mathbf{Y}, \mathbf{Z}_k^*) \,, \tag{8}$$

at satisfying the constraints $\mathbf{Y}^{(l)} \le \mathbf{Y} \le \mathbf{Y}^{(u)}$.

Here $\mathbf{Z}_k^*$ are the vectors of variables which are the optimal solution of the first level problems.

The constraint on bounds of the vector $\mathbf{Y}$ is added into the problem to provide a finite value of the goal function $f(\mathbf{Y}^*)$ at solving the second level problem.

The iterative algorithm can be represented in the following steps:

1. Start with an initial coordination vector $\mathbf{Y}^*$.
2. Solve the K first level problems Eq.(7) and find the optimal vectors $\mathbf{Z}_k, \quad k = 1,2, \ldots K$.

3. Solve the first level problem Eq.(8) and find new vector $\mathbf{Y}^*$.
4. Check for the convergence of goal function f and the vector $\mathbf{Y}^*$ by comparison their values with those obtained earlier in iteration process.
5. If the process has not converged, go to step 2 and repeat the process until convergence.

5. Design optimization of stiffened panels with buckling constraints

Satisfying to buckling constraints at structural optimization is important requirement at aircraft design. Usually such constraints are prescribed to structural panel which generally consists of plate elements modeling skin of the aircraft lifting surface and bar elements modeling stiffening stringers. Consider rectangular in plan form stiffened panel which is subjected simultaneously to compressive/tension normal loads along x and y axes and uniform shear loads (Figure 1).

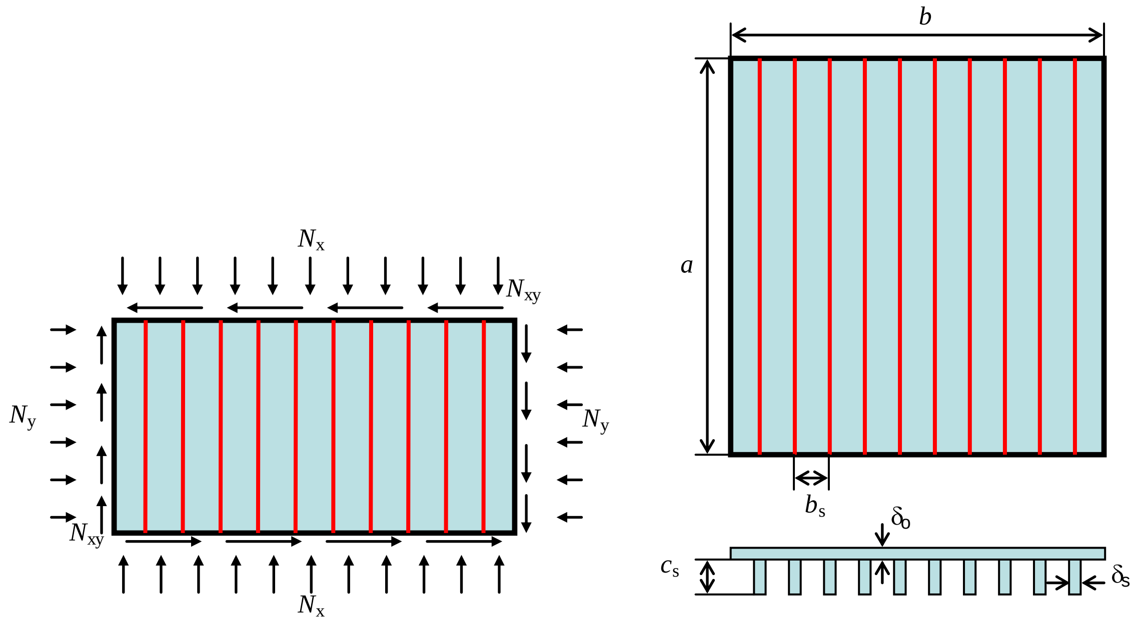

Figure 1: Stiffened panel (applied stress resultants and geometric parameters)

Different buckling shapes can be encountered for the panel under these loads. The following constraints should be taken into account to avoid the following phenomena:

- local buckling of skin between stringers

$$\varphi_1 = 1 + \frac{N_x}{\delta_o \sigma_{xl}} + \frac{N_y}{\delta_o \sigma_{yl}} - \left(\frac{N_{xy}}{\delta_o \tau_l} \right)^2 \geq 0 \,; \tag{9}$$

- local buckling of stringer elements

$$\varphi_2 = 1 + \frac{\sigma_s}{\sigma_s^{cr}} \geq 0 \,; \tag{10}$$

- overall buckling of panel

$$\varphi_3 = 1 + \frac{N_x}{\delta_o \sigma_{xg}} + \frac{N_y}{\delta_o \sigma_{yg}} - \left(\frac{N_{xy}}{\delta_o \tau_g} \right)^2 \geq 0, \tag{11}$$

where N_x, N_y, N_{xy} are the applied stress resultants in skin, σ_{xl}, σ_{yl}, τ_l are critical stresses for local buckling of skin, σ_{xg}, σ_{yg}, τ_g are critical stresses for overall (global) buckling of panel, σ_s, σ_l^{cr} are acting and critical stresses in stringer elements. Critical stresses in the constraints Eq.(9)–Eq.(11) are calculated by the formula given in book [5]. Also the critical stresses can be computed by minimization of potential energy with Ritz-Timoshenko method [6]. They are functions of the following parameters: a (length of the panel in the stringer direction), b (panel width), N (number of stringers), δ_0 (skin thickness), δ_s (stringer thickness), c_s (stringer depth). Here for simplicity we consider stringer as a rib with the shape of the simple web. However, the different geometry of stringers can be taken into consideration by introducing the shape parameter defining from the equality of critical stresses in the stringer elements. The stress resultants acting in a panel are calculated by averaging the stress

resultants in the skin elements which belong to the panel. Note that if we use finite element models in this approach we consider finite elements modeling the panel skin of the structure.
In the optimization problem we consider skin thickness δ_o, stringer thickness δ_s and stringer depth c_s as design variables. The following gauge constraints are imposed on the design variables of panel:

$$\varphi_4 = c_s^{\max} - c_s \geq 0, \qquad \varphi_5 = \delta_s - \delta_s^{\min} \geq 0,$$
$$\varphi_6 = \delta_o - \delta_o^{\min} \geq 0, \qquad \varphi_7 = c_s - c_s^{\min} \geq 0. \qquad (12)$$

Additional constraints on strength of structure $\varphi_8 = 1 - r \geq 0$ should be taken to structural optimization. Overload factor r is defining in dependence on the type of used material in structure to satisfy the corresponding strength criterion.
Let us introduce the two-level approach to solution of the optimization problem with taking into account the strength and buckling constraints. The mass of the optimizing material includes masses of all structural panels. Therefore the initial problem can be formulated as:
Find minimum of $f(\mathbf{x}) = \sum_k m_k(\mathbf{x}_k)$ at satisfying the constraints $\varphi_i \leq 1, \quad i = 1,2,\ldots 8$, where m_k is the mass of the k-th panel which is function of the vector of design variable for the panel $\mathbf{x}_k$. This vector includes skin thickness, stringer thickness and stringer web depth.
The design variables which influence on the static analysis results are thicknesses of elements modeling the panel skin and cross section areas of one-dimensional elements modeling stringers. The strength evaluation by the criterion $\varphi_8 \leq 1$ can be done after each structural analysis for all elements of panel. But the account of the buckling criteria $\varphi_i \leq 1, \quad i = 1,2,3$ can be performed by the consideration of the panel parameters δ_o, δ_s and c_s. The cross-sectional areas of elements F_e are related with the stringer design variables by the equality constraints $\varphi_9 = F_e - N\delta_s c_s / N_L = 0$, where N_L is the number of lines of the elements modeling the panel stringers.
Decompose the vector of design variables $\mathbf{x}$ into two subvectors $\mathbf{Y}=\{\mathbf{t}, \mathbf{F_e}\}^T$ and $\mathbf{Z}=\{\mathbf{Z_1}, \mathbf{Z_2},\ldots, \mathbf{Z_K}\}^T$, where $\mathbf{Z_k}=\{\delta_{ok}, \delta_{sk}, c_{sk}\}$ is row vector including the variables of k-th panel, $\mathbf{t}$, $\mathbf{F_e}$ are row vectors of design variables in full structural model.
In this case *the first level optimization problem* at fixed vector $\mathbf{Y}=\mathbf{Y}^*$ is to find the vectors $\mathbf{Z}_k$ minimizing the function $m_k(\mathbf{Y}^*, \mathbf{Z}_k)$ subject to the constraints:

$$\varphi_i^{(k)}(\mathbf{Y},\mathbf{Z}_k) \leq 0, i = 1,2,3, \qquad \varphi_9^{(k)}(\mathbf{Y},\mathbf{Z}_k) = 0, \quad \mathbf{Z}_k^{(l)} \leq \mathbf{Z}_k \leq \mathbf{Z}_k^{(u)}, \quad k = 1,2,\ldots K.$$

These problems can be solved by using different methods of nonlinear programming which are implemented in the multidisciplinary optimization system ARGON. It should be noted that gauge constraints Eq.(12) are replaced in this problems by upper and lower bounds for the components of the vector $\mathbf{Z_k}$.
The second level optimization problem is to find new vector $\mathbf{Y}^*$ minimizing the function $f(\mathbf{Y}) = \sum_{k=1}^{K} f^{(k)}(\mathbf{Y},\mathbf{Z}_k^*)$ at satisfying the constraints $\varphi_8(\mathbf{Y},\mathbf{Z}_k^*) \leq 0, \quad \mathbf{Y}^{(l)} \leq \mathbf{Y} \leq \mathbf{Y}^{(u)}$. In this problem $\mathbf{Z}_k^*$ are the fixed vectors of variables including optimal panel parameters. The lower bound of element thicknesses can be optimal panel skin thicknesses obtained from the solution of the first level problem.
The coordination optimization problem of the second level is solved by using finite element model where optimal parameters such as thicknesses and cross section areas of structure are calculated. Since the solution of the problems of two levels should be accomplished iteratively until convergence it is unnecessary to solve the coordination problem with large accuracy and it is sufficiently to do several iterations for determination of new vector $\mathbf{Y}^*$ in intermediate iterations. Often it is reasonably to use fully stressed design algorithm at solving the second level problem. This can essentially increase the performance of the optimization process at design of large-scale structures.

6. Aeroelasticity requirements

It is very important to take into consideration the influence of structural elasticity on changing aerodynamic loads and find the restricted number of extreme loads. The design process with inclusion of aeroelasticity problems becomes essentially more complex. Different mathematical models should be used for performing such process. The presented above method could be used also if additional aeroelasticity constraints would be added to the coordination second level optimization problem. But in this case the coordination problem should be solved accurately at each iteration and the same structural model as for stress analysis should be used for aeroelasticity evaluation. Often for the aeroelasticity analysis and loads calculation the coarser fidelity model can be used. For instance, in the multidisciplinary design system ARGON [7] the problems of aeroelasticity and loads calculation are solved by using the discrete-continual model of prescribed forms (first level model). The finite element model (second level model) is used for detailed evaluation of stresses and displacements of structure. So the approach for

structural optimization is based on two-level modeling of aircraft. Different multilevel structural optimization schemes are implemented in the software to treat aeroelasticity constraints together with strength and buckling constraints [6, 7].
Here we describe one of design procedures based on the two-level approach. The flowchart of the procedure is shown in Fig. 1. The aeroelastic/strength design cycle starts with calculation of aerodynamic/inertial loads for various parameters of maneuvers. Optimization under both stress constraints (for these loads) and aeroelasticity constraints is performed in the first level model. Loads for the optimized elastic structure are recalculated again, and new optimization is carried out. The optimization procedures are done up to the full convergence. The obtained results on the first-level model can be used for determination of the extreme load cases for structural parts with their corresponding load distribution, determination stiffness requirements and preliminary structural sizing of lifting surfaces structure.

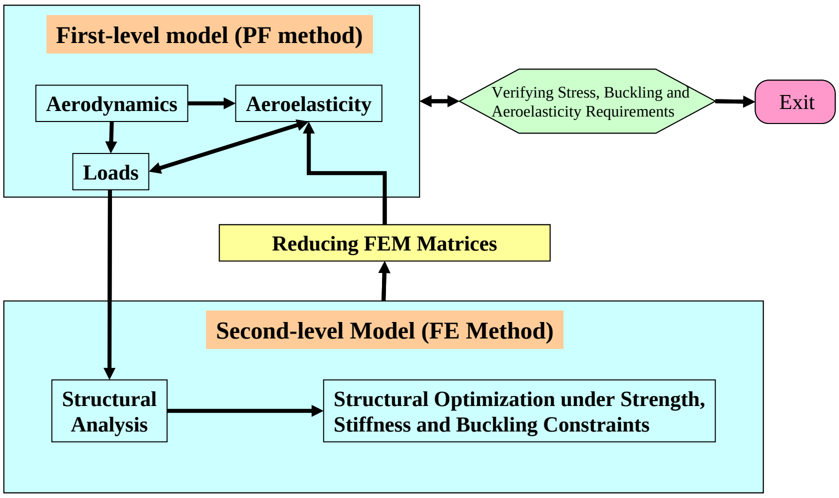

Figure 2: Flowchart of two-level model approach

The first-level model results are used to form initial data for detailed structural design by the second-level model. Optimization of design variables under stress, displacement, frequency and buckling constraints is also performed by using this model for chosen extreme load cases. The finite element stiffness/mass matrices for the structure with the optimal design variables can be transformed into the corresponding stiffness/mass matrices of the first-level model. This makes it possible to verify aeroelastic characteristics of aircraft after the optimization on the finite element model. The design cycle is completed if strength, buckling and aeroelasticity constraints are satisfied.
The static and dynamic aeroelasticity relationships which are used for calculation of the flutter, divergence and control surface effectiveness constraints are given in [7]. The developed optimization methods used for aeroelastic optimization are also presented there.

7. Numerical example of wing structural optimization

The design optimization problem of supersonic airplane wing is presented to demonstrate the application of the method. The complete finite element and aerodynamic models of airplane have been developed. Skins of fuselage, wing and vertical tail, webs of spars, ribs and frames are modeled by plate elements. Caps of spars, ribs and frames are modeled by rod elements. In Figure 3 the structural model of wing (main airplane component under research) is presented. The panel data include the information on finite elements of panels, number of stringers, gauge constraints for panel elements. The panel stringer set is modeled by membrane elements with the nominal Young modulus along stringer direction and near-zero one in transverse direction. Initial values of panel parameters were those obtained by using optimization on the first-level model with taking into account only strength conditions.
The baseline wing structure is made of three materials: front part is of aluminum alloy AK4, main part of the wing-box is of titanium alloy BT6 and tip part of wing is of composite material KMU-7T (high strength) or KMU-7L (high stiffness). The composite material has the following stacking of layers $0°_{40\%}/90°_{10\%}/\pm45°_{50\%}$ with the 0° being directed along stringers. Minimum thickness of the elements made of aluminum is 1.2mm, and of titanium is 0.8mm. For buckling optimization the minimum thickness of stringer is 0.8mm for titanium panels and 1.2mm is for aluminum panels. The minimum stringer depth is 10mm and the maximum one is 100mm.
Ten extreme load cases have been determined by loads analysis with using the first-level model. They include five maneuver loads with load factor 2.5, four maneuver loads with load factor -1 at different flight numbers M. One more load case corresponds to cruise flight with M=2.

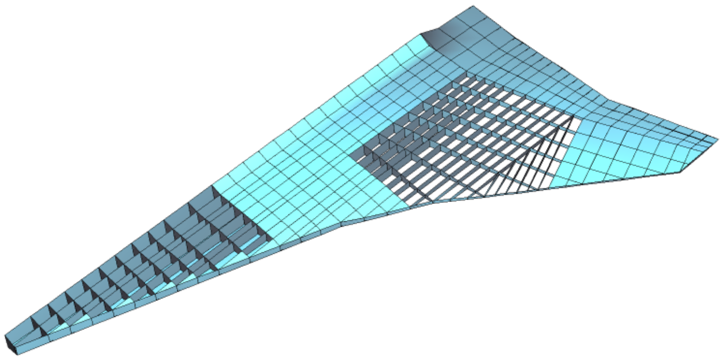

Figure 3: Structural model of wing

The finite elements have been linked to form design variables of the specified type. The cross section areas and thicknesses of the elements are being recalculated in dependence on type of design variable. They can have the same values for one design variable or be fixed or they can define the parameters of a stiffened panel with taking into consideration the strength and buckling constraints. The optimization problem in this example includes 235 design variables which are illustrated in Figure 4 by different colors.

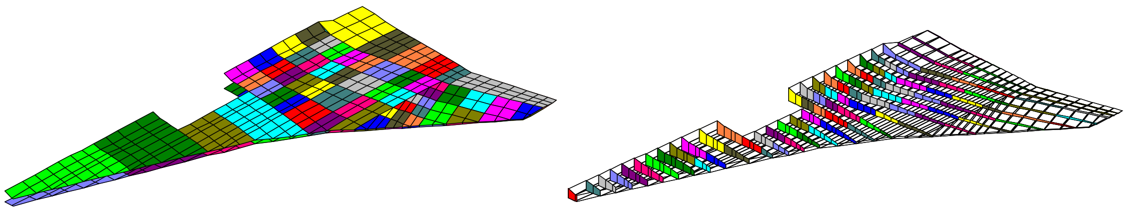

Figure 4: Design variables of wing panels and spars

The important parameters determined by using the proposed two-level approach are the number of stringers in panels and the panel sizes. A lot of optimization runs to design wing have been done at changes of the parameters. Note that the necessary number of finite element analyses was not greater than 10 to reach optimal feasible solution. The dependence of structural weight on number of stringers in the wing-box is shown in Figure 5. The structural weight of the wing-box was reduced on 17% compared with the baseline structure. It was obtained that optimum value of stringer step in titanium wing-box was about 80 mm, optimal number of stringers was 14.

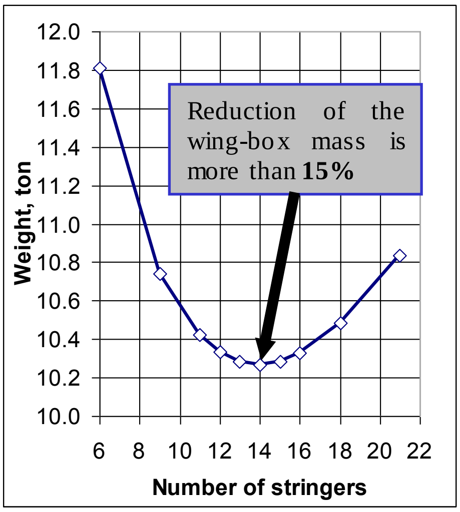

Figure 5: Weight versus stringer numbers in panels

Preliminary analysis of composite tip part of wing showed that initial structural layout has unlucky location of ribs and spars and the needed thicknesses to compensate buckling constraints achieved to 20-25 mm. It was due to relatively little structural depth, low elasticity modulus of composite laminate, using of unstiffened panel and large ribs step. Thus the problem of determination of optimal ribs step in the tip part of the wing was considered. The optimization runs were done for models with different additional partitioning of panels. The obtained optimal rib step is 250 mm. In this case the thickness of composite skin can be reduced more than two times in comparison with initial structural layout. The dependence of structural weight on rib step in the tip part of wing is shown in Figure 6.

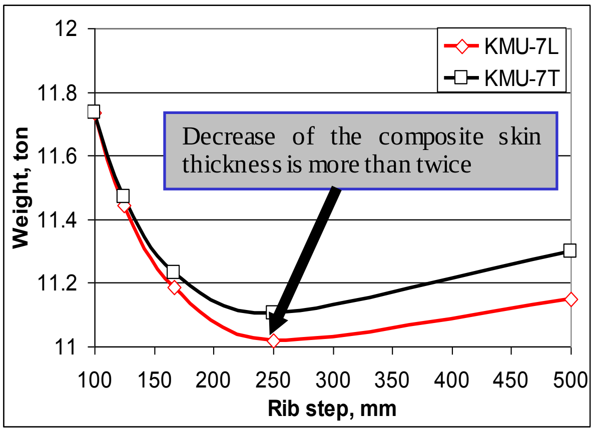

Figure 6: Weight versus rib step in tip part of wing

The equivalent von Mises stresses in upper wing skin for optimized structures are shown in Figure 7 for extreme load case. Maximum stresses are observed practically in all titanium part of wing-box in the case of account of only stress constraints. Meanwhile, in the case of satisfying both stress and buckling constraints the maximum stresses take place only in central part of wing-box.

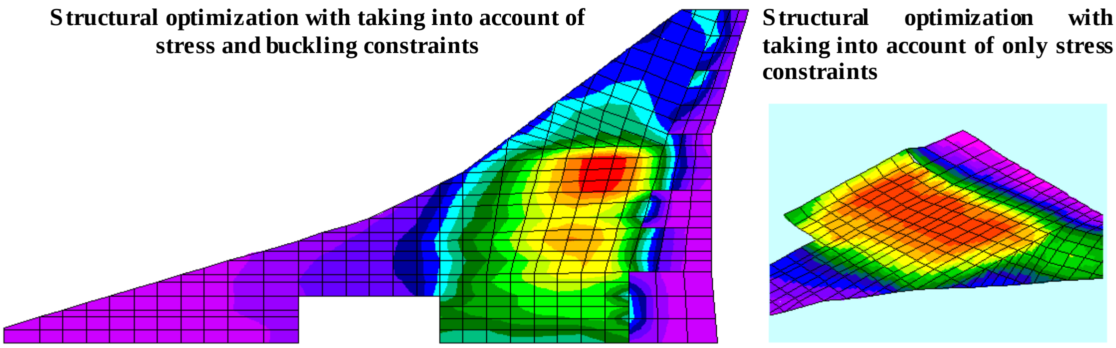

Figure 7: Von Mises stress distribution in upper skin for optimized structures

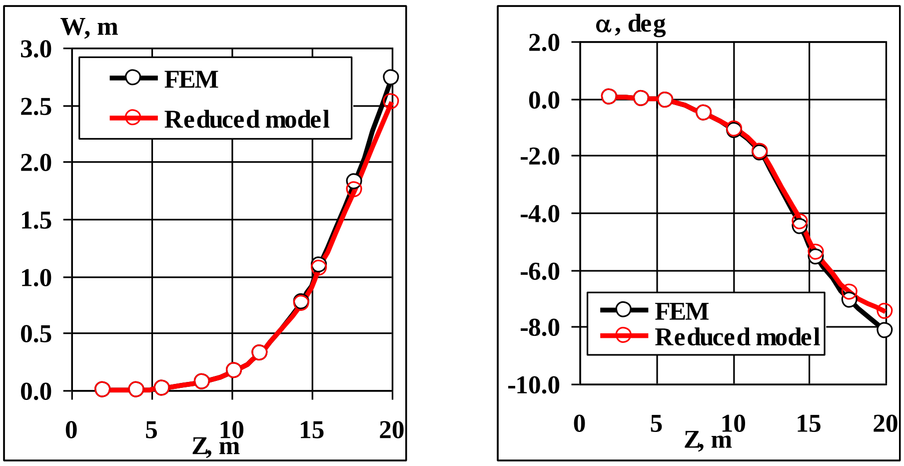

Figure 8: Comparison of the wing displacements and the stream-wise twist angles for two models

To perform aeroelastic analysis of airplane the FE stiffness and mass matrices of optimized wing were reduced to ones of equivalent plate model. The comparison of the wing displacements and the stream-wise twist angles for both two models along conventional stiffness axis is shown in Figure 8. The difference in the deflection at the wing end for two models is less than 4 percents for all considered load cases. Also good agreement in the analysis results is observed for the the stream-wise twist angles. The reduced matrices for optimal wing structure with taking into consideration stress and buckling constraints was used for aeroelasticity analysis of airplane.
The first analyses showed that aeroelasticity constraints were not satisfied: poor effectiveness of elevons and low value of flutter speed. Sensitivity analysis showed that aeroelasticity characteristics are mainly defined by stiffness of root and middle sections. Structural optimization procedure with using equivalent plate model allows finding optimal increase of skin thicknesses in metallic part of wing. The increase in mass was less than 5 percent.

8. Conclusion

The two-level approach for aircraft structural optimization under stress/aeroelastic and buckling constraints has been developed. Both two-level optimization scheme and analysis by using structural models of two levels are included into multidisciplinary design procedure. The optimization approach is shown in details on the design optimization problem with strength/buckling constraints.
The application of the proposed two-level method has been demonstrated by the design optimization of wing. Such important structural wing parameters as optimal panel sizes, number of stringers in panels and ribs steps are determined.
The developed numerical optimization method can serve as valuable tool in design investigations of advanced airplanes. It helps to experienced designers and engineers to find needed structural design parameters satisfying to an objective and different multidisciplinary constraints.

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