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Uncertainty quantification in aircraft load calibration

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1. Abstract
Flight tests are an important part of the certification process for aircraft. In particular, determination of flight loads allows validating predictions from the design office. The method of flight load determination needs a first step of ground load calibration. Measurements from this calibration are subjected to uncertainties. These uncertainties have to be taken into account to analyse impacts of uncertainties on the calibration. For this aim, the problem is modified. Uncertainties are included in the problem by following a method called "error in the equation". This method allows considering measurement errors on both design points and responses of the problem. The quality of this new model is then validated.
2. Keywords: Flight, tests, measurement, error, quantification.

3. Introduction
A current method for determining flight loads experimentally relies on a method developed by NASA (ex-NACA) in 1954 [1]. Flight loads cannot be directly measured. The most common method consists in measuring the effects of flight loads on the structure, which means measuring local strains. Another method is by measuring pressure distribution [2].

Different applications of the first method have been carried out. It has been applied to different types of aircraft (military [2][3][4][5][6][7][8], civil, space shuttles [9][10][11][12], drones [13]), on different structures (wing, wing tip [7], horizontal and vertical tail planes [4][8], canard, elevon and control surfaces [7][12]) and in different conditions (clean or high lift configurations [12], different altitudes, speeds and manoeuvres [2][4][8][12][14], in static and dynamic conditions [2][4][7]). In addition to real aircraft and prototypes [2][7], the method has also been applied to FE model of aircraft [13][6].

For certification requirements, structural loads of aircraft have to be demonstrated with required margins. These structural loads can be predicted by calculations. Estimations of these loads can only be checked and confirmed with real flight measurements. But these estimations are also subjected to uncertainties. To reduce margins to the minimum, real flight loads have to be known with accuracy.

Some studies have been done on the accuracy of this load determination method [15]. These works were focused on the meta-modelling approach and the mathematical resolution of the problem. Other studies are focused on the temperature impact on measurements [9].

This paper seeks to characterize the model uncertainty on parameters β_k by taking into account various sources of uncertainty in the ground load calibration. These uncertainties are both on strain measurements μ, which are the coordinates of the design points, and on loads $F^{'}$, which are the responses. Traditional approaches for multiple linear regression usually only take into account uncertainty on the responses [16][17].

Since the system representing the problem is highly over determined, a criterion has been first implemented to make a systematic and reliable selection of the relevant coefficients β_k and thus of the relevant strain measurements [16]. This criterion tests the significance of the coefficients β_k in the model. The next step is the propagation of the uncertainties on both strains and loads in the ground load calibration. For this, a specific regression model will be implemented using the so-called "errors-in-variables" method [18].

In the first part, the criterion used to reduce the number of strain variables in the problem is presented. The current data used and the new data use are described and compared. The new data use takes into account all available data. In the second part, uncertainties on both load and strain measurements are taken into account. The current problem formulation is first reminded and then the new formulation taking into account uncertainties on both responses and design points is presented. Results for the two formulations are discussed.

4. Parameter selection criterion

The direct problem can be modelled by a response surface approximation defined by Eq.(1).

$$F'^{flight} = (\mu_k)^T(\beta_k) \tag{1}$$

where F' is the load, μ_k is the measurement of the strain at location k and β_k is the parameter associated to the strain at location k. These parameters β_k represent the response of the structure. Here, the studied structure is a civil aircraft wing, considered as a cantilever beam as defined in Euler-Bernoulli beam theory. The first step of the flight load determination method is the identification of the model parameters β_k. These coefficients are identified by a load calibration. The second step is its application in flight. The paper is focused on the first step.

A ground load calibration allows characterizing the behaviour of the structure by determining the parameters β_k. The aircraft, in a hangar, is on jacks or on its landing gears. Strains sensors are positioned at different locations on the structure. Chosen loads are applied on hard points of the structure with jacks. Different load cases are applied to the structure. Corresponding local strains are measured. The structural response is thus characterized by the resolution of the inverse problem. Because the system is over determined, the resolution of the inverse problem is done by the resolution of the least squares normal equation described by Eq.(2) according to [1]:

$$(\tilde{\beta}_k) = ([\mu_{ik}]^T[\mu_{ik}])^{-1}[\mu_{ik}]^T(F_i'^{ground}) \tag{2}$$

where i is the index for the load cases and $\tilde{\beta}_k$ is the estimation of the parameter β_k. However both measured loads and strains are affected by various sources of uncertainty, which are not taken into account until now.

4.1. Current data use

The current formulation of the problem is a reduced one. During the ground test, the structure is not directly loaded to the chosen values of loads. In order not to damage the wings and to have reliable measurements, the loads are applied step by step up to the chosen values. A cycle of loading and unloading is done for each load case. The measurements are done during stabilized steps.

For the data exploitation, a linear regression is done between applied load cases i and strain measurements k for the different loading steps j defined by Eq.(3).

$$\begin{pmatrix} \mu_{i1k} \\ \vdots \\ \mu_{ijk} \\ \vdots \\ \mu_{in_{step}k} \end{pmatrix} = \begin{bmatrix} 1 & F_{i1} \\ \vdots & \vdots \\ 1 & F_{ij} \\ \vdots & \vdots \\ 1 & F_{in_{step}} \end{bmatrix} \begin{pmatrix} A0_{ik} \\ A1_{ik} \end{pmatrix} \tag{3}$$

where n_{step} is the number of loading steps, $A0$ and $A1$ are the constant and the slope of the linear regression between the load case F_i and the strain measurement at location k μ_k. Only one set of values per load case i is extracted from this regression to write the system. The set of values extracted are values corresponding to the maximum value $F'^{\,max}$ for the considered load case i as shown in Eq.(4).

$$\overline{\mu}_{ik}^{max} = A1_{ik}F_i^{\,max} + A0_{ik} \tag{4}$$

The system to solve, defined by Eq.(5), is accordingly under-determined.

$$(F_i'^{\,max}) = [\overline{\mu}_{ik}^{max}]\,(\beta_k) \tag{5}$$

Because of the beam hypothesis, the main loads which are seen by the structure are bending, shear and torsion. So all strain measurements are not necessary to characterize the structure. Based on AIRBUS expertise and experience, a selection of the best regressors, which are the strain measurements, is applied.

By reduction of the number of parameters β_k, the system become over-determined allowing its resolution given in Eq.(2).

This approach has proven its efficiency. The linear regression done between applied load and strain measurements allows minimizing noise measurement and so uncertainty. But, it is not sufficient in order to take into account impacts of uncertainties in the problem. Indeed, all available data are not used. For a more accurate quantification of uncertainties, as much data as possible should be used.

4.2. Full data use

For quantification of uncertainties, a new approach has been implemented. This new approach takes into account all measurements done for each loading step. For each load case i, each loading step j becomes a new set of data in the system. This allows using more data for a more reliable quantification of uncertainties. Furthermore, a complete system with all regressors, which are strain measurements, can be written as defined by Eq.(6).

$$\left(F'_{ij}\right) = \left[\mu_{ijk}\right] \left(\beta_k\right) \tag{6}$$

The previous linear regression on measurements Eq.(3) is thus included in the multiple linear regression. This formulation allows taking into account more information for uncertainties quantification. Furthermore, the system is now highly over-determined and every parameter β_k can be calculated.

It is known that all strain measurements are not necessary for characterizing the structure. The previous criteria from AIRBUS expertise and experience would be too expensive in terms of calculation time and analysis. A classical criterion has been implemented to make a systematic and reliable selection of the relevant coefficients β_k and thus of the relevant strain measurements. This criterion tests the significance of the parameters β_k in the model [16][17].

The test of significance of coefficients is applied on the β_k coefficients. The test statistic for each β_k coefficient is described in Eq.(7).

$$t_0(\tilde{\beta}_k) = \frac{\tilde{\beta}_k}{\sqrt{\tilde{\sigma}^2 C_{kk}}} = \frac{\tilde{\beta}_k}{se(\beta_k)} \tag{7}$$

where σ^2 is the variance of the residual e, C_{kk} is the diagonal element of $C = ([\mu_{ik}]^T[\mu_{ik}])^{-1}$ and se is the standard error of the regression coefficient β_k. An estimation of the variance of the residual e is given by Eq.(8).

$$\tilde{\sigma}^2 = \frac{S\tilde{S}_E}{n-(p+1)} = \frac{\tilde{e}^T\tilde{e}}{n-(p+1)} \tag{8}$$

where $SS_E = e^T e$ is the residual sum of squares, $\tilde{e} = F' - \tilde{F}' = F' - \mu\tilde{\beta}$ is an estimate of the residual e, n is the number of new load cases and p is the number of strain variables in the system. The test of significance is defined by Eq.(9).

$$\text{If} \quad |t_0(\beta_k)| > t_{1-\frac{\alpha}{2},n-(p+1)} \quad \text{then} \quad \beta_k \neq 0. \tag{9}$$

where $t_{P,N} = F^{-1}_{t,N}(P)$ is the inverse of the cumulative distribution function F for the Student's t-distribution with N degrees of freedom and for the probability P. α is the risk level and is often taken equal to 5% which is equivalent to have 95% of confidence on the test.

Non significant coefficients are deleted from the response surface which is recalculated until all remaining regressors are significant.

4.3. Comparison

Results from this new calculation are compared with the previous calculation to check the validity of the criterion.

The new calculation stops to a solution with 4 strain variables (Figure 1). For the current calculation (section 4.1), a solution with 3 strain variables has been retained. Among this 4 strain measurements, 3 out of 4 are the same as the current choice. Some coefficients β_k found by the two approaches (4.1 and 4.2) may be quite different because the system is ill-conditioned. But, the coefficients β_k, which are associated to the strain type corresponding to the type of applied load, are very close, less than 5% of difference on coefficient values. The other coefficients β_k may be very different.

If the current calculation is forced to 4 strain measurements, the results are the same even if the variance slightly increases compared with the current calculation with 3 strain measurements which are retained.

5. Taking into account uncertainty

Several sources of uncertainties have been identified during the ground load calibration. But, here, uncertainties are present both on the coordinates of the design points, μ, and on the responses, F'. Taking into account this particularity, a specific model is implemented using the so-called "error in the equation" method [18].

5.1. New problem formulation

Classical approaches usually only take into account uncertainties on responses. The considered problem is described by Eq.(10).

$$F'^{flight} = (\mu_k^*)^T (\beta_k) + e = \mu^{*T}\beta + e \tag{10}$$

where F' is the "observed" load which means real load, μ_k^* is the "true" value of the strain measurement k, β_k is the parameter related to μ_k^* variable and e is the residual.

The solution of the inverse problem minimizes the sum of squared residuals. The solution is the previous one.

In fact, uncertainties are also present on strain measurements μ. The load and strain measurements are now described by Eq.(11).

$$\left\{ \begin{array}{lcl} \mu & = & \mu^* + u \\ F' & = & F'^* + w \end{array} \right. \tag{11}$$

where F' and μ are the "observed" values of load and strains which means real measurements with uncertainties, F'^* and μ^* are the "true" values of load and strains which means values if there was no uncertainty and w and u are respectively uncertainties on load and strain measurements. The new problem is changed as defined in Eq.(12).

$$F'^{flight*} = \mu^{*T}\beta + q \tag{12}$$

where μ are the strain measurements and q is called the error in the equation.

An estimation of the solution of the inverse problem associated to this problem reformulation taking into account uncertainties on both load and strain measurements is given by Eq.(13) [18]:

$$(\tilde{\beta}_k) = (M_{\mu\mu} - S_{uu})^{-1} (M_{\mu F'} - S_{uw}) \tag{13}$$

with:

$$M_{\mu\mu} = \frac{\mu^T \mu}{n} \tag{14}$$

$$M_{\mu F'} = \frac{\mu^T F'}{n} \tag{15}$$

and S_{uu} and S_{uw} are the covariance matrix of measurement errors. Here, S_{uu} is diagonal because strain measurements are supposed independent and $S_{uw} = 0$ because load and strain measurement are supposed independent.

According to [18], an estimate of the covariance matrix of the parameters $\tilde{\beta}$ is provided by the following equation Eq.(16).

$$\tilde{V}(\tilde{\beta}) = \tilde{M}_{\mu^*\mu^*}^{-1} \left(\frac{M_{\mu\mu} s_{vv} + \tilde{S}_{uv}\tilde{S}_{vu}}{n} + 2\tilde{R}_{uu} \right) \tilde{M}_{\mu^*\mu^*}^{-1} \tag{16}$$

with:

$$\tilde{M}_{\mu^*\mu^*} = M_{\mu\mu} - S_{uu} \tag{17}$$

$$\tilde{S}_{uv} = -S_{uu}\tilde{\beta} \tag{18}$$

$$\tilde{R}_{uu} = diag\left(\frac{\tilde{\beta}_k^2 S_{uu,kk}}{d_{f,k}} \right) \tag{19}$$

$$s_{vv} = \frac{(F' - \mu^T \beta)^2}{n - (p+1)} \tag{20}$$

where $d_{f,k}$ is the degrees of freedom of $S_{uu,kk}$ distributed as χ^2.

The previous equations allow determining the new parameters ($\tilde{\beta}_k$) taking into account uncertainties on load and strain measurements and give the impact of measurement uncertainties in terms of standard error on these parameters. Finally, an estimation of the variance of q is defined by Eq.(21) according to [18].

$$\tilde{\sigma}_{qq}^2 = s_{vv} - \frac{\left(S_{ww} - 2\tilde{\beta}^T S_{uw} + \tilde{\beta}^T S_{uu}\tilde{\beta}\right)}{n} \tag{21}$$

where $S_{ww} = s_{ww}$ is the variance of load measurement error and $S_{uw} = 0$.

5.2. Application

The previous reformulation of the problem allows estimating parameters β taking into account uncertainties on both load and strain measurements. Furthermore, the "error in the equation" method provides the variance on these parameters. To check the quality of this new model, estimations from this model are compared with the real measured values. According to [19], the new prediction error for each load case is defined by Eq.(22).

$$r = \tilde{F}'^* - F'^* = \mu^T\tilde{\beta} - \left(\mu^{*T}\beta + q\right) = \mu^{*T}(\tilde{\beta} - \beta) + u^T\tilde{\beta} - q \tag{22}$$

The expected value of the prediction error also is defined by Eq.(23).

$$E(r) = \mu^{*T}E(\tilde{\beta} - \beta) \tag{23}$$

The variance of the prediction error is then given by the following equation Eq.(24).

$$V(r) = \sigma_{qq}^2 + \mu^{*T}V(\tilde{\beta})\mu^* + E(\tilde{\beta}^T S_{uu}\tilde{\beta}) \tag{24}$$

According to [19], the prediction error variance can be estimate by Eq.(25).

$$\tilde{V}(r) = \tilde{\sigma}_{qq}^2 + \mu^T\tilde{V}(\tilde{\beta})\mu - trace(\tilde{V}(\tilde{\beta})S_{uu}) + \tilde{\beta}^T S_{uu}\tilde{\beta} \tag{25}$$

To validate the model, measurements used for determining the parameters β_k are also used to check the validity of the model. For each load case ij described in 4.2., the variance of the prediction by the model is defined by Eq.(26).

$$\tilde{V}(r_{ij}) = \tilde{\sigma}_{qq}^2 + (\mu_{ij})^T\tilde{V}(\tilde{\beta})(\mu_{ij}) - trace(\tilde{V}(\tilde{\beta})S_{uu}) + \tilde{\beta}^T S_{uu}\tilde{\beta} \tag{26}$$

5.3. Results

The covariance matrix of the strain measurement errors S_{uu} and the variance of load measurement error s_{ww} have been determined by experiments.

First, the full model using every data as described in 4.2. by Eq.(6) is calculated. Because some strain measurements are redundant, the criterion presented in 4.2. by Eq.(9) is applied to reduce the number of strain variables in the system. The obtained model is the optimized one without uncertainties on design points, which are the strain measurements, according to the previous criterion (Figure 1).

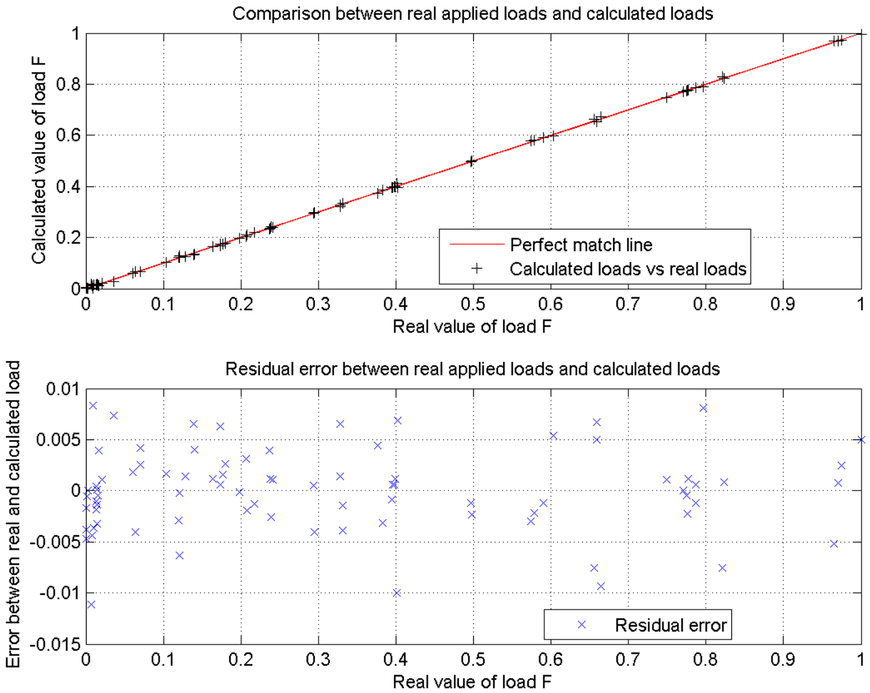

Figure 1: Optimized model without strain measurement errors after the application of the criterion developed in section 4.2

After determining the significant strain variables using the approach described in section 4.2, the new system taking into account uncertainties on design points and responses, which means on strain and load measurements, is written. Its resolution is given by Eq.(13). This new model is the optimized one taking into account uncertainties on design points and responses (Figure 2). Results from this new model are compared with the previous one, without uncertainties on strain measurements.

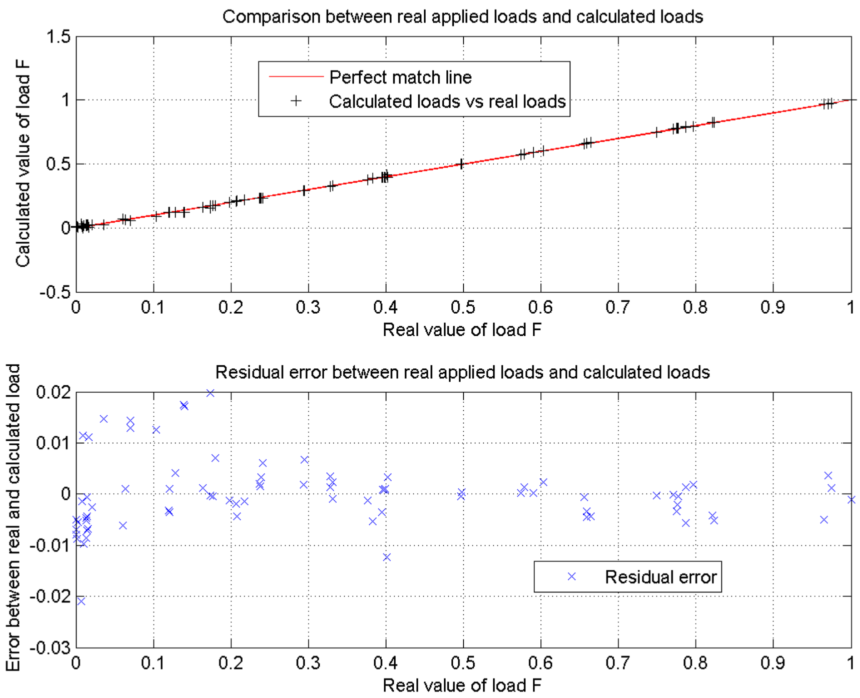

Figure 2: Optimized model (without strain measurement errors after the application of the criterion developed in section 4.2) while adding measurement errors on strains

The results on the residual errors show that the gain is greater for the model which takes into account uncertainties on strain and load measurements from 20% of the maximum value of applied loads. This is an encouraging result for the extension of the method to flight loads which are bigger than applied loads during the ground load calibration.

The coefficients β_k for the two models are different. Those which are close for the two models have as expected a small standard error Eq.(16), at most $\pm 1\%$ of its value. The others have a non negligible standard error, from $\pm 10\%$ to $\pm 50\%$ of their values. For the two models, the standard errors on coefficients β_k are very close, less than 3 points of difference. It appears that standard errors on coefficients β_k for the model with uncertainties on strains are greater than for the model without strain uncertainties, for all coefficients.

The models are then validated with the real measurements used for determining the coefficients β_k as described in 5.2. The equation Eq.(26) allows determining the variance at any new load point and is thus useful for including the model uncertainty when determining in-flight loads.

6. Conclusions

In flight load determination, the aircraft load calibration is a necessary step to characterize the parameters of the response surface representing the problem. Even if the system is over-determined, a preliminary criterion has been implemented to delete redundant strain measurements. This criterion shows a correspondence with the current results. The criterion allows making a systematic and reliable selection of the strain measurements based on the significance of associated parameters.

Then, knowing measurement errors for every variable in the problem, the impact of these uncertainties has been quantified. The method called "error in the equation" has been followed to take into account uncertainties on design points and responses, which means here strain and load measurements. The results are consistent with those expected. Indeed, the standard error on the response increases while taking into account uncertainties in the problem. But, the impact of measurement errors on load calculation remains acceptable in comparison with applied loads during ground tests.

7. Acknowledgements

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